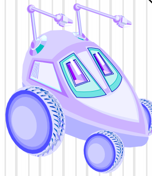
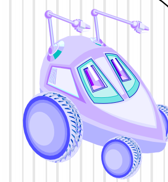


LECTURE 5-1

Mobile Robot Kinematics



Quote of the Week

Asimov's Robot Laws

A robot may not injure a human being, or, through inaction, allow a human being to come to harm

A robot should obey a human being, unless this contradicts the first law

A robot should not harm another robot and protect its own existence unless the contradicts the first or second law.

From Handbook of Robotics, 56th Edition, 2058 A.D., as quoted in I, Robot.

ANNOUNCEMENTS

- Lab 5 - Vision Sensing is due on *Tuesday, 4/13/10*
- Lab 5 memo and code due on Angel by midnight on *Thursday, 4/15/10*
- Quiz 9 on Lecture 5-1 on *Thursday, 4/15/10*

OBJECTIVES

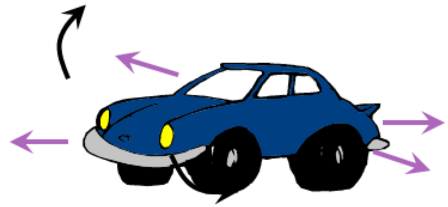
Upon completion of this lecture the student should be able to:

- Find the number of DOF for a differential drive mobile robot
- Define the terms: holonomic, redundant, mobile robot kinematics
- Describe the difference between forward and inverse kinematics
- Describe the forward kinematic model and how to use the ICR for pose estimation

DEGREES OF FREEDOM (DOF)

- A ***DOF*** is the minimum number of coordinates to completely specify the motion of a mechanical system
- A ***DOF*** is the way in which a robot can move
- The ***translational DOF*** are x, y, z
- The ***rotational DOF*** are roll, pitch, yaw
- A DOF can be uncontrollable if there is no actuator for it

MOBILE ROBOT DOF



- Wheels have one degree of freedom
- A system in 3D space has 6 DOF (translational, rotational)
- The Traxster has only 2 controllable DOF (forward/reverse, rotation) or (translation in x, rotational yaw)
- Some motions cannot be done like moving sideways to parallel park
- The robot can get into any position and orientation in 2D space by using a
  - Continuous trajectory
  - Discontinuous velocity

THREE TYPES OF SYSTEMS

- Holonomic
  - Controllable DOF is equivalent to the total DOF
- Nonholonomic
  - Controllable DOF is less than the total DOF
- Redundant
  - Controllable DOF is greater than the total DOF

TRAJECTORY AND MOTION PLANNING

- There are 2 concerns in locomotion, how to
  - Move the robot to a particular ***goal location***
  - Move the robot along a certain ***trajectory***
- ***Navigation*** is concerned with getting to a goal
- ***Motion/path (trajectory) planning*** is more difficult than navigation. This is related to forward and inverse kinematics.

MOBILE ROBOT KINEMATICS

- *Mobile robot kinematics* is the dynamic model of a how a robot behaves
- *Mobile robot kinematics* is a description of the mechanical behavior of the robot for design and control
- *Mobile robot kinematics* is used for
  - Position estimation
  - Motion estimation

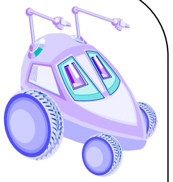
- Mobile robots move unbounded with respect to their environment
  - There is no way to measure robot position
  - Position must be integrated over time and this leads to inaccuracies in position and motion estimation

FORWARD VS. INVERSE KINEMATICS

- Forward Kinematics
  - Estimating a mobile robot's motion and/or pose given the angular and linear velocity
- Inverse Kinematics
  - Determining the robot's angular and linear velocity to achieve a given robot motion and/or pose

RELATIVE POSITIONING

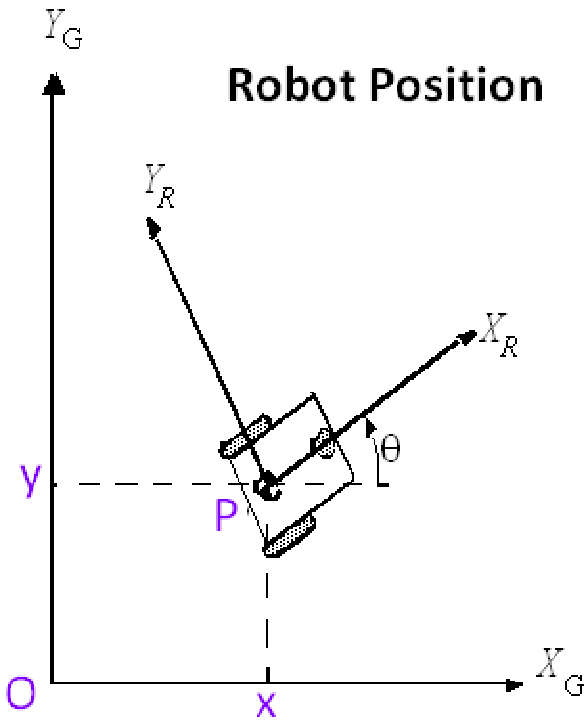
- Given wheel velocities at any given time, compute position/orientation for a future time
- Advantages
  - Self-contained
  - Can get positions anywhere along curved paths
  - Always provides an “estimate” of position
- Disadvantages
  - Requires accurate measurement of wheel velocities over time, including measuring acceleration and deceleration
  - Position error grows over time



CONSTRAINTS

- Each robot wheel contributes to the robot’s motion and imposes constraints on the robot’s motion
- All of the constraints must be expressed with respect to the reference frame (global inertial frame)

ROBOT REFERENCE FRAME



$$^{G}P_{R} = \begin{bmatrix} x & y & \theta \end{bmatrix}^{T}$$

- The robot's reference frame is three dimensional including position on the plane and the orientation, **$\{X_R, Y_R, \theta\}$**
- The axes **$\{X_G, Y_G\}$**, define the inertial global reference frame with origin, ***O***
- The angular difference between the global and reference frames is θ
- Point **P** on the robot chassis in the global reference frame is specified by coordinates **(x, y)**
- The vector **$^{G}P_{R}$** describes the location of the robot with respect to the inertial global reference frame.

ORTHOGONAL ROTATION MATRIX

- The ***orthogonal rotation matrix*** is used to map motion in the global reference frame $\{X_G, Y_G\}$ to motion in the robot's local reference frame $\{X_R, Y_R\}$
- The ***orthogonal rotation matrix*** is used to convert robot velocity in the global reference frame $\{X_G, Y_G\}$ to components of motion along the robot's local axes $\{X_R, Y_R\}$
- The vector P_R describes the location of the robot with respect to the local reference reference frame.

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

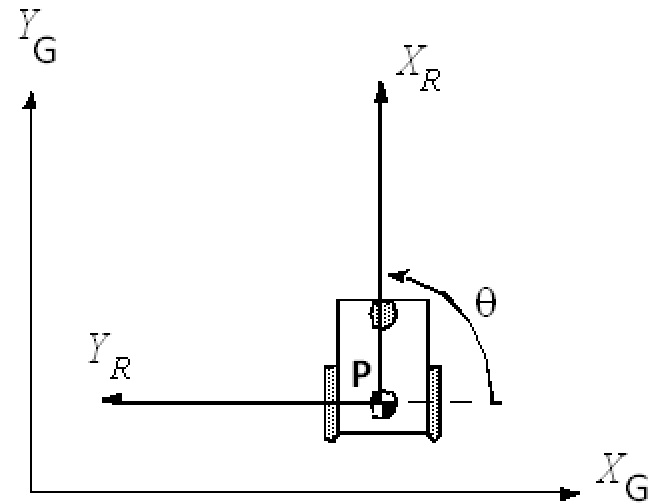
$$\dot{P}_R = R(\theta)\,{}^G\dot{P}_R = R(\theta)\begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} \end{bmatrix}^T$$

ROTATION EXAMPLE: (GLOBAL TO LOCAL REFERENCE FRAME)

- Suppose that a robot is at point **P** and θ = π /2 and the robot's velocity with respect to the global reference frame is $(\dot{x}, \dot{y}, \dot{\theta})$
- Find the robot's motion with respect to the local reference frame

  {X_R, Y_R}

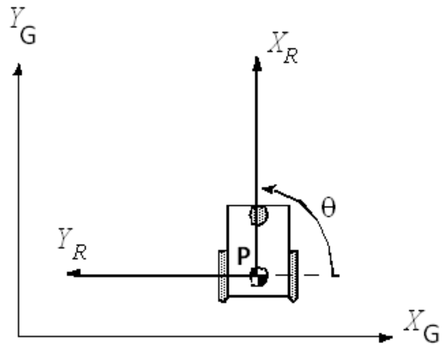
- The motion along X_R and Y_R due to θ is



$$\dot{P}_R = R\left(\frac{\pi}{2}\right) {}^G\dot{P}_R = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ -\dot{x} \\ \dot{\theta} \end{bmatrix}$$

ROTATION EXAMPLE: (LOCAL TO GLOBAL REFERENCE FRAME)

- Now suppose that a robot is at point **P** and θ = π /2 and the robot's velocity with respect to its local frame is $(\dot{x}, \dot{y}, \dot{\theta})$
- Find the robot's motion in the global reference frame $\{X_G, Y_G\}$
- The motion along X_G and Y_G due to θ is



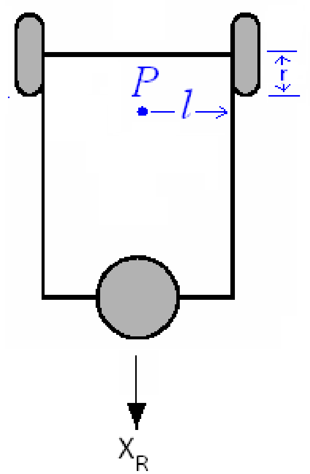
$$ {}^{G}\dot{P}_R = R(\theta)^{-1}\,\dot{P}_R $$

$$ {}^{G}\dot{P}_R = R\left(\frac{\pi}{2}\right)^{-1} V_R = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -\dot{y} \\ \dot{x} \\ \dot{\theta} \end{bmatrix} $$

FORWARD KINEMATICS MODEL: DIFFERENTIAL DRIVE ROBOT

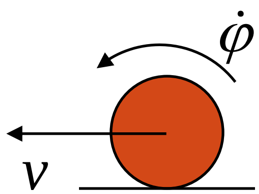
- Consider a differential drive robot which has 2 wheels with radius ***r***, a point ***P*** centered between the 2 drive wheels and each wheel is a distance ℓ from ***P***
- If the rotational speed of the 2 wheels is $\dot{\varphi}_1$ and $\dot{\varphi}_2$ then the forward kinematic model is

$$^{G}\dot{P}_R = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(l, r, \theta, \dot{\varphi}_1, \dot{\varphi}_2)$$



FORWARD KINEMATICS MODEL: DIFFERENTIAL DRIVE ROBOT LINEAR VELOCITY

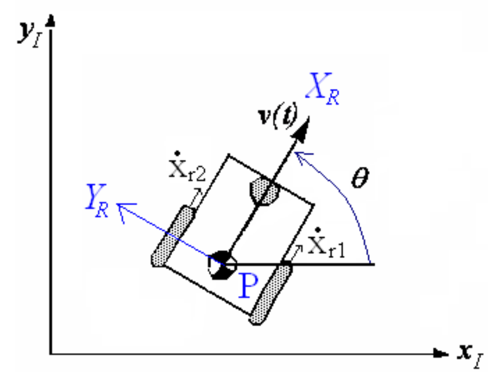
- To find the ***linear velocity*** in the direction of $+X_R$ each wheel contributes one half of the total speed.



$$\dot{x}_{r1} = (1/2) r \dot{\varphi}_1 \qquad \dot{x}_{r2} = (1/2) r \dot{\varphi}_2$$

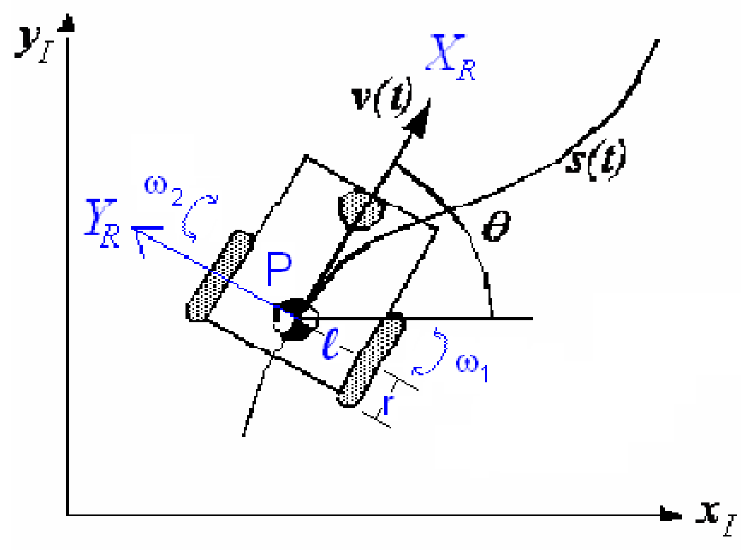
$$v(t) = \dot{x}_{r1} + \dot{x}_{r2}$$

- Since the wheels cannot move sideways, the velocity in the direction of $\mathbf{Y_R}$ is zero.



FORWARD KINEMATICS MODEL: DIFFERENTIAL DRIVE ROBOT ANGULAR VELOCITY

- The ***angular velocity*** about θ is calculated from the contribution from each of the two wheels working alone.
- The right wheel contributes counterclockwise rotation ω_1 around the left wheel.
- The left wheel contributes clockwise rotation ω_2 about the right wheel.
- Each rotation has a radius of 2ℓ.



$$\omega_1 = \frac{r\dot{\varphi}_1}{2l}$$

$$\omega_2 = -\frac{r\dot{\varphi}_2}{2l}$$

COMPLETE FORWARD KINEMATICS MODEL: DIFFERENTIAL DRIVE ROBOT

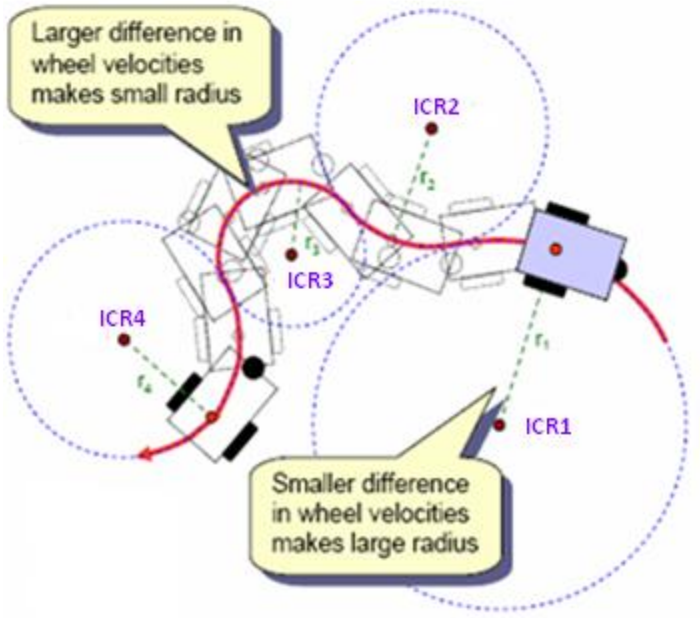
Given the robot’s rotation with respect to the global reference frame, wheel velocities, radius of the wheels and distance between the wheels it is possible to find the robot’s velocity with respect to the global reference frame. The ***complete forward kinematic model*** is

$$ {}^{G}\dot{P}_R = R(\theta)^{-1}\dot{P}_R = R(\theta)^{-1}\begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{bmatrix} $$

$$ {}^{G}\dot{P}_R = R(\theta)^{-1}\begin{bmatrix} \dot{x}_{r1} + \dot{x}_{r2} \\ 0 \\ \omega_1 + \omega_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}_2}{2} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l} - \frac{r\dot{\varphi}_2}{2l} \end{bmatrix} $$

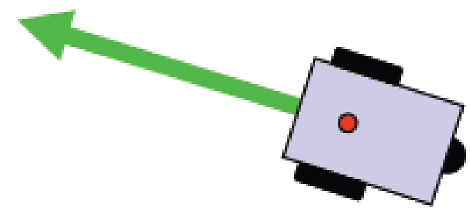
INSTANTANEOUS CENTER OF ROTATION (ICR)

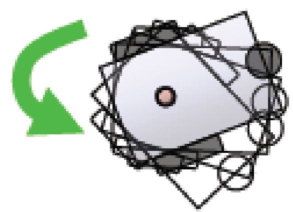
- The ***ICR*** has a ***zero motion line*** drawn through the horizontal axis perpendicular to the wheel plane
- The wheel moves along a radius R with center on the zero motion line, the center of the circle is the ***ICR***
- ***ICR*** is the point around which each wheel of the robot makes a circular course
- The ***ICR*** changes over time as a function of the individual wheel velocities



INSTANTANEOUS CENTER OF ROTATION (ICR) CONT.

- When ***R is infinity***, wheel velocities are equivalent and the robot moves in a straight line
- When ***R is zero***, wheel velocities are the negatives of each other and the robot spins in place
- All other cases, ***R is finite*** and ***non-zero*** and the robot follows a curved trajectory about a point which is a distance R from the robot's center
- Note that differential drive robot's are very sensitive to the velocity differences between the two wheels…making it hard to move in a perfectly straight line

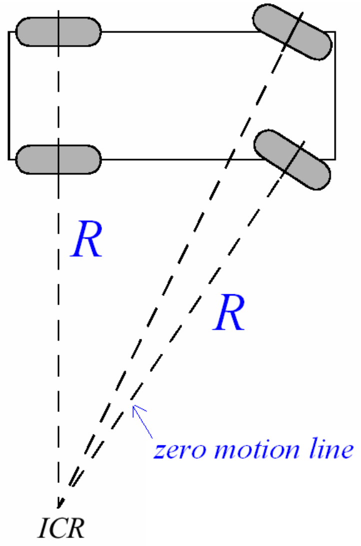


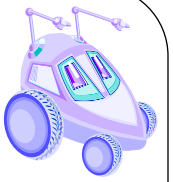


DEGREE OF MOBILITY

- The degree of mobility quantifies the degrees of controllable freedom of a mobile robot based on changes to wheel velocity

- The kinematic constraints of a robot with respect to the degree of mobility can be demonstrated geometrically by using the ***ICR***



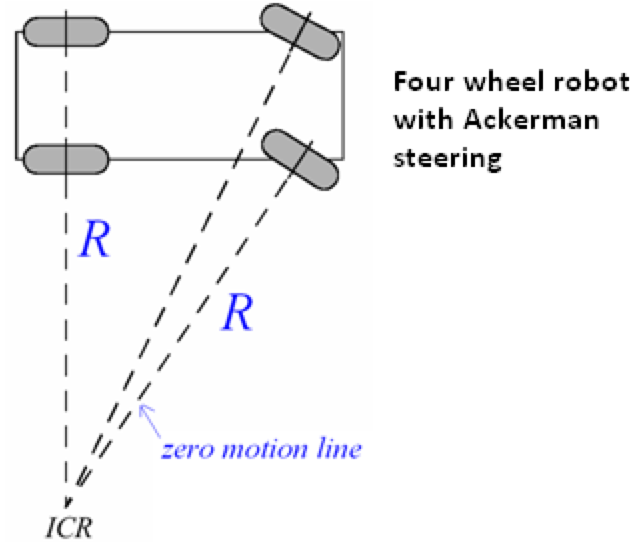


DEGREE OF MOBILITY CONT.

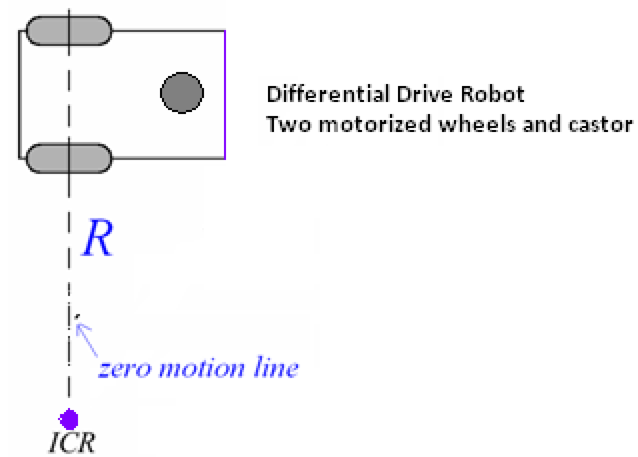
- *Robot mobility* is the ability of a robot chassis to directly move in the environment
- The *degree of mobility* quantifies the degrees of controllable freedom based on changes to wheel velocity
- *Robot mobility* is a function of the number of constraints on the robot's motion, not the number of wheels

DIFFERENTIAL DRIVE ROBOT DEGREE OF MOBILITY

- The *Ackerman vehicle* has two independent kinematic constraints because all of the zero motion lines meet at a single point. There is one single solution for robot motion.



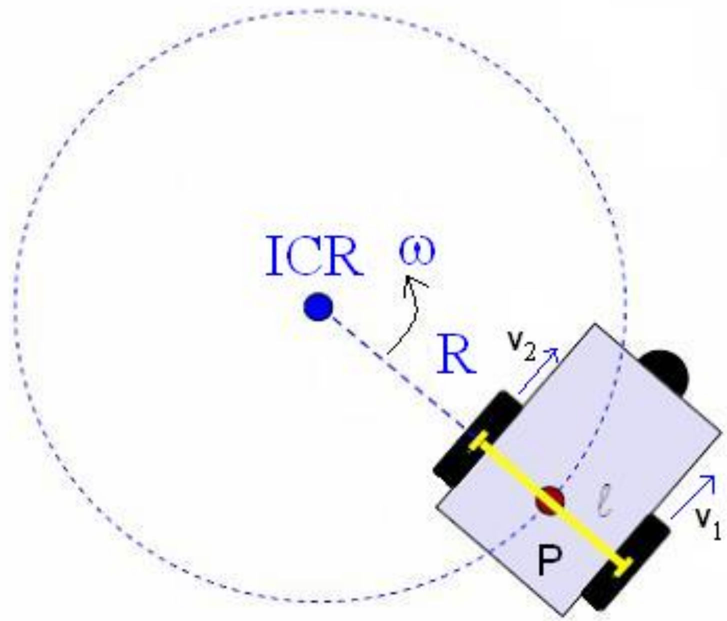
- A *differential drive robot* has one independent kinematic constraint because both of the zero motion lines are aligned along the same horizontal line . There are infinite solutions for robot motion. The castor wheel imposes no additional kinematic constraint



FORWARD KINEMATICS

Assume that at each instance of time, the robot is following the ***ICR*** with radius ***R*** at angular rate $\boldsymbol{\omega}$

$$\omega = \frac{(v_1 - v_2)}{2l} \qquad R = \frac{V}{\omega} = \frac{l(v_1 + v_2)}{(v_1 - v_2)}$$



V = robot forward velocity

v_1 – right wheel velocity

v_2 – left wheel velocity

ω - robot angular velocity

ℓ – distance from robot center to wheel

FORWARD KINEMATICS CONT.

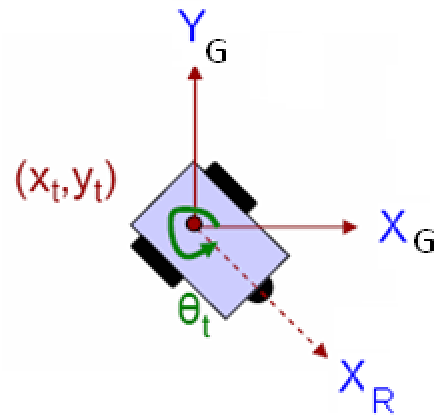
- Given some control parameters (e.g. wheel velocities) determine the pose of the robot
- The position can be determined recursively as a function of the velocity and position,

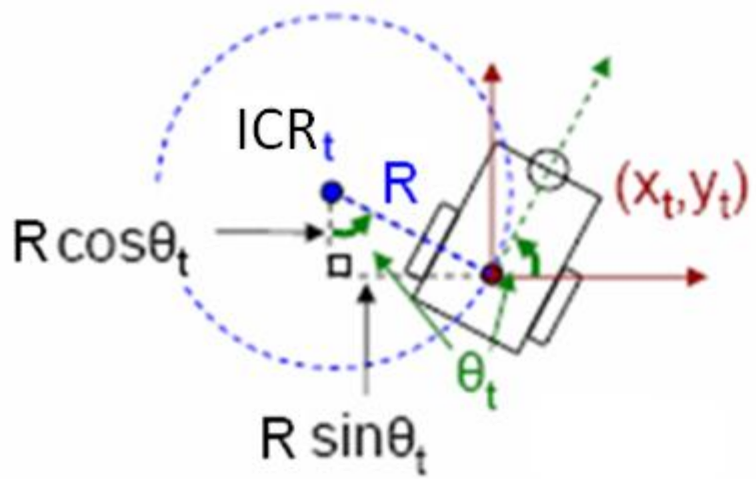
$$p_R(t + \Delta) = F(v_1, v_2)\, p_R(t)$$

- To solve for the ICR center at an instance of time use the following

$$ICR(t) = (ICR_x, ICR_y) =$$

$$(x_t - R \sin \theta_t,\ y_t + R \cos \theta_t)$$





FORWARD KINEMATICS: INSTANTANEOUS POSE

- At time t + Δ, the robot's pose with respect to the ICR is

$$ {}^{G}p_R(t+\Delta) = R(\omega\Delta)^{-1}\, p_R(t) + ICR(t) $$

$$ {}^{G}p_R(t+\Delta) = \begin{bmatrix} {}^{G}x_R(t+\Delta) \\ {}^{G}y_R(t+\Delta) \\ {}^{G}\theta_R(t+\Delta) \end{bmatrix} = \begin{bmatrix} \cos(\omega\Delta) & -\sin(\omega\Delta) & 0 \\ \sin(\omega\Delta) & \cos(\omega\Delta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_R(t) \\ y_R(t) \\ \theta_R(t) \end{bmatrix} + \begin{bmatrix} ICR_x \\ ICR_y \\ \omega\Delta \end{bmatrix} $$

$$ {}^{G}p_R(t+\Delta) = \begin{bmatrix} {}^{G}x_R(t+\Delta) \\ {}^{G}y_R(t+\Delta) \\ {}^{G}\theta_R(t+\Delta) \end{bmatrix} = \begin{bmatrix} \cos(\omega\Delta) & -\sin(\omega\Delta) & 0 \\ \sin(\omega\Delta) & \cos(\omega\Delta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R\sin\theta_t \\ -R\cos\theta_t \\ \theta_t \end{bmatrix} + \begin{bmatrix} ICR_x \\ ICR_y \\ \omega\Delta \end{bmatrix} $$

FORWARD KINEMATICS: INSTANTANEOUS POSE CONT.

Since ICR(t) = (ICR_x, ICR_y) = (x(t) – R sin θ, y(t) + R cos θ)

$$ {}^{G}p_R(t+\Delta) = \begin{bmatrix} x(t+\Delta) \\ y(t+\Delta) \\ \theta(t+\Delta) \end{bmatrix} = \begin{bmatrix} \cos(\omega\Delta) & -\sin(\omega\Delta) & 0 \\ \sin(\omega\Delta) & \cos(\omega\Delta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R\sin\theta_t \\ -R\cos\theta_t \\ \theta_t \end{bmatrix} + \begin{bmatrix} ICR_x \\ ICR_y \\ \omega\Delta \end{bmatrix} $$

$$ {}^{G}p_R(t+\Delta) = \begin{bmatrix} R\cos(\omega\Delta)\sin\theta_t + R\sin(\omega\Delta)\cos\theta_t + (x_t - R\sin\theta_t) \\ R\sin(\omega\Delta)\sin\theta_t - R\cos(\omega\Delta)\cos\theta_t + (y_t + R\cos\theta_t) \\ \theta_t + \omega\Delta \end{bmatrix} $$

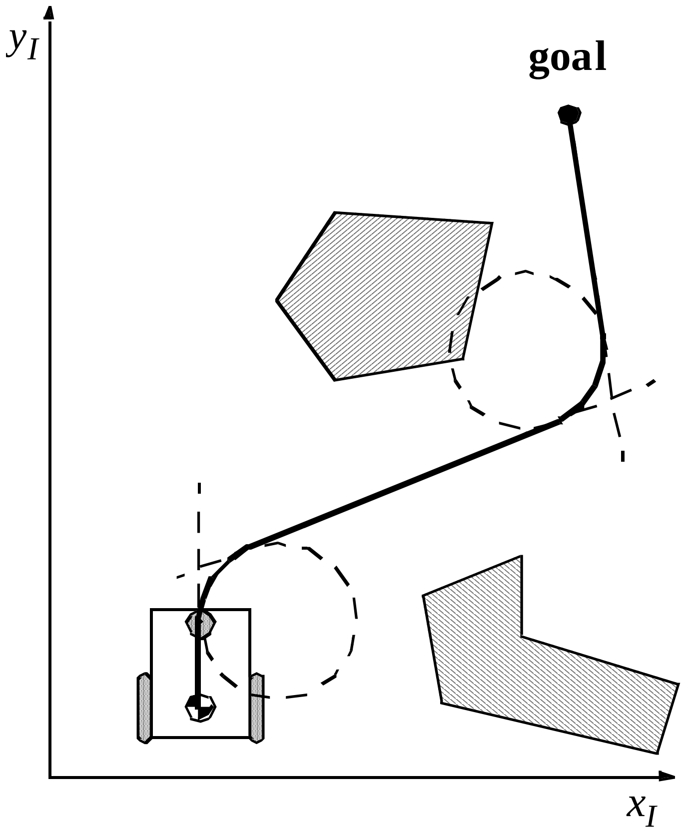
FORWARD KINEMATICS: LINEAR DISPLACEMENT

- When $v_1 = v_2 = v_t$, $R = \infty$, the robot moves in a straight line so ignore the *ICR* and use the following equations:
- $x(t + \Delta) = x_t + v_t \Delta \cos \theta_t$
- $y(t + \Delta) = y_t + v_t \Delta \sin \theta_t$
- $\theta(t + \Delta) = \theta_t$

KINEMATIC CONTROLLER

- The objective of a ***kinematic controller*** is to have the robot follow a ***trajectory*** described by its position and/or velocity profiles as function of time.
- A trajectory is like a path but it has the additional dimension of ***time***
- ***Motion control (kinematic control)*** is not straight forward because mobile robots are non-holonomic systems (and may require the derivative of a position variable).

KINEMATIC CONTROLLER CONT.



- One method is to divide the trajectory (path) into motion segments of clearly defined shape:
  - straight ***lines*** and segments of a ***circle*** (open loop control)
- control problem:
  - pre-compute a smooth trajectory
    based on line and circle segments

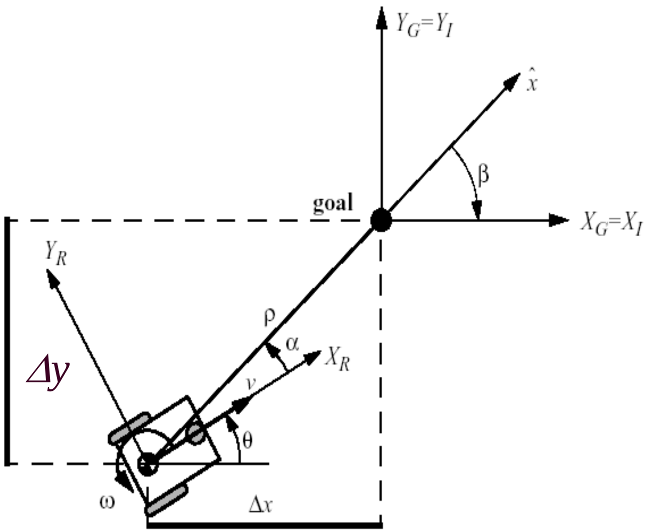
MOTION CONTROL
OPEN-LOOP CONTROL

- Disadvantages:
  - It is not easy to pre-compute a feasible trajectory
  - There are limitations and constraints on the robots velocities and accelerations
  - The robot does not adapt or correct the trajectory if dynamic changes in the environment occur.
  - The resulting trajectories are usually not smooth
  - There are discontinuities in the robot’s acceleration
- A more appropriate approach in motion control is to use a real-state feedback controller

KINEMATIC MODEL

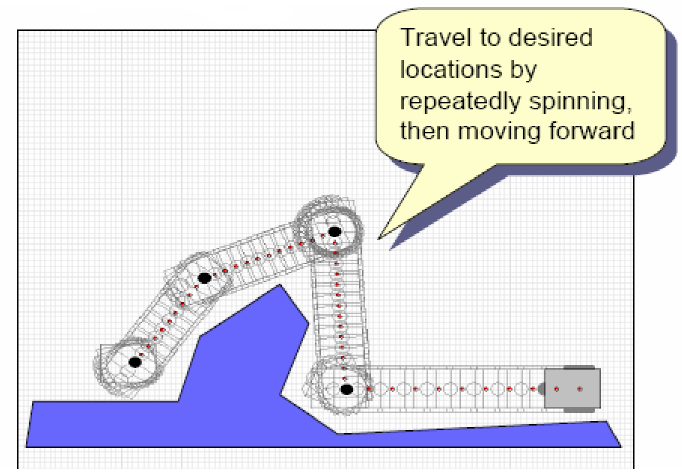
Assume that the goal of the robot is the origin of the global inertial frame. The ***kinematics*** for the differential drive mobile robot with respect to the global reference frame are:

$$ {}^{G}P_{R} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} $$



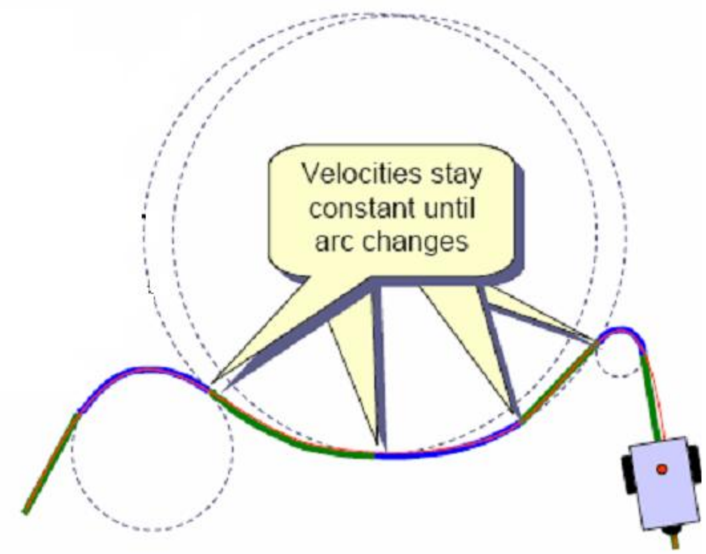
INVERSE KINEMATICS

- ***Inverse Kinematics*** is determining the control parameters (wheel velocities) that will make the robot move to a new pose from its current pose
- This is a very difficult problem
  - Too many unknowns, not enough equations and multiple solutions
- The easy solution is to
  - Spin the robot to the desired angle
  - Move forward to the desired location



INVERSE KINEMATICS CONT.

- Approximate a desired path with arcs based upon computing ICR values
- Result is a set of straight-line paths and ICR arc potions
- Either set the robot drive time and compute velocities for each portion of the path
- Or set velocities and compute drive time for each portion of the path



INVERSE KINEMATICS: SPIN TIME AND VELOCITIES

- The ***spin time*** is determined from the wheel velocities
  - $\theta(t + \Delta) = \theta(t) + \omega\Delta \rightarrow \Delta = [\theta(t + \Delta) - \theta(t)]/\omega$
  - Since $\omega = (v_1 - v_2)/(2\ell)$ and $v_1 = -v_2 \rightarrow \omega = v_1/\ell$
  - $\Delta = \ell\,[\theta(t + \Delta) - \theta(t)]/v_1$
- Alternately, set the spin time and calculate the ***wheel velocities***
  - $v_1 = \ell\,(\theta(t + \Delta) - \theta(t))/\Delta$

INVERSE KINEMATICS: FORWARD TIME

- The ***forward time*** is determined by the velocity ($v_t = v_1 = v_2$)
- Since $x(t + \Delta) = x_t + v_t\Delta \cos(\theta_t)$ and $y(t + \Delta) = y_t + v_t\Delta \sin(\theta_t)$
  - if $x(t + \Delta) \neq x_t$
    - $\Delta = (x(t+ \Delta) - x_t)/(v_t\cos(\theta_t))$, or
  - if $x(t + \Delta) = x(t)$
    - $\Delta = (y(t+ \Delta) - y_t)/(v_t\sin(\theta_t))$

INVERSE KINEMATICS: FORWARD VELOCITIES

- Conversely, the ***wheel velocities***, $v_t = v_1 = v_2$, can be determined by setting the forward time
- Since $x(t + \Delta) = x_t + v_t\Delta \cos(\theta_t)$ and $y(t + \Delta) = y_t + v_t\Delta \sin(\theta_t)$
  - if $x(t + \Delta) \neq x_t$
    - $v_t = (y(t+ \Delta) - y_t)/(\Delta \cos (\theta_t))$
  - if $x(t + \Delta) = x(t)$
    - $v_t =(y(t+ \Delta) - y_t)/(\Delta \sin(\theta_t))$