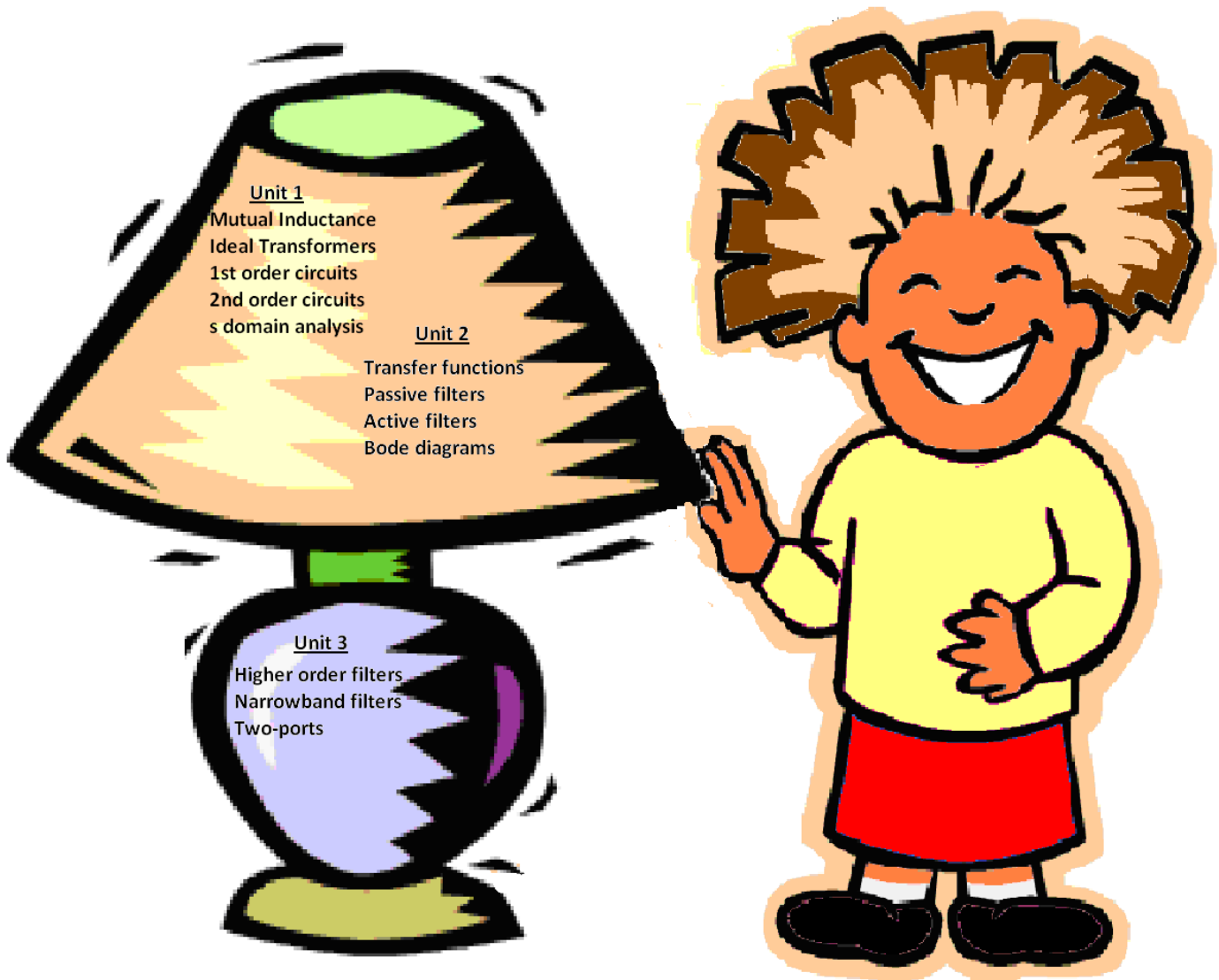
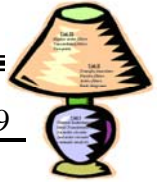


ECE200 Circuits & Systems Study Guide

C.A. Berry





Course Calendar

Class	Day	Date	Topic	Reading	Assignments Due
1-1	M	12/01	Mutual inductance	6.4	
1-2	T	12/02	Mutual inductance	6.5	Pre- Lab 11
1-L	W	12/03	<i>Lab 11. Instrumentation Amplifier</i>		
1-3	R	12/04	The Ideal transformer	9.11	HW 1
2-1	M	12/08	Natural response of 1 st order circuits	7.1 - 2	Quiz 2
2-2	T	12/09	Step response of 1 st order circuits	7.3 - 4	Pre-Lab 12
2-L	W	12/10	<i>Lab 12. Self & Mutual Inductance</i>		
2-3	R	12/11	Natural & Step response of 2 nd order circuits	8.1-4	HW 2
3-1	M	12/15	Natural & Step response of 2 nd order circuits	8.1-4	Quiz 3
3-2	T	12/16	Circuit elements and analysis in the s domain	13.1 - 2	Pre-Lab 13
3-L	W	12/17	<i>Lab 13. Step Response Design</i>		
3-3	R	12/18	Circuit analysis in the s domain	13.2	HW 3
Winter Break					
4-1	M	1/05	Applications in s domain	13.3 & 12.6	Quiz 4
4-2	T	1/06	Applications in s domain	13.3 & 12.6	Pre-Lab 14
4-L	W	1/07	<i>Lab 14. Gyrator</i>		HW 4
4-3	R	1/08	Exam 1 (up through 4-2)		
5-1	M	1/12	Initial/final value theorem, Transfer function & the steady-state sinusoidal response	13.4 & 12.8-9 13.7	Quiz 5
5-2	T	1/13	Introduction to filters	14.1-3	Pre-Lab 16
5-L	W	1/14	<i>Lab 16. Phase-Shift Oscillator</i>		
5-3	R	1/15	Low-pass Filters passive & active	14.2 & 15.1	HW 5
6-1	M	1/19	High-pass Filters passive & active	14.3 & 15.1	Quiz 6
6-2	T	1/20	First order filters & Bode diagrams	E.1-2	Pre-Lab 15
6-L	W	1/21	<i>Lab 15. Steps & Steady State</i>		
6-3	R	1/22	Second order filters & Bode diagrams	14.4-5	HW 6
7-1	M	1/26	Bandpass filters & Bode diagrams	14.4	Quiz 7
7-2	T	1/27	Bandreject filters & Bode diagrams	14.5	
7-L	W	1/28	<i>Lab 17. Lab Practical Test</i>		
7-3	R	1/29	Filter scaling	15.2	HW 7
8-1	M	2/02	Op Amp Bandpass & Bandreject filters	15.3	Quiz 8
8-2	T	2/03	Higher order op amp filters	15.4	Pre-Lab 18
8-L	W	2/04	<i>Lab 18. Bode Diagram Design</i>		
8-3	R	2/05	Exam 2 (up through 7-3)		HW 8-due 2/6
9-1	M	2/09	Butterworth filters	15.4	Quiz 9
9-2	T	2/10	Narrowband bandpass filters	15.5	Pre-Lab 19
9-L	W	2/11	<i>Lab 19. Low-Pass Filter Design</i>		
9-3	R	2/12	Narrowband bandreject filters	15.5	HW 9
10-1	M	2/16	Terminal equations and Two-port parameters	18.1-2	Quiz 10
10-2	T	2/17	Terminal equations and Two-port parameters	18.1-2	Pre-Lab 20
10-L	W	2/18	<i>Lab 20. Poles on Planes</i>		
10-3	R	2/19	Analysis of terminated two-port circuits	18.3	HW 10
Finals Week			FINAL EXAM (up through 10-3)		

Course Information**Instructor:**

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carlotta.berry@rose-hulman.edu
<http://www.rose-hulman.edu/~berry123>

Description:

Electrical and computer engineering (ECE) deals with the application of electrical and electronic technology to the daily needs of people. Almost every aspect of their lives and occupations are controlled by ECE. To understand the details of ECE it is necessary to study circuit analysis. The reason for this is that it is the one thing that ties all branches of ECE together; as such, it lays the foundation for analysis, design, and operation of electrical devices and systems.

This class helps provide a foundation for you to build on. You will continue, in subsequent ECE courses, to update the knowledge you gain here. Since this course is the prerequisite for just about everything you will be doing in ECE, it is vital that you gain a confident grasp of circuit and systems fundamentals.

The Calendar gives details of what we'll be covering. You should follow it closely.

Text : J.W. Nilsson & S.A. Riedel, *Electric Circuits*, 8th edition, Prentice Hall, 2008.

Office Hours: M – R, 7th hour, 1:35 – 2:25 pm or by appointment

What is expected of You:

First and foremost, professional work is the norm in this course. All of your written work and your conduct in class are to be at the level of one who is studying a profession—the profession of engineering. This means a number of things:

1. Your work is neatly done in a professional manner, using formats specified.
2. Your work is honestly done. You are encouraged to discuss course material with classmates to help each other understand and assimilate the concepts. Nevertheless, I distinguish between helping someone understand concepts and providing them with specific answers. I expect you'll work individually on homework without reference to others' work.
3. Your work is done on time.

Attendance:

Regardless of whether formal attendance is taken, attendance at each class is expected. As a rule of thumb you should consider yourself seriously behind if you miss more than **four classes** in a four credit-hour course. According to our Academic Rules and Procedures, "A student whose total absences in a course, excused or unexcused, exceed two per credit is liable to fail

the course.” ***Eight absences in this course are grounds for failure.*** Missing an attendance check due to lateness may be counted as an absence.

If you miss a lab with an excused absence you need to make it up within 1 week without penalty. If you miss a lab without an excused absence, you need to make it up within 1 week and you will receive a grade of zero. ***If you come to lab more than 15 minutes late you need to complete the lab on your own.***

Grading:

Grades will be assigned at the end of the quarter based on the grade weights and grading scale shown below:

Midterm Test I	20%	90 – 100	A
Midterm Test II	20%	85 – 89	B+
Final Exam	30%	80 - 84	B
Homework and Quizzes	10%	75 - 79	C+
Labs and Memos	15%	70 - 74	C
Lab Practical Test	5%	65 - 79	D+
		60 - 64	D
		Below 60	F

Independent of point totals:

- Satisfactory completion of each of the ten lab projects is required in order to receive a passing grade in the course,
- You must earn a passing **overall weighted midterm and final exam average** in order to receive a passing grade in the course, and

Homework:

The homework is intended to help you to understand the concepts presented in the course, and to provide you with practice in problem solving.

- **Problem sets are due each Thursday in class before the bell rings at the beginning of class.** Assignments and solutions will be distributed using ANGEL.
- Homework turned in after the bell rings is late and will incur a **20%** penalty.
- Homework turned in after 8 am on the day following the due date will **not** be accepted.
- Arrange to turn in your homework early if you will be away for job interviews, athletic events, etc.
- The required format is described in the *Sophomore Engineering Curriculum Guidelines and Standards for Writing Assignments*. It is your responsibility to make your methods and results clear to the grader.

Homework will be graded using a restricted scale of 0, 1, 2, 3, or 4:

- 4 Problem worked completely correctly.
- 3 Problem worked with minor errors.
- 2 Problem worked with substantial errors.
- 1 Problem attempted, but with no understanding.
- 0 Problem not attempted or unacceptable.

Preparation for the laboratory sessions:

Pre-lab exercises are due each Tuesday in class before the bell rings at the beginning of lab.

Each student should do the pre-lab in their notebook and make a photocopy to turn-in. The solutions to the pre-lab may be presented at the start of the lab period. Any team that has not completed the pre-lab, must do it at the beginning of lab for zero credit. This team must still finish the laboratory project within the allotted time.

Laboratory notebooks:

Laboratory notebooks will be collected at the conclusion of each laboratory period. The laboratory notebook will be graded and both members of the team share the notebook grade. Each team member must alternate submission of the lab notebook as well as circuit building on a weekly basis. A memorandum will be submitted for one laboratory projects and this will be an individual grade.

Exams:

On both tests you will be allowed to use one 8½ x 11 page of notes (both sides) that you have made but may not use anyone else's notes. You should prepare for tests as though they were closed-book so that you can finish on-time; reference material should only be a back-up. In the final exam, you are permitted to use two 8½ x 11 pages of notes (both sides).

Missed exams will not be made up. ***The final exam grade will be used to replace a missing test grade in the case of excused absences.*** Excused absence from an examination normally requires advance approval or formal documentation of an emergency. An examination that is missed for an unexcused reason will be given a grade of zero. Students are not excused from scheduled exams for intramural athletics or fraternity events.

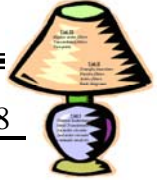
Academic accommodation:

Those students with documented special needs may request extra time on timed tests. Students need to contact me at least 2 business day prior to each exam to make the necessary arrangements.

Unit I



- **Mutual Inductance**
- **Ideal Transformers**
- **1st order circuits**
- **2nd order circuits**
- **s domain analysis**

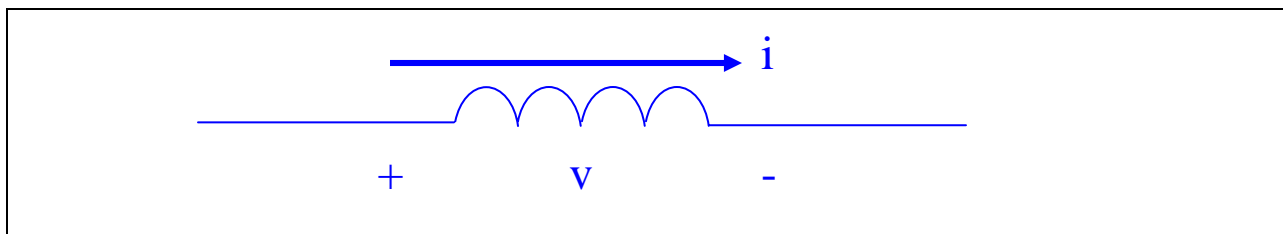


Lecture 1-1: Mutual Inductance

Reading: 6.4

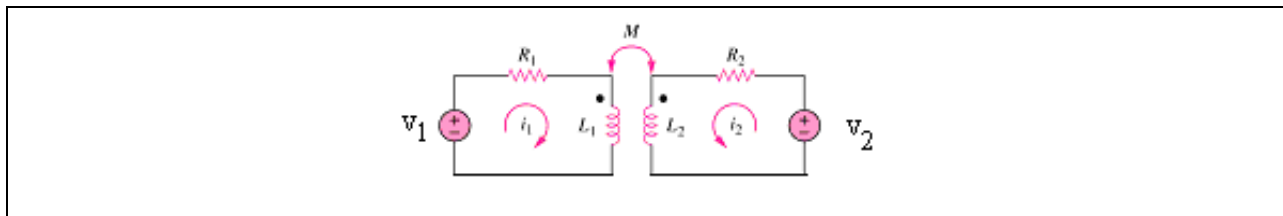
Objectives: To define inductance, self-inductance and mutual inductance
To apply the dot convention to write the mesh equations of a mutual inductance circuit

Inductance is a circuit parameter used to describe an inductor. Inductance is symbolized by the letter L and is measured in henrys (H). It is represented graphically as a coiled wire. It is a conductor linking a magnetic field.



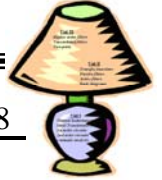
self - inductance is a parameter that relates a voltage to a time-varying current in the same circuit.

When two circuits are linked by a magnetic field, mutual inductance is the parameter that relates to the voltage induced in a second circuit by the time-varying current in the first circuit.



The self-inductances of the two coils are labeled L_1 and L_2 and the mutual inductance is labeled M .

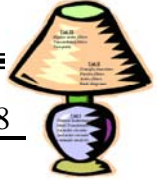
The dot convention states that when the reference direction for a current enters the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is positive at its dotted terminal.



	<p>i_1 enters the dotted terminal of coil 1 so v_2 is <u>positive</u> at the dotted terminal of coil 2.</p> $v_2 = +M di_1/dt$
	<p>i_1 enters the dotted terminal of coil 1 so v_2 is <u>positive</u> at the dotted terminal of coil 2.</p> $v_2 = -M di_1/dt$
	<p>i_2 leaves the dotted terminal of coil 2 so v_1 is <u>negative</u> at the dotted terminal of coil 1.</p> $v_1 = -M di_2/dt$
	<p>i_2 leaves the dotted terminal of coil 2 so v_1 is <u>negative</u> at the dotted terminal of coil 1.</p> $v_1 = M di_2/dt$

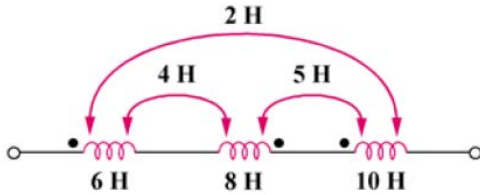
There are also two dot conventions for coupled coils in series

<p>Series – adding</p>		<p>i enters the dotted terminal of coil 1 and coil 2</p> $V_{ab} = (L_1 + L_2 + 2M) di/dt$ $L_{tot} = L_1 + L_2 + 2M$
<p>Series - opposing</p>		<p>i enters the dotted terminal of coil 1 and leaves the dotted terminal of coil 2</p> $V_{ab} = (L_1 + L_2 - 2M) di/dt$ $L_{tot} = L_1 + L_2 - 2M$



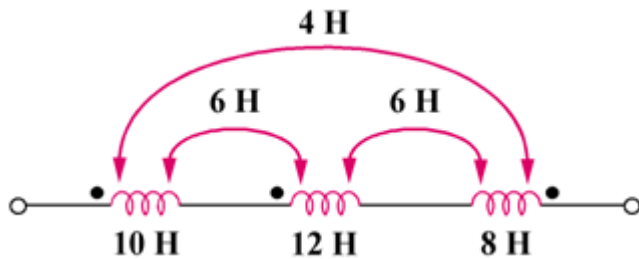
Example 6.4.1:

Calculate the total inductance for the following circuit.



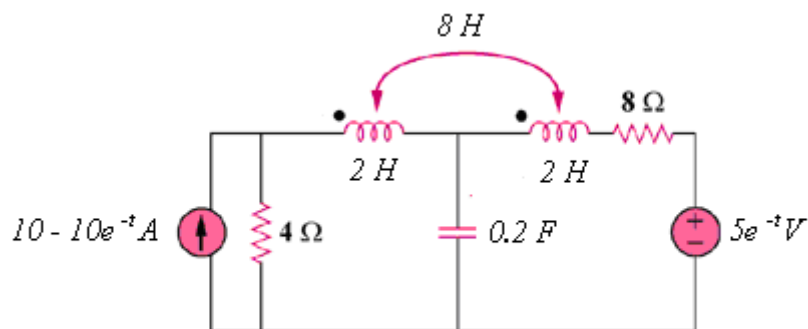
Example 6.4.2:

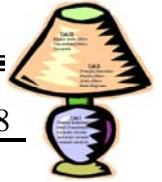
Determine the inductance of the three series-connected inductors in the following figure.



Example 6.4.3:

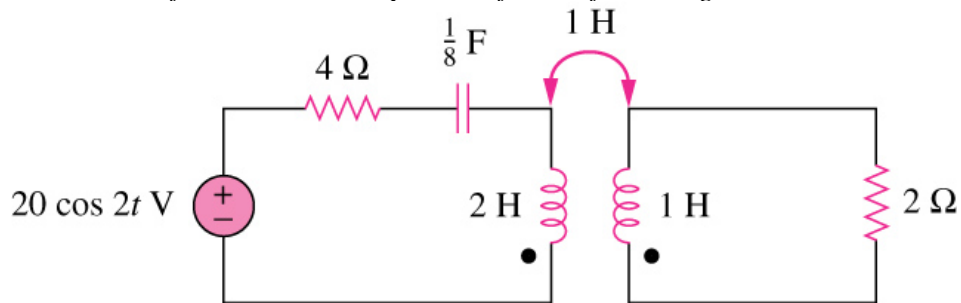
Write a set of mesh-current equations for the following circuit.

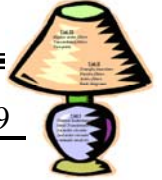




Example 6.4.4:

Write a set of mesh-current equations for the following circuit.





Lecture 1-2: A Closer Look At Mutual Inductance

Reading: 6.5

- Objectives:
- To define self-inductance and the coefficient of coupling
 - To relate the coefficient of coupling to mutual inductance
 - To calculate energy for a mutual inductance circuit

Faraday's law states that $v = d\lambda/dt$, where λ is the flux in weber-turns.

The flux linkage is $\lambda = N\phi$, where ϕ is the magnetic field in Webers (Wb) and the number of turns linked by the field (N).

The magnitude of the flux, ϕ is related to the magnitude of the coil current by $\phi = PNi$

P is the permeance and is a quantity that describes the magnetic properties of the space in the inductors measured in Webers/Ampere (Wb/A).

For circuits with two magnetically coupled coils $M_{12} = M_{21} = M$

$$L_1 = \frac{N_1^2 P_1}{}$$

$$L_2 = \frac{N_2^2 P_2}{}$$

In a linear system, $\frac{N_1}{N_2} = \sqrt{\frac{L_1}{L_2}}$ because $P_1 = P_2$

The coefficient of coupling, k , can be used to relate self-inductance and mutual inductance.

$$M = k\sqrt{L_1 L_2}$$

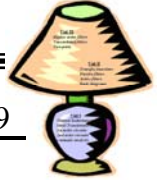
The coefficient of coupling must lie between 0 and 1 ($0 \leq k \leq 1$)

$k = 0$	No common flux, NOT coupled
$k = 1$	Perfectly coupled
$k < 0.5$	Loosely coupled
$k > 0.5$	Tightly coupled

The total energy stored in coupled coils is defined by $w(t) = .5L_1i_1^2 + .5L_2i_2^2 \pm Mi_1i_2$

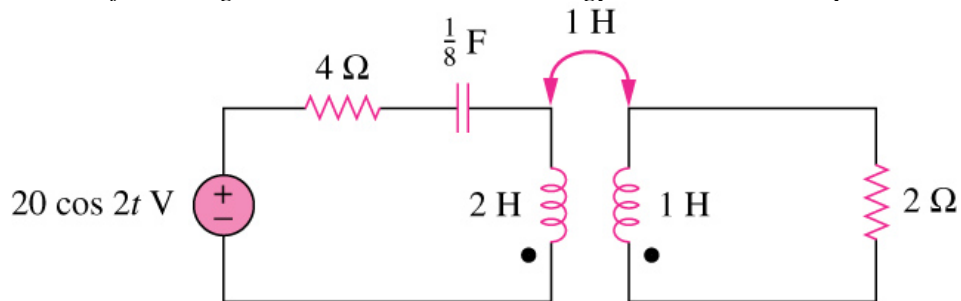
+ Mi_1i_2 - both currents enter or leave the dotted terminal

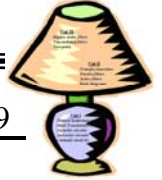
- Mi_1i_2 - one current enters and one current leaves the dotted terminal



Example 6.5.1:

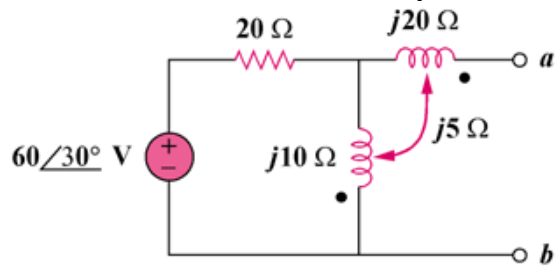
For the following circuit, determine the energy stored in the coupled inductors at $t = 1.5$ seconds.

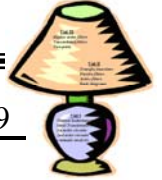




Example 6.5.2:

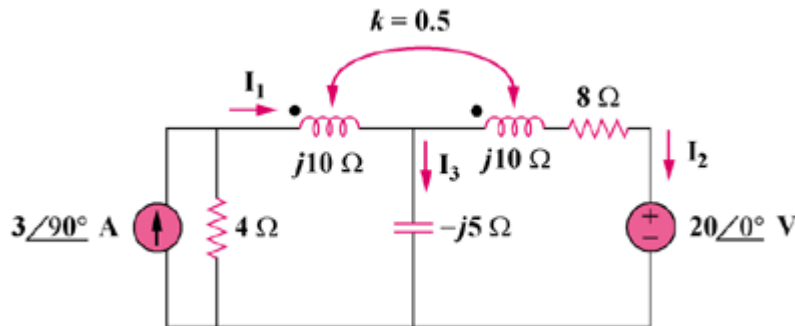
Find the Thevenin and Norton equivalent across terminals a and b for the following circuit.

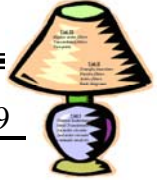




Example 6.5.3:

Determine currents I_1 , I_2 , and I_3 in the following circuit. Assume that $\omega = 1000$ rad/s and find the energy stored in the coupled coils at $t = 2$ ms.





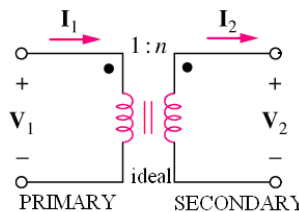
Lecture 1-3: The Ideal Transformer

Reading: 9.11

- Objectives:
- To define the ideal transformer and terminal characteristics
 - To define the dot convention for the analysis of circuits with ideal transformers
 - To apply reflection to calculate voltage and currents in an ideal transformer circuit

An [ideal transformer](#) consists of two magnetically coupled coils having N_1 and N_2 turns that exhibits the following three properties:

- i. the coefficient of coupling is unity ($k = 1$)
- ii. the self inductance of each coils is infinite ($L_1 = L_2 = \infty$)
- iii. coil losses due to resistance are negligible



There are two characteristics for the terminal behavior of an ideal transformer.

1. $\left| \frac{V_1}{N_1} \right| = \left| \frac{V_2}{N_2} \right|$
2. $|I_1 N_1| = |I_2 N_2|$

The primary winding coil and its circuit are called the [primary side](#) of the transformer

The secondary winding coil and its circuit are called the [secondary side](#) of the transformer

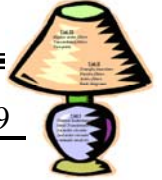
The turns ratio for a transformer is $n = N_2/N_1$ where the primary winding has N_1 turns and the secondary winding has N_2 turns.

$n = 1$	Isolation transformer
$n > 1$	Step up transformer (voltage is increased from the primary to secondary side)
$0 < n < 1$	Step down transformer (voltage is decreased from the primary to secondary side)

The convention for determining the polarity for the currents and voltages on the transformer use the following rules.

- i. if the coil voltages V_1 and V_2 are both positive or negative at the dot-marked terminals, use a plus sign in the voltage relationship otherwise use a negative
- ii. if the coil currents I_1 and I_2 are both directed into or out of the dot-marked terminal, use a minus sign in the current relationship otherwise use a positive.

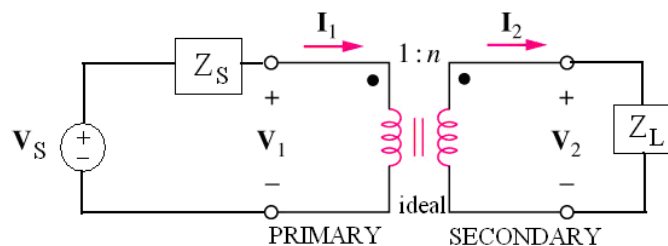
The graphical symbol for an ideal transformer is given in the following table for the voltage and current conventions



Circuit	Voltages	Currents
	$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n$	$\frac{I_2}{I_1} = -\frac{N_1}{N_2} = -\frac{1}{n}$
	$\frac{V_2}{V_1} = -\frac{N_2}{N_1} = -n$	$\frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{n}$
	$\frac{V_2}{V_1} = -\frac{N_2}{N_1} = -n$	$\frac{I_2}{I_1} = -\frac{N_1}{N_2} = -\frac{1}{n}$
	$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n$	$\frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{n}$

Reflected Impedance

Ideal transformers can also be used to raise or lower the impedance level of a load.



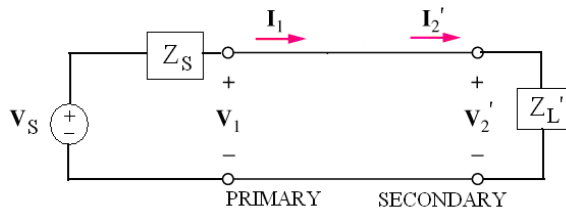
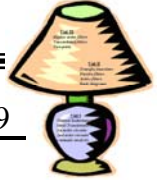
The input impedance from the primary side is

$$Z_{in} = \frac{V_1}{I_1} = \left(\frac{1}{n^2}\right) \left(\frac{V_2}{I_2}\right) = \left(\frac{1}{n^2}\right) (Z_L)$$

It is possible to simplify the analysis of an ideal transformer by reflecting the secondary impedance to the primary side and vice versa.

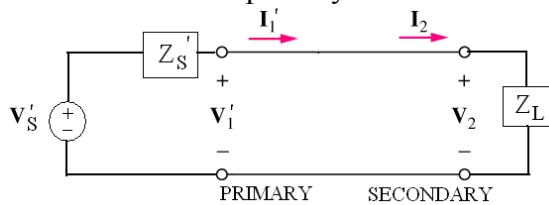
To reflect the secondary side to the primary side:

1. the primary side remains the same
2. the secondary impedance is reflected to the primary side as $Z_L' = Z_L/n^2$
3. the secondary voltage becomes $V_2' = V_2/n$
4. the secondary current becomes $I_2' = nI_2$



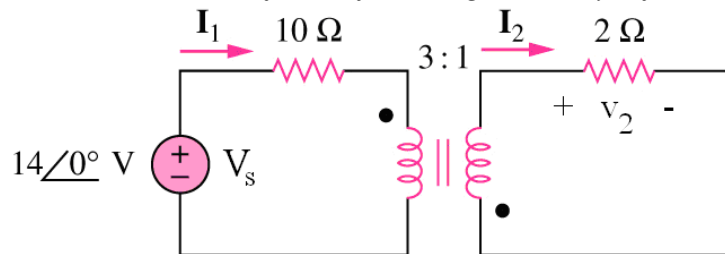
To reflect the primary side to the secondary side:

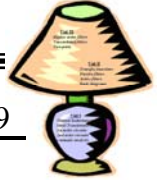
1. the secondary side remains the same
2. the primary impedance is reflected to the secondary side as $Z_s' = n^2 Z_s$
3. the primary voltages become $V_s' = nV_s$ and $V_1' = nV_1$
4. the primary current becomes $I_1' = (1/n)I_1$



Example 9.11.1:

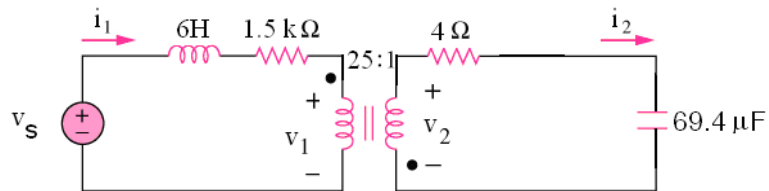
Determine I_1 and I_2 for the following circuit by reflecting the secondary to the primary side.

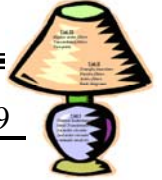




Example 9.11.2:

Determine the steady state expressions i_1 , i_2 , v_1 , and v_2 for the following circuit given that $v_s(t) = 25\cos 1000t$ V.





Lecture 2-1: The Natural response of RL and RC Circuits

Reading: 7.1-2

- Objectives:
- To analyze the natural response of 1st order systems
 - To analyze the step response of 1st order systems
 - To demonstrate the general solution for step and natural responses

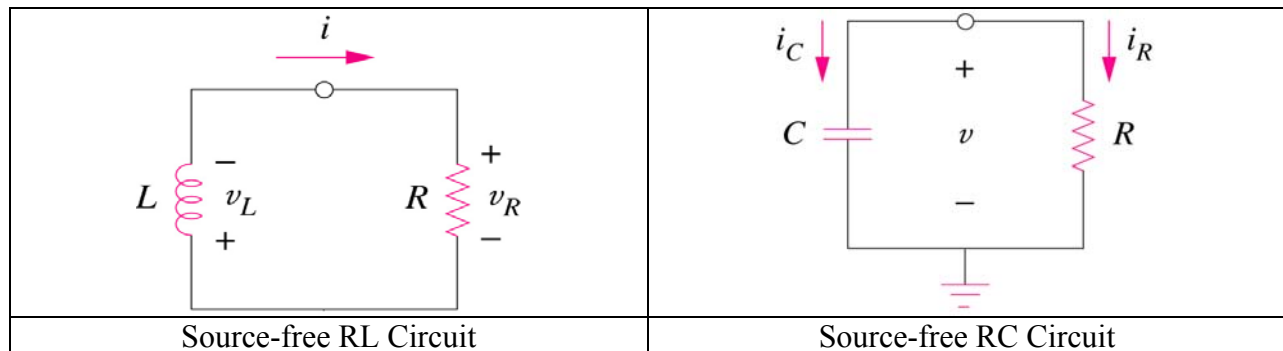
The purpose of this week's lecture will be to analyze RL (resistor-inductor) and RC (resistor-capacitor) circuits.

The first method of analysis is the **natural response** which occurs when an inductor or capacitor is connected to a DC source and is suddenly disconnected and the stored energy is released to a resistive network.

The second method of analysis is the **step response** which occurs when a DC source is suddenly connected to an inductor or capacitor and it begins to store energy.

The third method of analysis describes the general method that can be used to find the step and natural responses of RL or RC circuits.

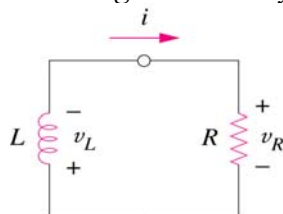
Since RL and RC circuits can also be described by first-order differential equations they are also known as **first-order circuits**.

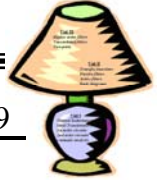


7.1 The natural response of an RL circuit

Example 7.1.1:

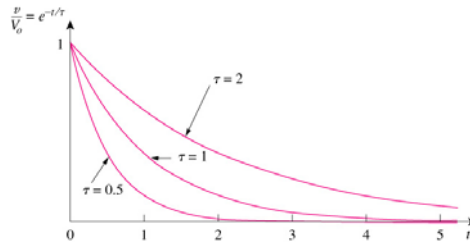
Use KVL to find the expression for the current through the inductor in the following circuit assuming it is initially charged to I_0 at $t = 0$, (i.e. $i_L(0) = I_0$).





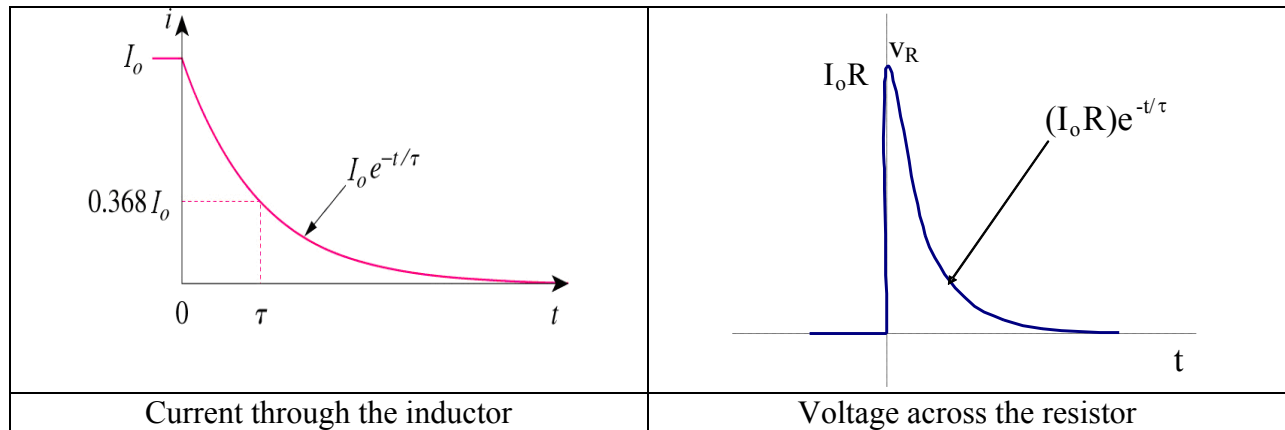
The time constant for an RL circuits is $\tau = L/R$. This is significant because when the time since the DC source was removed exceeds 5 time constants, the current through the inductor is less than 1% of its initial value. The time when the current through the inductor is changing or discharging before 5 time constants is referred to as the **transient response**. The response that exists a long time after DC switching is the **steady-state response**.

The following figure demonstrates the affect the value of the time constant has on the response.



It should be noted that current through an inductor cannot change abruptly $[i(0^-) = i(0^+) = I_0]$ although the voltage can change abruptly. The voltage will change abruptly dependent upon whether the inductor is storing or discharging energy. The current is an exponential decaying curve after the DC source has been removed. Recall that under DC conditions an inductor acts like a **short circuit**.

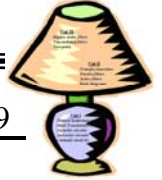
Using Ohm's Law, the voltage across the resistor is given by $v_R(t) = I_0 R$ for $t \geq 0^+$ because the voltage across the inductor relates to the first derivative thus $v_R(0^-) = 0$, $v_R(0^+) = I_0 R$.



The power dissipated in the resistor is given by $p = I_0^2 R e^{-2(R/L)t}$ (W), $t \geq 0$
 The energy delivered to the resistor is given by $.5LI_0^2(1 - e^{-2(R/L)t})$ (J), $t \geq 0$
 The initial energy stored in the inductor is given by $w = 0.5LI_0^2$ (J)

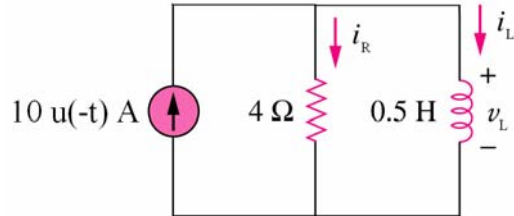
To find the natural response of an RL circuit,

- i. Find the initial current I_0 through the inductor
- ii. Find the time constant of the circuit, $\tau = L/R_{eq}$
- iii. Use the expression, $i(t) = I_0 e^{-(R/L)t}$, to find $i(t)$ using I_0 and τ .



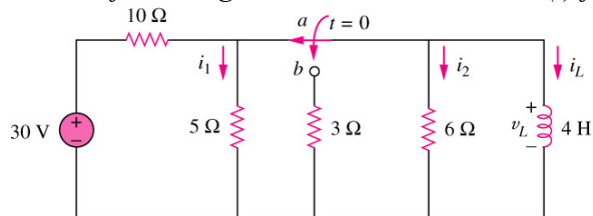
Example 7.1.2:

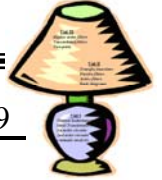
For the following circuit determine, $i_L(t)$, $v_L(t)$ and $i_R(t)$ for $t \geq 0$.



Example 7.1.3

For the following circuit, determine the $i_L(t)$ for $t \geq 0$.

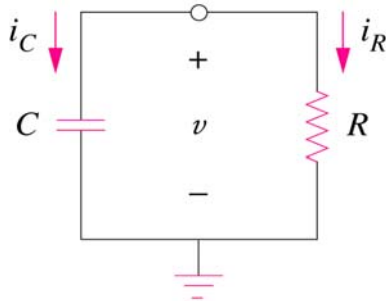




7.2 The natural response of an RC circuit

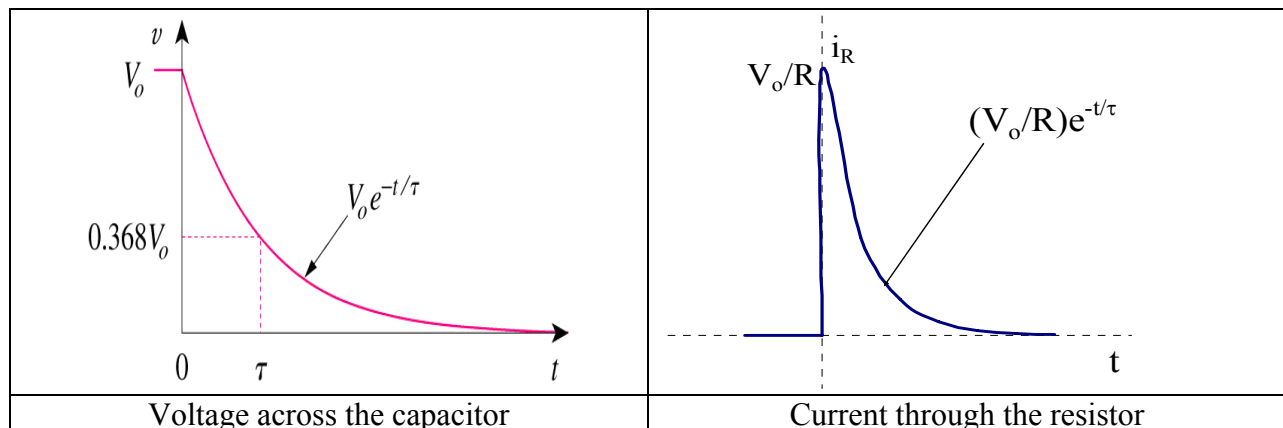
Example 7.2.1:

Use KCL to find the expression for the voltage across the capacitor in the following circuit assuming it is initially charged to V_o at $t = 0$, (i.e. $v_L(0) = V_o$).



It should be noted that voltage across a capacitor cannot change abruptly [$v(0^-) = v(0^+) = V_o$] although the current can change abruptly. The current will change abruptly dependent upon whether the capacitor is storing or discharging energy. The voltage is an exponential decaying curve after the DC source has been removed. Recall that under DC conditions a capacitor acts like an **open circuit**.

Using Ohm's Law, the current through the resistor is given by $i_R(t) = V_o/R$ for $t \geq 0^+$ because the current through the capacitor relates to the first derivative thus $i_R(0^-) = 0$, $i_R(0^+) = V_o/R$.



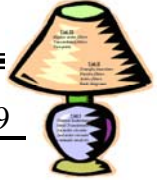
The power dissipated in the resistor is given by $p = V_o^2/Re^{-2[1/(RC)]t}$ (W), $t \geq 0$

The energy delivered to the resistor is given by $.5CV_o^2(1 - e^{-2[1/(RC)]t})$ (J), $t \geq 0$

The initial energy stored in the inductor is given by $w = 0.5CV_o^2$ (J)

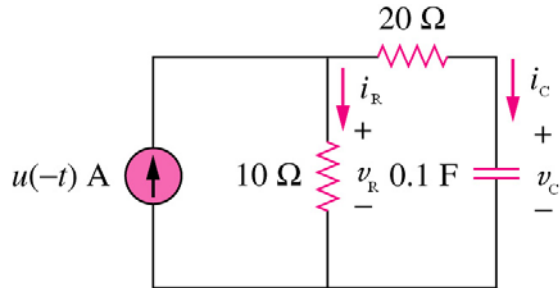
To find the natural response of an RC circuit,

- i. Find the initial voltage V_o across the capacitor
- ii. Find the time constant of the circuit, $\tau = R_{eq}C$
- iii. Use the expression, $v(t) = V_o e^{-1/(RC)t}$, to find $v(t)$ using V_o and τ



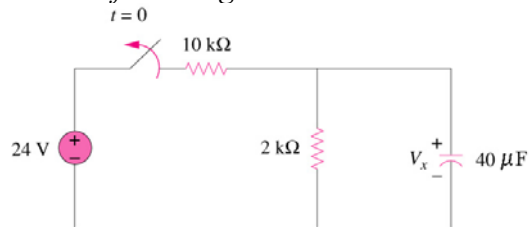
Example 7.2.2:

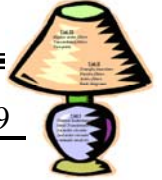
For the following circuit, determine $v_C(t)$, $i_C(t)$, $v_R(t)$, $i_R(t)$ for $t \geq 0$.



Example 7.2.3:

For the following circuit determine the voltage across the capacitor for $t \geq 0$.





Lecture 2-2: The step response and general solution of 1st order circuits

Reading: 7.3 - 4

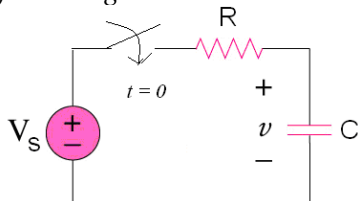
7.3 The step response of RL and RC circuits

As previously mentioned the step response of an RL or RC circuit is the voltage or current that results when a DC source is suddenly added.

The general solution for the natural and step response of RL and RC circuits is given by the following, $x(t) = x(\infty) + [x(0) - x(\infty)]e^{-t/\tau}$

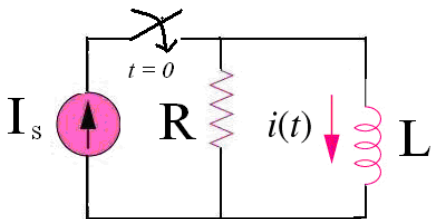
Example 7.3.1:

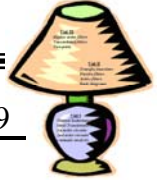
For the following circuit assume $v(0) = V_o$ and use KVL to derive the step response $v(t)$ for the following circuit.



Example 7.3.2:

For the following circuit assume $i(0) = I_o$ and use KCL to derive the step response $i(t)$ for the following circuit.





7.4 A general solution for step and natural responses

Since all of these circuits are described by a first order differential equation, it is possible to define a general solution as

$$\underline{x(t) = x_f + [x(t_0) - x_f]e^{-(t-t_0)/\tau}}$$

where x_f is the final value of the variable and $x(t_0)$ is the initial value of the variable

The complete response of a circuit can be decomposed into the **natural** and the **step**. The **natural response** is due to the stored energy and the **step** is due to the independent source.

$$x(t) = x_{\text{natural}}(t) + x_{\text{step}}(t)$$

$$x_{\text{natural}} = \underline{x(t_0) e^{-(t-t_0)/\tau}} \text{ and } x_{\text{step}} = \underline{x_f - x_f e^{-(t-t_0)/\tau}}$$

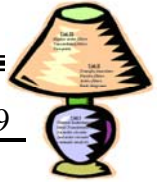
The natural response eventually dies out and the steady-state component remains. Therefore, the response can also be characterized by the temporary part and the permanent part. The temporary part is the **transient response**. The permanent part is the **steady state response**. The transient response is the circuit's temporary response that will die out with time. The steady-state response is the behavior of the circuit after a long time after an external excitation is applied.

$$x(t) = x_{\text{transient}}(t) + x_{\text{steadystate}}(t)$$

$$x_{\text{transient}} = \underline{[x(t_0) - x_f]e^{-(t-t_0)/\tau}} \text{ and } x_{\text{steadystate}} = \underline{x_f}$$

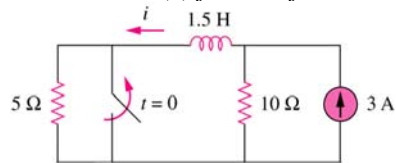
To compute the step and natural response of circuits use the following steps

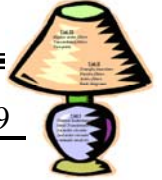
- i. identify the variable of interest (i.e. capacitive voltage or inductive current)
- ii. determine the initial voltage or current for the capacitor or inductor
- iii. calculate the final value of the variable as $t \rightarrow \infty$
- iv. calculate the time constant for the circuit
- v. use the general formula to find the variable of interest



Example 7.4.1:

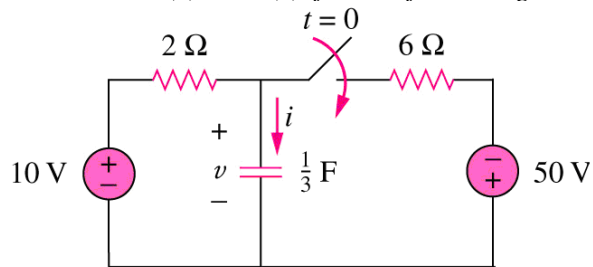
Determine $i(t)$ for the following circuit.

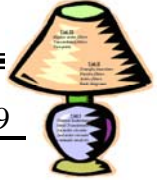




Example 7.4.2:

Determine $v(t)$ and $i(t)$ for the following circuit.



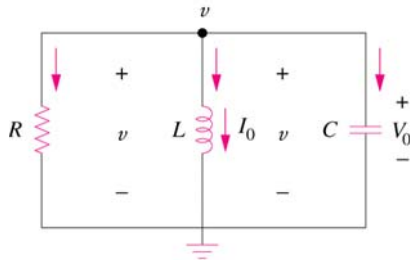


Lecture 2-3: The natural response of 2nd order circuits

Reading: 8.1, 8.2, 8.4

- Objectives:
- To define the types of responses: overdamped, underdamped, critically damped
 - To define the terms: damping factor, resonant frequency, undamped frequency
 - To define the characteristic equation for parallel and series RLC circuits
 - To calculate the natural response of parallel and series RLC circuits

8.1 The Natural Response of a parallel RLC circuit



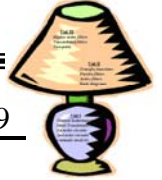
Example 8.1.1:

Use KCL to derive the 2nd order differential equation for the natural response of the above parallel RLC circuit.

Since this is a second-order differential equation, circuits with resistors, inductors and capacitors are referred to as second-order circuits.

Using Laplace analysis and assuming that the solution is of the form Ae^{st} , the differential equation simplifies to the characteristic equation, $s^2 + 2\alpha s + \omega_0^2 = 0$

Where the damping factor, $\alpha = 1/(2RC)$ (rad/s) and the resonant frequency, $\omega_0 = 1/\sqrt{LC}$ (rad/s)



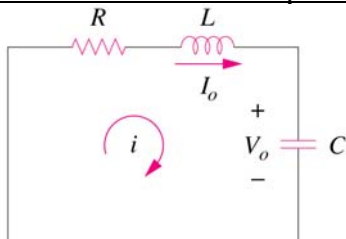
The roots of the characteristic equation are the complex frequencies, s_1 and s_2 (rad/s).

The complex frequencies can be found by using the quadratic formula

$$s_1, s_2 = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

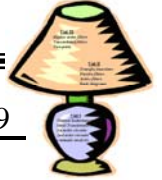
These roots can be real or complex and there can also be one or two distinct roots of the characteristic equation. The nature of the roots of the characteristic equation determines the type of natural response for the RLC circuit. The damping ratio, $\zeta = \alpha/\omega_o$, can also be used to determine the type of natural response for the RLC circuit.

8.4 The natural responses of a series RLC circuit



Example 8.4.1:

Use KVL to derive the 2nd order differential equation for the natural response of the above series RLC circuit.

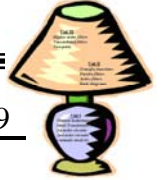


The following table provides a summary of the results for the analysis of the natural response for parallel and series RLC circuits.

Circuit	Parallel	Series
Second order D.E.	$\frac{d^2v}{dt^2} + \frac{1}{RC} \frac{dv}{dt} + \frac{v}{LC} = 0$	$\frac{d^2i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = 0$
Characteristic equation $s^2 + 2\alpha s + \omega_o^2$	$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$	$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$
Roots, complex frequencies $-\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$	$s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$	$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$
Damping factor	$\alpha = \frac{1}{2RC}$	$\alpha = \frac{R}{2L}$
Resonant radian frequency (undamped frequency)	$\omega_o = \frac{1}{\sqrt{LC}}$	
Initial conditions	Using KCL: $i_R(0) + i_L(0) + i_C(0)$ $C \frac{dv(0)}{dt} + I_o + \frac{V_o}{R} = 0$ $\frac{dv(0)}{dt} = -\frac{V_o}{RC} - \frac{I_o}{C}$	using KVL: $v_R(0) + v_L(0) + v_C(0)$ $I_o R + L \frac{di(0)}{dt} + V_o = 0$ $\frac{di(0)}{dt} = -\frac{I_o R}{L} - \frac{V_o}{L}$

8.2 Types of responses

<p>Overdamped $\alpha > \omega_o$ $\zeta > 1$</p> <p>2 real, distinct roots</p>		<p>Slow response and long settling time</p>
<p>Critically Damped $\alpha = \omega_o$ $\zeta = 1$</p> <p>2 equal real roots</p>		<p>Fast response and short settling time</p>
<p>Underdamped $\alpha < \omega_o$ $\zeta < 1$</p> <p>2 complex roots that are complex conjugates</p>		<p>Fastest response and long settling time</p>



Type of Response	Parallel	Series
Overdamped $\alpha > \omega_0$ 2 distinct real roots	$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ $v(0^+) = A_1 + A_2$ $\frac{dv(0^+)}{dt} = s_1 A_1 + s_2 A_2$	$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ $i(0) = A_1 + A_2$ $\frac{di(0)}{dt} = s_1 A_1 + s_2 A_2$
Critically Damped $\alpha = \omega_0$ 2 equal real roots	$v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$ $v(0^+) = D_2$ $\frac{dv(0^+)}{dt} = D_1 - \alpha D_2$	$i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$ $i(0) = D_2$ $\frac{di(0)}{dt} = D_1 - \alpha D_2$
Underdamped $\alpha < \omega_0$ 2 complex conjugate roots	$v(t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t$ damped radian frequency $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$ $v(0^+) = B_1$ $\frac{dv(0^+)}{dt} = -\alpha B_1 + \omega_d B_2$	$i(t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t$ damped radian frequency $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$ $i(0) = B_1$ $\frac{di(0)}{dt} = -\alpha B_1 + \omega_d B_2$

There is also an **undamped response** which occurs when $\alpha \rightarrow 0$. There is a persistent oscillation that approaches the undamped frequency, ω_0 . The response for this type of systems would be **sinusoidal**.

Team Concept Question 8.2.1:

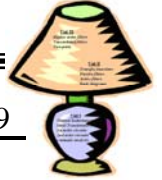
In a source-free parallel RLC circuit, if the resistor is replaced with a wire then the voltage across the capacitor is

- a) underdamped
- b) overdamped
- c) critically damped
- d) undamped

Team Concept Question 8.2.2:

In a source-free parallel RLC circuit, if the resistor is removed from the circuit then the voltage across the capacitor is

- a) underdamped
- b) overdamped
- c) critically damped
- d) undamped



Team Concept Question 8.4.1:

In a source-free series RLC circuit, if the resistor is replaced with a wire then the voltage across the capacitor is

- a) underdamped
- b) overdamped
- c) critically damped
- d) undamped

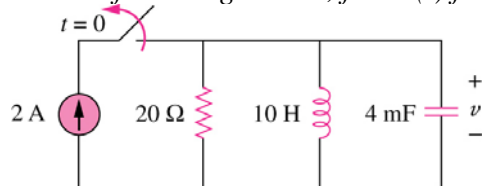
Team Concept Question 8.4.2:

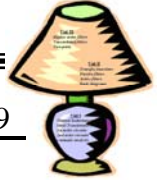
In a source-free series RLC circuit, if the resistor is removed from the circuit then the voltage across the capacitor is

- a) underdamped
- b) overdamped
- c) critically damped
- d) none of the above

In-class activity 8.2.2:

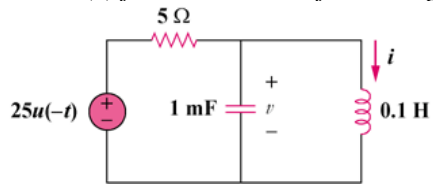
For the following circuit, find $v(t)$ for $t > 0$.





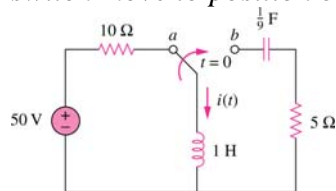
In-class activity 8.2.3:

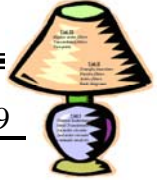
Find $v(t)$ for $t > 0$ in the following circuit.



In-class activity 8.4.1:

The circuit in the following figure has reached steady state at $t = 0^-$. If the make-before-break switch move to position b at $t = 0$, calculate $i(t)$ for $t > 0$.



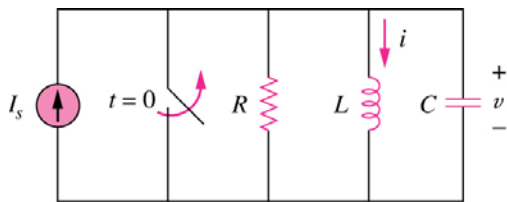


Lecture 3-1: The step response of 2nd order circuits

Reading: 8.3 - 4

Objectives: To define the types of responses: overdamped, underdamped, critically damped
To calculate the step response of parallel and series RLC circuits

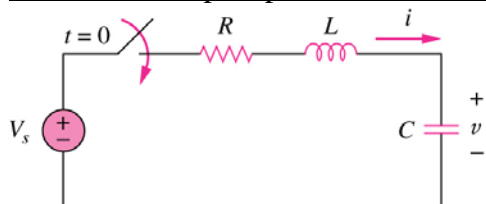
8.3 The Step response of a parallel RLC circuit



Example 8.3.1:

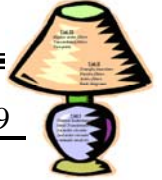
To find the step response of the above parallel RLC circuit, use KCL to derive the 2nd order differential equation.

8.4 The Step response of a series RLC circuit



Example 8.4.1:

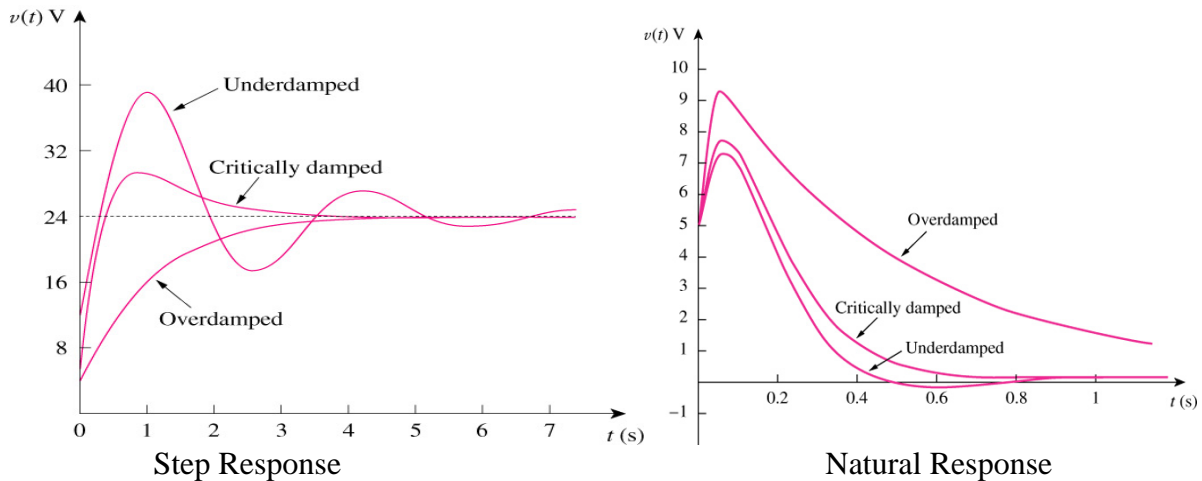
To find the step response of the above series RLC circuit, use KVL to derive the 2nd order differential equation. Observe that it is a similar derivation to the parallel RLC circuit.

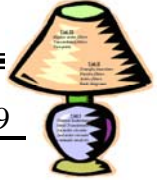


Circuit	Parallel	Series
Second order D.E.	$\frac{d^2 i}{dt^2} + \frac{1}{RC} \frac{di}{dt} + \frac{i}{LC} = \frac{I_s}{LC}$	$\frac{d^2 v}{dt^2} + \frac{R}{L} \frac{dv}{dt} + \frac{v}{LC} = \frac{V_s}{LC}$
Characteristic equation $s^2 + 2\alpha s + \omega_o^2$	$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$	$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$
Roots, Complex frequencies $-\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$	$s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$	$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$
Damping factor	$\alpha = \frac{1}{2RC}$	$\alpha = \frac{R}{2L}$
Resonant radian frequency (undamped frequency)	$\omega_o = \frac{1}{\sqrt{LC}}$	
Initial conditions	using parallel voltages: $v_L(0^+) = v_C(0^+)$ $\left. \frac{di}{dt} \right _{t=0^+} = \frac{v_C(0^+)}{L}$	using series current: $i_C(0^+) = i_L(0^+)$ $\left. \frac{dv}{dt} \right _{t=0^+} = \frac{i_L(0^+)}{C}$

8.3 - 4 Types of responses

Recall that the type of response of a 2nd order circuit is related to the roots of the characteristic equation or ζ . A comparison of the 3 types of responses is shown here.





Type of Response	Parallel	Series
Overdamped $\alpha > \omega_0$ $\zeta > 1$ 2 real, distinct roots	$i(t) = I_s + A_1' e^{s_1 t} + A_2' e^{s_2 t}$ $i(0) = I_s + A_1' + A_2'$ $\left. \frac{di}{dt} \right _{t=0^+} = s_1 A_1' + s_2 A_2'$	$v(t) = V_s + A_1' e^{s_1 t} + A_2' e^{s_2 t}$ $v(0) = V_s + A_1' + A_2'$ $\left. \frac{dv}{dt} \right _{t=0^+} = s_1 A_1' + s_2 A_2'$
Critically Damped $\alpha = \omega_0$ $\zeta = 1$ 2 equal real roots	$i(t) = I_s + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}$ $i(0) = I_s + D_2'$ $\left. \frac{di}{dt} \right _{t=0^+} = D_1' - \alpha D_2'$	$v(t) = V_s + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}$ $v(0) = V_s + D_2'$ $\left. \frac{dv}{dt} \right _{t=0^+} = D_1' - \alpha D_2'$
Underdamped $\alpha < \omega_0$ $\zeta < 1$ 2 complex roots that are complex conjugates	$i(t) = I_s + B_1' e^{-\alpha t} \cos \omega_d t + B_2' e^{-\alpha t} \sin \omega_d t$ damped frequency $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$ $i(0) = I_s + B_1'$ $\left. \frac{di}{dt} \right _{t=0^+} = -\alpha B_1' + \omega_d B_2'$	$v(t) = V_s + B_1' e^{-\alpha t} \cos \omega_d t + B_2' e^{-\alpha t} \sin \omega_d t$ damped frequency $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$ $v(0) = V_s + B_1'$ $\left. \frac{dv}{dt} \right _{t=0^+} = -\alpha B_1' + \omega_d B_2'$

Team Concept Question 8.3.1:

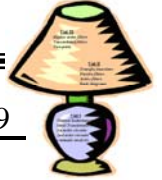
In a parallel RLC circuit with a DC source, if L equals C, determine the range of resistor values that will achieve an overdamped response.

- a) $0 < R < .5 \Omega$
- b) $R > .5L \Omega$
- c) $R > .5/L \Omega$
- d) none of the above

Team Concept Question 8.4.1:

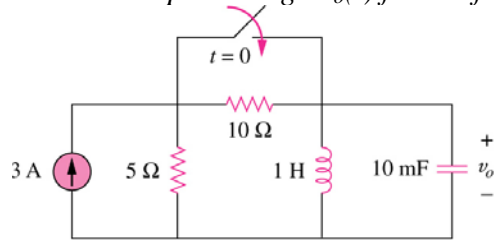
In a series RLC circuit with a DC source, if L equals C then determine the value of R such that the voltage across the capacitor is critically damped.

- a) $R = .5\Omega$
- b) $R = 2\Omega$
- c) $R = .5/L$
- d) none of the above



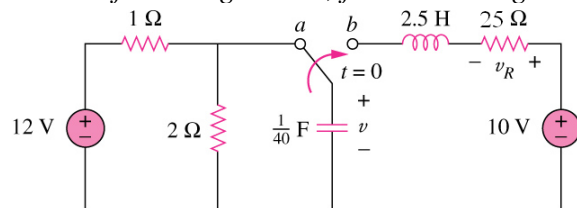
In-class activity 8.3.1:

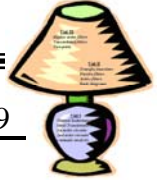
Find the output voltage $v_o(t)$ for the following circuit.



In-class activity 8.4.1:

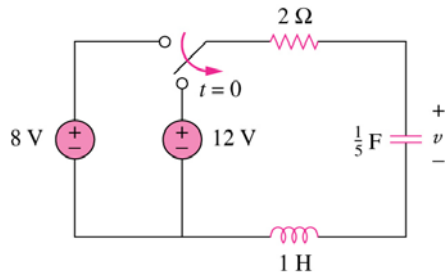
For the following circuit, find the voltages $v(t)$ and $v_R(t)$.

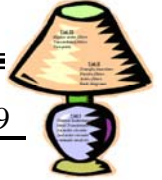




In-class activity 8.4.2:

For the following circuit, find the voltage $v(t)$.





Lecture 3-2: Circuit elements and analysis in the s domain

Reading: 13.1 – 13.2

- Objectives:
- To define circuit elements in the s domain
 - To redraw 1st and 2nd order circuits in the s domain using initial conditions
 - To calculate the natural response of circuits in the s-domain

13.1 – 13.2 Circuit Elements and Analysis in the s Domain

The benefit of Laplace transforms is that it transforms differential equations to algebraic equations that may be easier to solve.

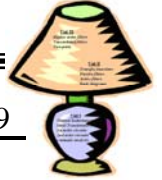
The following table summarizes the relationship between resistors, inductors, capacitors at the frequency domain.

Note that $\mathbf{V} = \mathcal{A}\{v\}$, $\mathbf{I} = \mathcal{A}\{i\}$ and impedance relationships can be related to the phasor relationships where $s = j\omega$ for sinusoidal steady state where the transient has died off or $\alpha=0$.

Time Domain	Voltage Current Relationships	Laplace Transform	Complex Frequency Domain (s-domain)
	$v = iR$ $i = v/R$	$\mathbf{V} = \mathbf{I}R$ $\mathbf{I} = \mathbf{V}/R$ $Z = R$	
	$v = L \frac{di}{dt}$ $i = \frac{1}{L} \int_0^t v dx + I_o$	$\mathbf{V} = s\mathbf{L}\mathbf{I} - \mathbf{L}I_o$ $\mathbf{I} = \frac{\mathbf{V}}{sL} + \frac{I_o}{s}$ $Z = sL$	
	$i = C \frac{dv}{dt}$ $v = \frac{1}{C} \int_0^t i d\tau + V_o$	$\mathbf{I} = s\mathbf{C}\mathbf{V} - \mathbf{C}V_o$ $\mathbf{V} = \frac{\mathbf{I}}{sC} + \frac{V_o}{s}$ $Z = \frac{1}{sC}$	

Note that the two frequency domain circuits for the inductor and capacitor initial conditions represent the Thevenin and Norton equivalent circuits with respect to the terminals for those circuits.

Ohm's law in the frequency domain is stated as $\mathbf{V} = \mathbf{I}Z$.



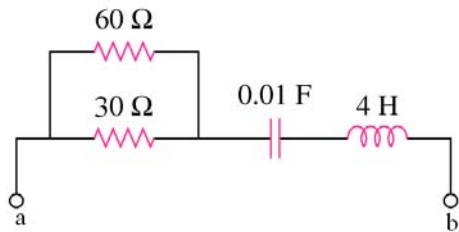
Element	Impedance (Ω)	Admittance (S)
R	R	1/R
L	sL	1/(sL)
C	1/(sC)	sC

All of the circuit analysis techniques in the frequency domain are applicable to circuits in the s-domain including KVL, KCL, nodal analysis, mesh analysis, source transformations, and Thevenin-Norton equivalents.

Example 13.2.1:

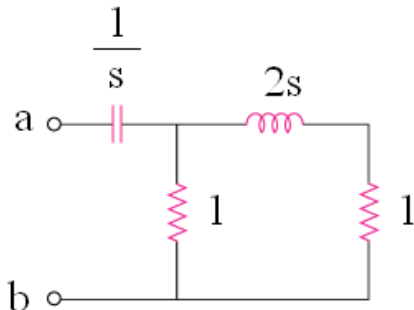
For the following network,

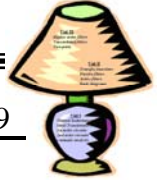
- a) find the impedance and admittance across terminals a and b as a rational function of s
- b) compute the numerical values of the zeros and poles of the impedance
- c) plot the zeros and poles of the impedance on a s-plane



Example 13.2.2:

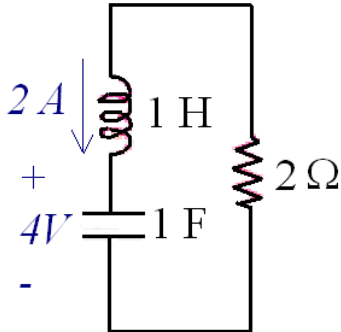
For the following network, find the input impedance across terminals a and b as a rational function of s.





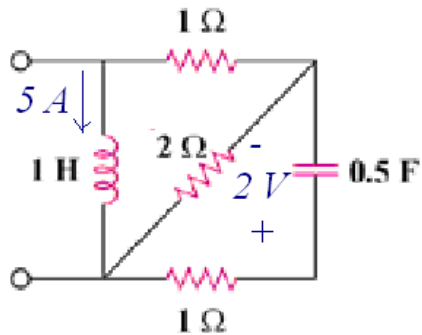
Example 13.2.3:

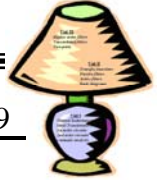
For the following network, redraw the circuit in the s -domain.



Example 13.2.4:

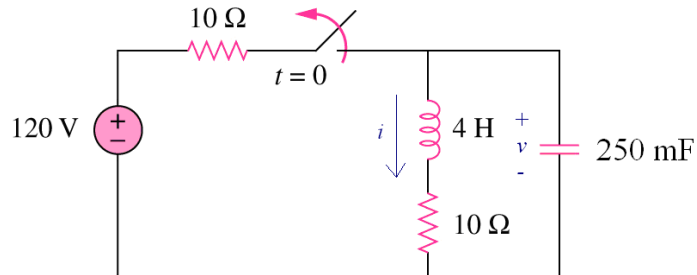
For the following network, redraw the circuit in the s -domain.

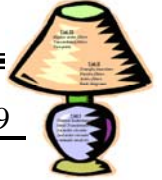




EXAMPLE 13.2.5 (natural response):

For the following circuit, redraw the circuit in the s -domain at $t = 0^+$. If the circuit has reached steady state at $t = 0^-$ and the switch opens at $t = 0$, determine $i(t)$.





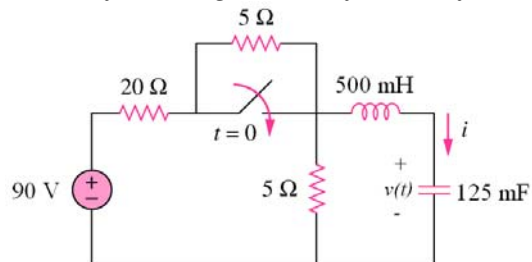
Lecture 3-3: Circuit analysis in the s domain
Reading: 13.2

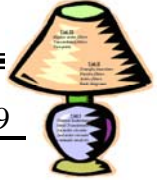
Objectives: To analyze the natural response of circuits in the s-domain
To analyze the step response of circuits in the s-domain

13.2 Circuit Analysis in the s-domain

In-class activity 13.2.1(step response):

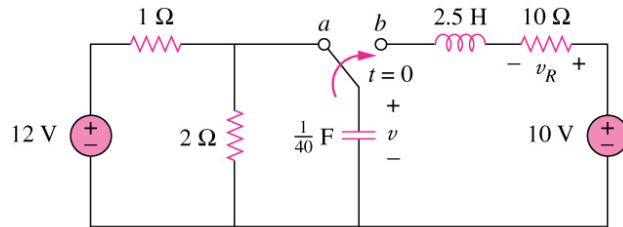
For the following network find $i(t)$ for $t > 0$.

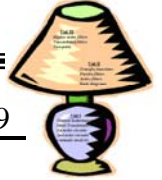




In-class activity 13.2.2 (step response):

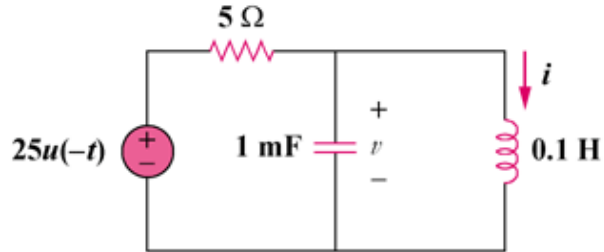
Having been in position *a* for a long time, the switch in the following circuit moves to position *b* at $t = 0$. Find $v(t)$ for $t > 0$.

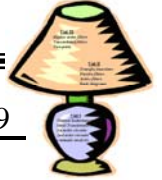




In-class activity 13.2.3 (natural response):

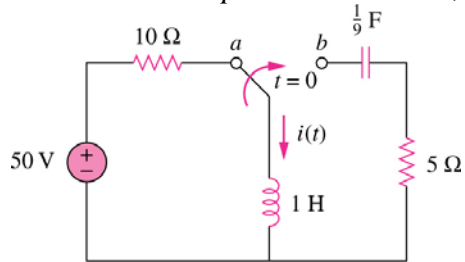
Find $v(t)$ for $t > 0$ in the following circuit.

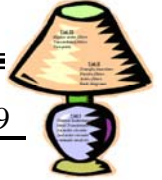




In-class activity 13.2.4 (natural response):

The circuit in the following figure has reached steady state at $t = 0^-$. If the make-before-break switch moves to position b at $t = 0$, calculate $i(t)$ for $t > 0$.



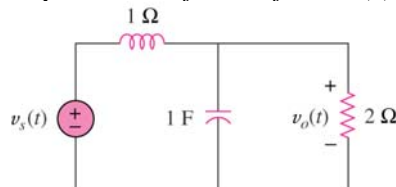


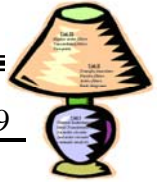
Lecture 4-1: Applications in the s domain
Reading: 13.3

Objective: To calculate the steady-state and transient response of circuits in the s-domain
 To analyze circuits in the s-domain using Thevenin's theorem
 To analyze circuits in the s-domain using superposition
 To analyze mutual inductance circuits in the s-domain

In-class activity 13.3.1 (Steady State and Transient Response):

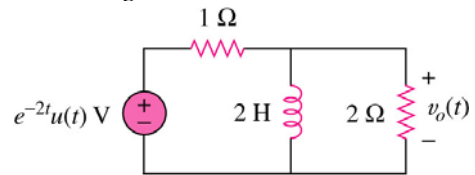
For the following circuit determine the steady state and transient response for $v_o(t)$ using Laplace transforms if the $v_s(t) = 2 \cos t$ V

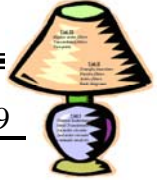




In-class activity 13.3.2 (Steady State and Transient Response):

Assuming zero initial conditions, determine $v_o(t)$ in the following circuit.



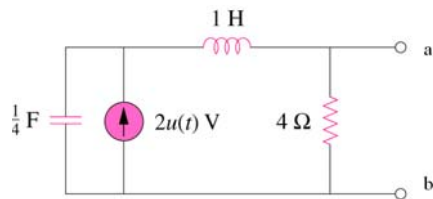


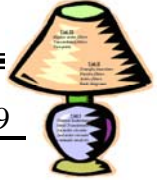
Lecture 4-2: Applications in the s domain
Reading: 13.3

Objective: To calculate the steady-state and transient response of circuits in the s-domain
 To analyze circuits in the s-domain using Thevenin's theorem
 To analyze circuits in the s-domain using superposition
 To analyze mutual inductance circuits in the s-domain

In-class activity 13.3.3 (Thevenin and Norton equivalent):

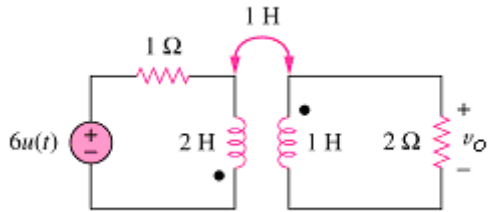
Find the s-domain Thevenin and Norton equivalent circuit with respect to terminals a and b for the following circuit.

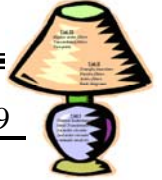




In-class activity 13.3.4 (Mutual Inductance):

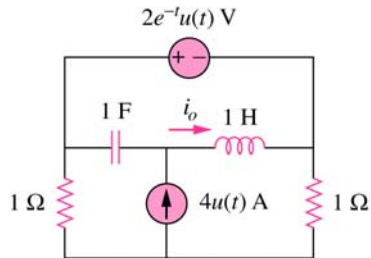
Use mesh analysis to find $v_o(t)$ for $t > 0$ for the following circuit.





In-class activity 13.3.5 (superposition):

Use superposition to find the current through the inductor for all time $t > 0$ in the following circuit.

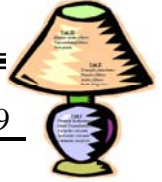


ECE200 Circuits & Systems

Unit II



- **Transfer Functions**
- **Passive Filters**
- **Active Filters**
- **Bode diagrams**



**Lecture 5-1: The Initial and Final Value Theorem, Transfer functions,
Steady-State response
Reading: 13.4, 13.7**

Objectives: To be able to define and clearly explain the transfer function of a circuit
 To calculate the transfer function of a circuit
 To apply the initial value and final value theorem to find voltage and current values in an electric circuit an electric circuit
 To use the transfer function instead of phasor analysis to determine the steady-state response of a circuit.

13.4 The Initial Value Theorem and Transfer function

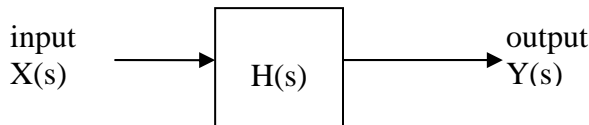
The **initial and final value theorems** can be used to find the initial and final values of a voltage or current in the time-domain by using the s-domain equivalent of the current or voltage.

The initial value theorem states that $\lim_{t \rightarrow 0^+} f(t) = \lim_{s \rightarrow \infty} sF(s)$

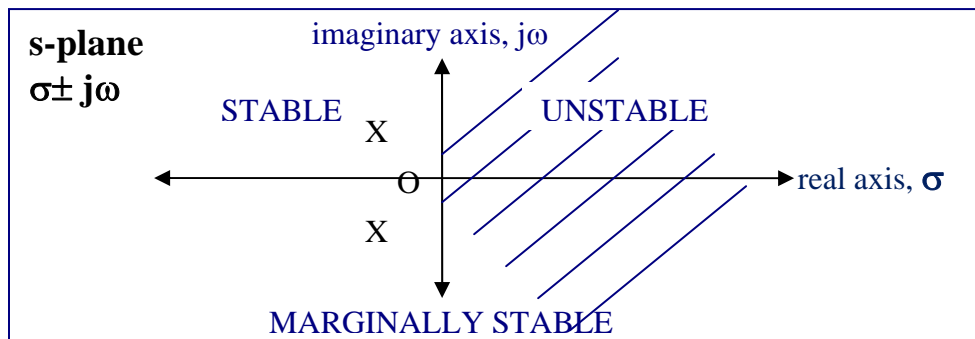
The final value theorem states that $\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$

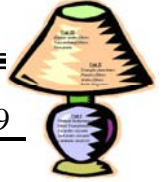
The **transfer function** is defined as the s-domain ratio of the Laplace transform of the output (response) to the Laplace transform of the input (source). To find the transfer function of a circuit, all of the **initial conditions must be zero**.

The transfer function is $\underline{H(s) = \frac{\text{output}}{\text{input}} = \frac{Y(s)}{X(s)}}$



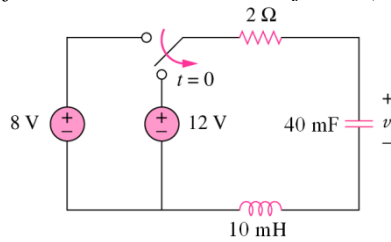
H(s) is always a rational function of s. If there are any complex poles, they always appear in conjugate pairs. The poles of H(s) must lie in the left half of the s plane if the response is to be bounded. The zeros may lie in either the right half or the left half of the s plane. the poles are represented with “X’s” and the zeros are represented with “O’s”.





In-class activity 13.4.1 (Initial, Final Value Theorem):

For the following circuit, (a) Find $V(s)$, (b) Use the initial value theorem to find $v(0)$, (c) Use the final value theorem to find $v(\infty)$. [hint: $v(t) = 12 - 4.3e^{-13.4t} + 0.3e^{-187t}$ V]



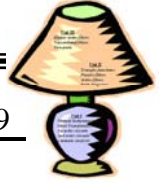
In-class activity 13.4.2 (Transfer functions):

For each of the following transfer functions, find K (gain), α (damping factor/coefficient), ω_o (undamped frequency), ω_d (damped frequency), ζ (damping ratio), poles, zeros, type of response. Is it stable?

$$H(s) = \frac{5}{s(s^2 + 2s + 1)}$$

$$H(s) = \frac{3s + 12}{s^2 + 5s + 6}$$

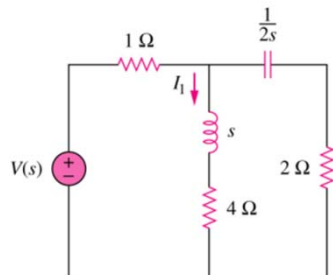
$$H(s) = \frac{8s^2 + 24s}{2s^2 + 10s + 18}$$

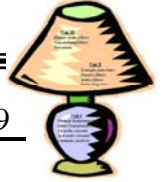


In-class activity 13.4.3 (Transfer function):

Assume the following circuit has zero initial conditions, determine the transfer function

$$H(s) = I_1(s)/V(s).$$





13.7 Transfer function and Steady-State Sinusoidal Response

For a steady state sinusoidal response, $\mathbf{x(t) = A \cos(\omega t + \phi)}$

Using trigonometric properties, $\mathbf{x(t) = A \cos \omega t \cos \phi - A \sin \omega t \sin \phi}$

The Laplace transform of the input, x(t) is $\mathbf{X(s) = \frac{A \cos \phi}{s^2 + \omega^2} - \frac{A \omega \sin \phi}{s^2 + \omega^2} = \frac{A(s \cos \phi - \omega \sin \phi)}{s^2 + \omega^2}}$

The output is $\mathbf{Y(s) = H(s) X(s) = H(s) \frac{A(s \cos \phi - \omega \sin \phi)}{s^2 + \omega^2}}$

Using partial fraction expansion, $\mathbf{Y(s) = \text{terms from poles of } H(s) + \frac{K_1}{s - j\omega} + \frac{K_1^*}{s + j\omega}}$

Only the $\mathbf{X(s)}$ terms contribute to the steady-state response.

The terms due to the transfer function $\mathbf{H(s)}$ are transient and approach 0 as t approaches ∞ .

The terms from the poles of H(s) are in the open left-half of the s-plane (they have to be for stability)!!

The poles of the sinusoidal input, $\mathbf{x(t)}$ are on the imaginary axis of the s-plane.

$$K_1 = \left. \frac{H(s)A(s \cos \phi - \omega \sin \phi)}{s + j\omega} \right|_{s = j\omega} = \frac{1}{2} \mathbf{H(j\omega)} A e^{j\phi} = \frac{A}{2} |H(j\omega)| e^{j[\theta(\omega) + \phi]}$$

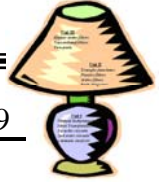
The steady-state solution for y(t) or the steady-state sinusoidal response is

$$\mathbf{Y_{ss}(s) = (H(j\omega))(A \angle \phi) = A |H(j\omega)| e^{j(\phi + \theta(\omega))}}$$

$$\mathbf{y_{ss}(t) = A |H(j\omega)| \cos[\omega t + \phi + \theta(\omega)]}$$

The amplitude of the solution is equal to the amplitude of the source, A, times the magnitude of the transfer function, |H(j ω)|.

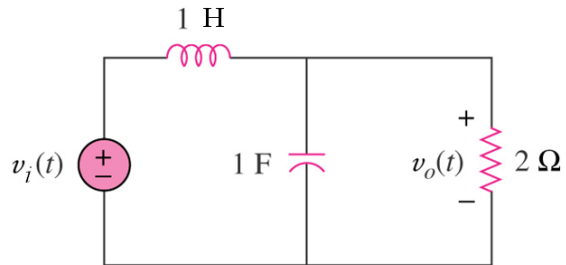
The phase angle of the response is equal to the phase angle of the source, ϕ , plus the phase angle of the transfer function, $\theta(\omega)$ at the frequency of the source, ω .

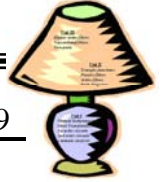


In-class activity 13.4.4 (Steady-state sinusoidal response):

For the following circuit $v_s(t) = 2 \cos t$ V,

- Use the transfer function to compute the steady-state expression for $v_o(t)$.
- Use Laplace transforms to find the complete solution $v_o(t) = v_{oss}(t) + v_{otran}(t)$





Lecture 5-2: Introduction to Filters

Reading: 13.4, 14.1-3

- Objectives:
- To introduce the 4 major types of passive filters
 - To illustrate the frequency response, magnitude and phase angle plots
 - To define the terms passband, stopband, cutoff and half-power frequency

The **frequency response** of a circuit is the analysis of the result of varying the source frequency on voltages and currents in the circuit.

Frequency-selective circuits are circuits that pass only certain signals in a desired-range of frequencies to the output of the system. (i.e. radios, telephone systems, car stereos, televisions)

Frequency-selective circuits are also called **filters**. Filters weaken or attenuate any input signals outside of particular frequency band.

The action of a filter on an input signal results in an output signal.



The signals passed from the input to the output fall within a band of frequencies called the **passband**. Frequencies not in the circuit's passband are in the **stopband**.

The **cutoff frequency** separates the passband from the stopband

The **frequency response** plot shows how a transfer function (amplitude and phase) changes as the source frequency changes.

The graph of $|H(j\omega)|$ versus frequency ω is the **magnitude plot**

The graph of $\theta(j\omega)$ versus frequency ω is the **phase angle plot**

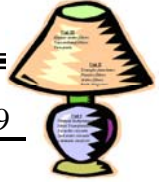
All of the filters in this lecture will be **passive filters** that depend only on the passive elements: resistors, capacitors, and inductors. All passive filters have a gain between 0 and 1.

A **low pass filter** passes low frequencies and stops high frequencies (i.e. woofer)

A **high pass filter** passes high frequencies and stops low frequencies (i.e. tweeter)

A **band pass filter** passes frequencies within a frequency band and blocks or attenuates frequencies outside the band (i.e. radio)

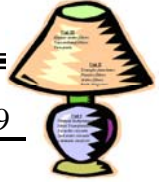
A **band reject filter** passes frequencies outside a frequency band and blocks or attenuates frequencies within the band. (i.e. unwanted noise)



The **cutoff frequency** is typically defined by engineers as the frequency for which the transfer function magnitude is decreased by **30%** or the factor $1/\sqrt{2}$ or **-3 dB**. (i.e. $|H(j\omega_c)| = 0.707 H_{max}$)

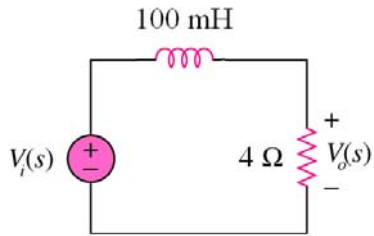
At the **cutoff frequency**, the average power delivered by the circuit is one half of the maximum average power. Thus ω_c is also called the **half-power frequency**. [$P(j\omega_c) = 0.5P_{max}$]

Type of Filter	Low - pass	High - pass	Bandpass	Bandreject
$H(0)$	1	0	0	1
$H(\infty)$	0	1	0	1
$H(\omega_c)$ or $H(\omega_0)$	$1/\sqrt{2}$	$1/\sqrt{2}$	1	0
Frequency response Ideal magnitude plot				
Example Circuit				
$H(s)$ Transfer function	$\frac{\omega_c}{s + \omega_c} = \frac{1/RC}{s + 1/RC}$	$\frac{s}{s + 1/RC} = \frac{s}{s + \omega_c}$	$\frac{(R/L)s}{s^2 + (R/L)s + (1/LC)}$	$\frac{s^2 + (1/LC)}{s^2 + (R/L)s + (1/LC)}$
$ H(j\omega) $	$\frac{1/RC}{\sqrt{\omega^2 + (1/RC)^2}}$	$\frac{\omega}{\sqrt{\omega^2 + (1/RC)^2}}$	$\frac{(R/L)\omega}{\sqrt{(1/LC) - \omega^2)^2 + ((R/L)\omega)^2}}$	$\frac{1/(LC) - \omega^2}{\sqrt{(1/LC) - \omega^2)^2 + ((R/L)\omega)^2}}$
Resonant frequency			$\omega_0^2 = 1/(LC)$ $\omega_0^2 = \omega_{c1}\omega_{c2}$	$\omega_0^2 = 1/(LC)$ $\omega_0^2 = \omega_{c1}\omega_{c2}$
bandwidth			$\beta = R/L = 2\alpha$ $\beta = \omega_{c2} - \omega_{c1}$	$\beta = R/L = 2\alpha$ $\beta = \omega_{c2} - \omega_{c1}$
Cut off frequencies	$\omega_c = \frac{1}{RC} = \frac{1}{\tau}$	$\omega_c = \frac{1}{RC} = \frac{1}{\tau}$	$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_0^2}$ $\omega_{c2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_0^2}$	$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_0^2}$ $\omega_{c2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_0^2}$
Quality factor			$Q = \omega_0/\beta$ $Q = \sqrt{\frac{L}{CR^2}}$	$Q = \omega_0/\beta$ $Q = \sqrt{\frac{L}{CR^2}}$



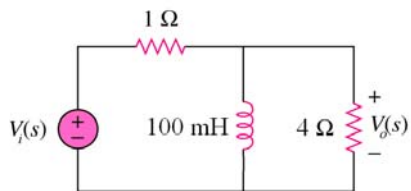
In-class activity 14.1.1 (Passive Filters):

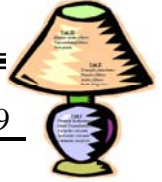
For the following circuit, use qualitative analysis, to determine the type of filter. What is the cutoff frequency? What is the gain?



In-class activity 14.1.2 (Passive Filters):

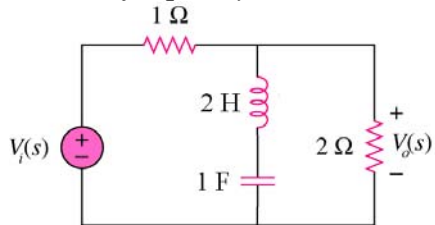
For the following circuit, use qualitative analysis, to determine the type of filter. What is the cutoff frequency? What is the gain?





In-class activity 14.1.3 (Passive Filters):

For the following circuit, use qualitative analysis, to determine the type of filter. What is the resonant frequency? What is the bandwidth? What is the gain?



In-class activity 14.1.4 (Passive Filters):

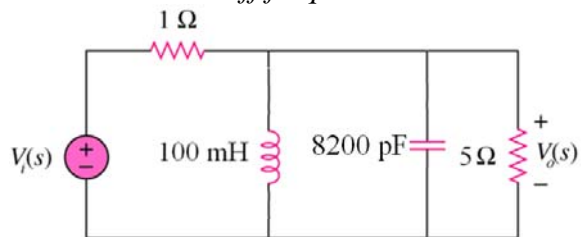
For the following circuit, use qualitative analysis, to determine the type of filter.

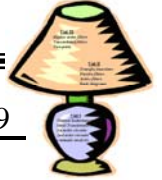
What is the gain?

What is the resonant frequency?

What is the bandwidth?

What are the cut off frequencies?





Lecture 5-3: Active Filters
Reading: 15.1 - 3

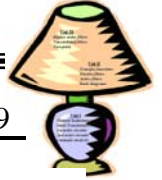
- Objectives:
- To analyze active filter circuits to determine the frequency response, gain, cut off frequencies, resonant frequency and bandwidth
 - To calculate the transfer function of passive and active filters
 - To design a passive or active filter given certain specifications

15.1 Active Filter Circuits

All the filters described so far have been passive, they only include resistors, inductors, and capacitors. The advantage of active filters over passive is that creating a filter with an operational amplifier may be less expensive because they do not require inductors and they can also have a maximum magnitude that can exceed one. Additionally, the addition of a resistive load will not alter the passband magnitude or the cutoff frequency of an active filter.

Low Pass Filter	High Pass Filter
$H(s) = -K \frac{\omega_c}{s + \omega_c}$	$H(s) = -K \frac{s}{s + \omega_c}$
$K = R_2/R_1$	$K = R_2/R_1$
$\omega_c = 1/(R_2C)$	$\omega_c = 1/(R_1C)$
integrating amplifier in the time domain	differentiating amplifier in the time domain

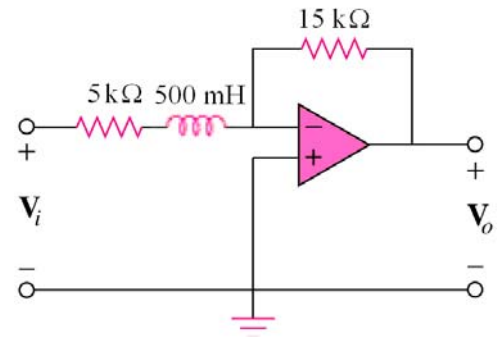
Bode plots or frequency response plots are used to graphically describe the behavior of passive and active filters. The Bode plot is used to graph the magnitude of the transfer function in decibels (dB) versus the log of the frequency. The cutoff frequency is the frequency at which the maximum magnitude of the transfer function has decreased by $1/\sqrt{2}$ or 3dB. The dB magnitude is defined as $20 \log |H(j\omega)|$.

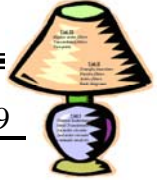


In-class activity 15.1.1

For the following circuit,

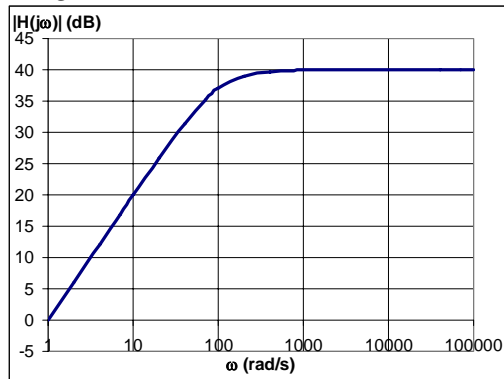
- a) derive the transfer function
- b) what is the step response?
- c) what type of filter is it?
- d) what is the cut off frequency?
- e) what is the DC gain?
- f) what is the AC gain at 2 kHz?

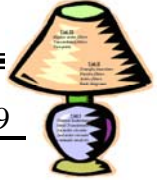




In-class activity 15.1.2 (active highpass filter):

Design a filter with the following Bode magnitude response. Use $0.5 \mu\text{F}$ capacitors in your design.

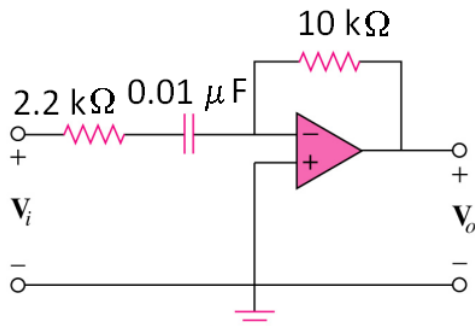


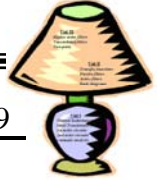


In-class activity 15.1.3

For the following circuit,

- a) derive the transfer function
- b) what is the step response?
- c) what type of filter is it?
- d) what is the cut off frequency?
- e) what is the AC gain at 2 kHz?
- f) Redesign the circuit to have a gain of 10 and the same cut off frequency
- g) Redesign the original circuit to have a cut of frequency of 5 kHz and the same gain.





Lecture 6-1: More Passive and Active Filters

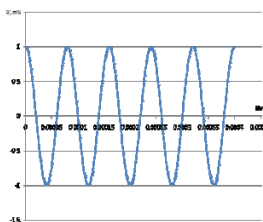
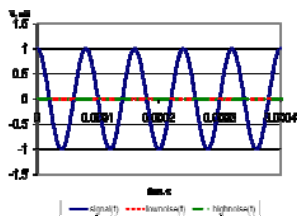
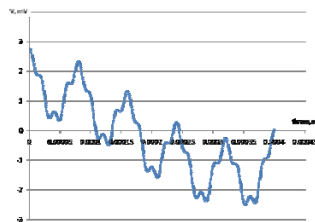
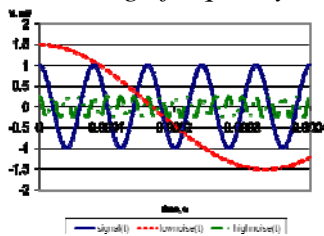
Reading: 14.1 – 5, 15.1 - 3

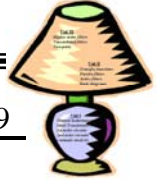
- Objectives: To analyze passive and active filter circuits to determine the frequency response, gain, cut off frequencies, resonant frequency and bandwidth
 To calculate the transfer function of passive and active filters
 To design a passive or active filter given certain specifications

In-Class Activity Signal to Noise Ratio Example:

In a power plant, a 1 mV communication signal has a resonant frequency of 12.5 kHz. However, the signal is corrupted by 1.5 mV power line noise 60 Hz and 0.25 mV machine vibration noise at 50 kHz.

- what is the signal to noise ratio for the low and high frequency noise?
- design a bandpass filter to “clean up” the noisy signal
- after passing the signal through the filter, what is the signal to noise ratio for the low and high frequency noise?

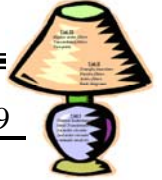




In-Class Activity 6.1.1:

Given the following transfer function, $H(j\omega) = \frac{160}{200 + 2j\omega}$

- a) What type of filter is it?
- b) What is the cutoff frequency?
- c) What is the DC gain?
- d) What is the magnitude of $H(j\omega)$ at $\omega = 10$? 100? 1000 rad/s?
- e) What is the phase of $H(j\omega)$ at $\omega = 10$? 100? 1000 rad/s?
- f) Using $0.01 \mu\text{F}$ capacitors, design a filter to satisfy this transfer function.

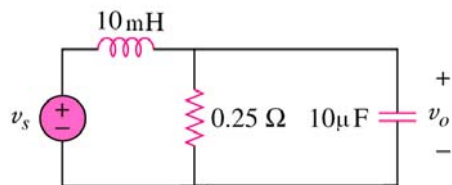


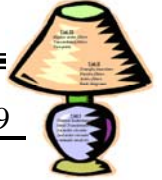
In-Class Activity 6.1.3:

For the following circuit,

- Using qualitative analysis, what is the low frequency gain?
- Using qualitative analysis, what is the high frequency gain?
- What type of filter is it?
- Find the transfer function V_o/V_s
- What is the cutoff frequency?
- What is the magnitude and phase of $H(j\omega)$ at $\omega = 20$ rad/s?

ans: $\omega_c = 251.57$ rad/s

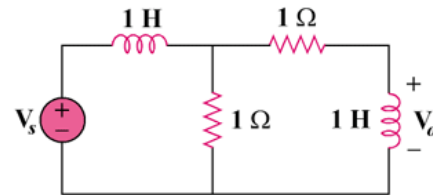


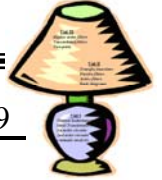


In-Class Activity 6.1.4:

For the following circuit,

- a) determine the transfer function,
- b) what type of filter is it?
- c) what is the center frequency?
- d) what is the bandwidth?
- e) what is the gain at resonance?





Lecture 6-2: Bode Diagrams
Reading: Appendix E.1 - 2

- Objectives:
- To create Bode diagrams for electric circuits and given transfer functions
 - To apply the steps to create Bode diagrams for first-order poles and zeros and poles and zeros at the origin
 - To be able to compute the transfer function of a circuit given the Bode diagram

The **bel** used to define power gain and is defined as $\log_{10}(P_{out}/P_{in})$.

The **decibel** is also used to measure power gain and is defined as $10 \log_{10}(P_{out}/P_{in})$.

Decibels can be used to define the voltage and current ratio as

$$10 \log_{10} \frac{P_{out}}{P_{in}} = 10 \log_{10} \left(\frac{v_{out}}{v_{in}} \right)^2 = 20 \log_{10} \frac{v_{out}}{v_{in}} \quad \text{and} \quad 20 \log_{10} \frac{i_{out}}{i_{in}}$$

Bode diagrams are graphical techniques that describe the frequency response of a circuit. There are two separate plots: amplitude and phase angle and they vary with the frequency.

The plots are on semi log graph paper with the frequency on the horizontal log scale and the amplitude and phase angle on the linear vertical scale.

There are several types of factors in a transfer function these include:

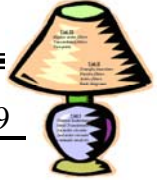
- i. a gain K
- ii. a zero $(j\omega)$ at the origin
- iii. a pole $(j\omega)^{-1}$ at the origin
- iv. a simple zero $1 + j\omega/z_1$
- v. a simple pole $1/(1+j\omega/p_1)$
- vi. a quadratic pole $1/[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]$
- vii. a quadratic zero $[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]$

This lecture will address creating a Bode diagram with the first 5 types of factors. To create the Bode diagram, plot each of these factors separately and then combine them graphically.

Standard Form

Given the following transfer function $H(s) = \frac{K(s + z_1)}{s(s + p_1)}$ and $H(j\omega) = \frac{K(j\omega + z_1)}{j\omega(j\omega + p_1)}$

The first step in creating a Bode diagram is to write $H(j\omega)$ in **standard form**. In order to write $H(j\omega)$ in standard form, divide out the poles and zeros.



STANDARD FORM:
$$H(j\omega) = \frac{Kz_1 \left(1 + j\omega/z_1\right)}{p_1(j\omega) \left(1 + j\omega/p_1\right)}$$

the amplitude is $|H(j\omega)| = \frac{K_o \left|1 + j\omega/z_1\right|}{\omega \left|1 + j\omega/p_1\right|}$, where $K_o = Kz_1/p_1$

The amplitude of $H(j\omega)$ in decibels is **$A_{dB} = 20\log_{10}|H(j\omega)|$**

$$A_{dB} = 20\log_{10} \frac{K_o \left|1 + j\omega/z_1\right|}{\omega \left|1 + j\omega/p_1\right|} = 20\log_{10}K_o + 20\log_{10}|1 + j\omega/z_1| - 20\log_{10}\omega - 20\log_{10}|1 + j\omega/p_1|$$

the phase is $\theta(\omega) = \psi_1 - 90^\circ - \beta_1$, where $\psi_1 = \tan^{-1}(\omega/z_1)$, $\beta_1 = \tan^{-1}(\omega/p_1)$

To create the Bode diagram, plot each term of the amplitude separately and then graphically combine the plots.

i. Constant K_o

The plot of the constant, K_o , is a horizontal straight line at the value **$20\log_{10}K_o$**

$H(\omega)$	H_{dB}	ϕ
K_o	$20 \log_{10} K_o $	0°
$-K_o$	$20 \log_{10} K_o $	$\pm 180^\circ$

ii. Zero at the origin

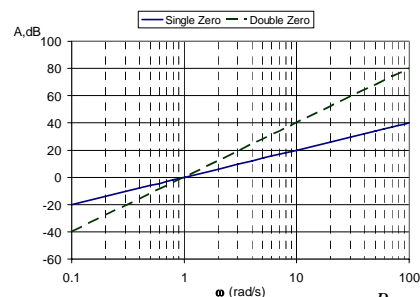
Another possible term in the numerator is a zero at the origin.

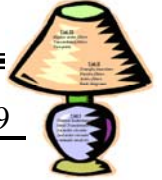
The term **$20\log_{10}|j\omega|$** is a zero at the origin. The plot of this term is a straight line with a slope of 20 dB/decade that crosses the 0dB axis at $\omega = 1$ rad/s. The plot of the phase is a horizontal line at 90° .

$H(\omega)$	H_{dB}	ϕ
$j\omega$	$20 \log_{10} \omega$	90°
$(j\omega)^N$	$20N \log_{10} \omega$	$90N^\circ$

for example, amplitude plot of a single and double zero at the origin

ω , rad/s	$20 \log_{10} \omega$	$40 \log_{10} \omega$
0.01	-40 dB	-80
0.1	-20 dB	-40
1.0	0 dB	0
10.0	20 dB	40
100.0	40 dB	80





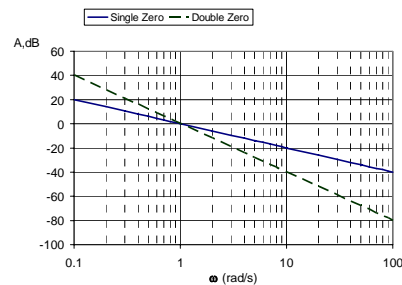
iii. Pole at the origin

The term $20\log_{10}|j\omega|$ is a pole at the origin. The plot of this term is a straight line with a slope of -20 dB/decade that crosses the 0dB axis at $\omega = 1$ rad/s. The plot of the phase is a horizontal line at -90° .

$H(\omega)$	H_{dB}	ϕ
$(j\omega)^{-1}$	$-20 \log_{10} \omega$	-90°
$(j\omega)^{-N}$	$-20N \log_{10} \omega$	$-90N^\circ$

for example, amplitude plot of a single and double pole at the origin

ω , rad/s	$-20 \log_{10} \omega$	$-40 \log_{10} \omega$
0.01	40 dB	80 dB
0.1	20 dB	40 dB
1.0	0 dB	0 dB
10.0	-20 dB	-40 dB
100.0	-40 dB	-80 dB



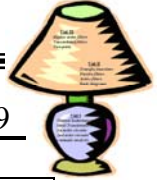
iv. First Order Zero

The amplitude term $20\log_{10}|1 + j\omega/z_1|$ is a first-order zero. The plot of this term is a horizontal line approximation from $\omega = 0$ to z_1 at 0° . At the **corner frequency** ($\omega = z_1$), the plot increases with a slope of 20dB/decade. The phase term, $\tan^{-1}(\omega/z_1)$, is a horizontal line approximation from $\omega = 0$ to $0.1z_1$ at 0° . At $0.1z_1$, the line increases with a slope of 45° /decade from $0.1z_1$ to $10z_1$. At $10z_1$, the line is a horizontal approximation at 90° .

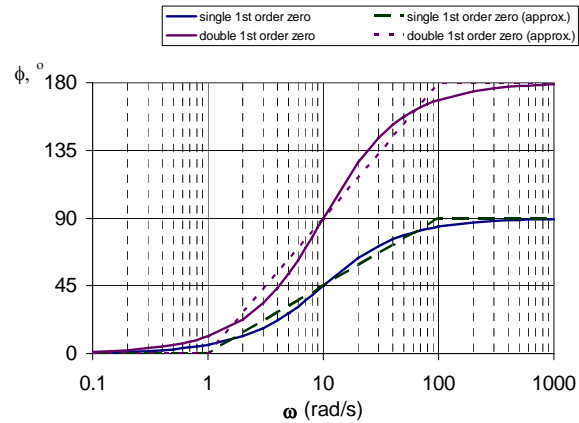
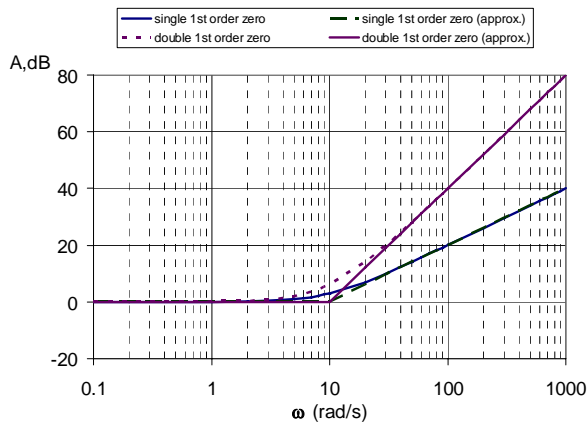
$H(\omega)$	H_{dB}	ϕ
$1 + j\omega/z_1$	$20 \log_{10} \sqrt{1 + \left(\frac{\omega}{z_1}\right)^2}$	$\tan^{-1}\left(\frac{\omega}{z_1}\right)$
$(1 + j\omega/z_1)^N$	$20N \log_{10} \sqrt{1 + \left(\frac{\omega}{z_1}\right)^2}$	$N \cdot \tan^{-1}\left(\frac{\omega}{z_1}\right)$

for example, amplitude and phase plot of a single and double first order zero ($1 + j\omega/10$)

ω , rad/s	$20 \log_{10} 1 + j\omega/10 $	$20 \log_{10} 1 + j\omega/10 $ approximate, dB	$20 \log_{10} 1 + j\omega/10 $	$20 \log_{10} 1 + j\omega/10 $ approximate, dB
0.1	0.0004	0	0.0009	0
1.0	0.0432	0	0.0864	0
10.0	3 dB	0	6 dB	0
100.0	20 dB	20	40 dB	40
1000.0	40 dB	40	80 dB	80



ω , rad/s	$\tan^{-1}(\omega/10), ^\circ$	$\tan^{-1}(\omega/10), ^\circ$ approximate	$2 \cdot \tan^{-1}(\omega/10), ^\circ$	$2 \cdot \tan^{-1}(\omega/10), ^\circ$ approximate
0.1	0.573	0	1.146	0
1.0	5.711	0	11.421	0
10.0	45	45	90	90
100.0	84.289	90	168.579	180
1000.0	89.427	90	178.854	180



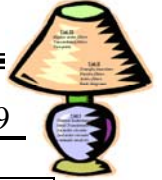
v. First Order Pole

The amplitude term $20 \log_{10} |1 + j\omega/p_1|$ is a first-order pole. The plot of this amplitude term is a horizontal line approximation from $\omega = 0$ to p_1 at 0 dB. At the **corner frequency** ($\omega = p_1$), the plot decreases with a slope of -20dB/decade. The phase term, $-\tan^{-1}(\omega/z_1)$, is a horizontal line approximation from $\omega = 0$ to $0.1z_1$ at 0° . At $0.1z_1$, the line decreases with a slope of $-45^\circ/\text{decade}$ from $0.1z_1$ to $10z_1$. At $10z_1$, the line is a horizontal approximation at -90° .

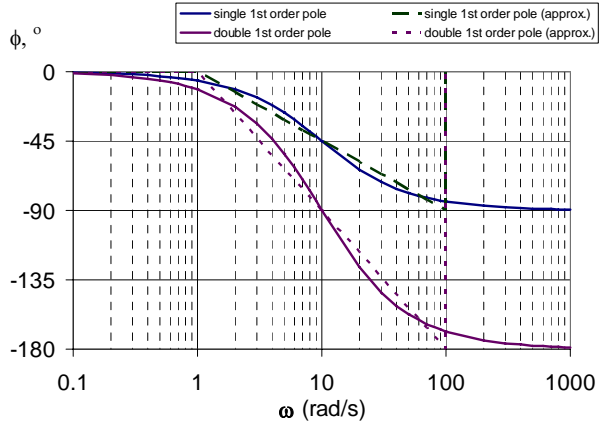
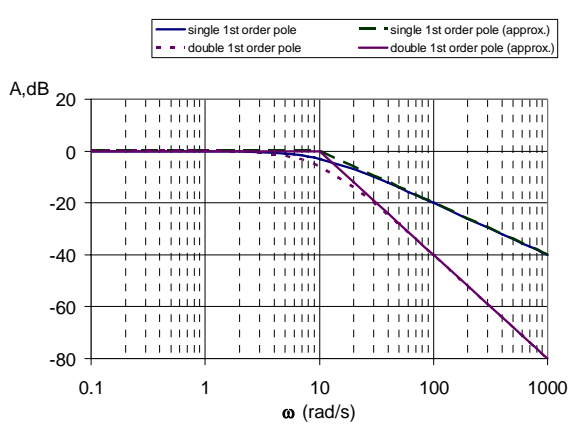
$H(\omega)$	H_{dB}	ϕ
$(1+j\omega/p_1)^{-1}$	$-20 \log_{10} \sqrt{1 + \left(\frac{\omega}{p_1}\right)^2}$	$-\tan^{-1}\left(\frac{\omega}{p_1}\right)$
$(1+j\omega/p_1)^{-N}$	$-20N \log_{10} \sqrt{1 + \left(\frac{\omega}{p_1}\right)^2}$	$-N \cdot \tan^{-1}\left(\frac{\omega}{p_1}\right)$

for example, amplitude and phase plot of a single and double first order pole $(1 + j\omega/10)^{-1}$

ω , rad/s	$-20 \log_{10} 1 + j\omega/10 $	$-20 \log_{10} 1 + j\omega/10 $ approximate, dB	$-20 \log_{10} 1 + j\omega/10 $	$-20 \log_{10} 1 + j\omega/10 $ approximate, dB
0.1	-0.0004	0	-0.0009	0
1.0	-0.0432	0	-0.0864	0
10.0	-3 dB	0	-6 dB	0
100.0	-20 dB	-20	-40 dB	-40
1000.0	-40 dB	-40	-80 dB	-80

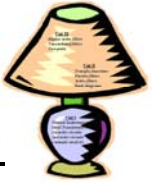


ω , rad/s	$-\tan^{-1}(\omega/10), ^\circ$	$-\tan^{-1}(\omega/10), ^\circ$ approximate	$-2 \cdot \tan^{-1}(\omega/10), ^\circ$	$-2 \cdot \tan^{-1}(\omega/10), ^\circ$ approximate
0.1	-0.573	0	-1.146	0
1.0	-5.711	0	-11.421	0
10.0	-45	-45	-90	-90
100.0	-84.289	-90	-168.579	-180
1000.0	-89.427	-90	-178.854	-180



Example E.1.1:

Construct the Bode amplitude and phase plot for the transfer function, $H(j\omega) = \frac{500(j\omega + 0.1)}{j\omega(j\omega + 5)}$



Lecture 6-3: Bode Diagrams

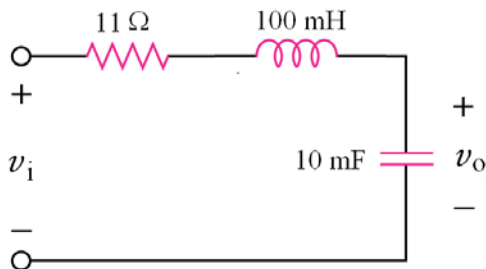
Reading: Appendix E.1 - 2

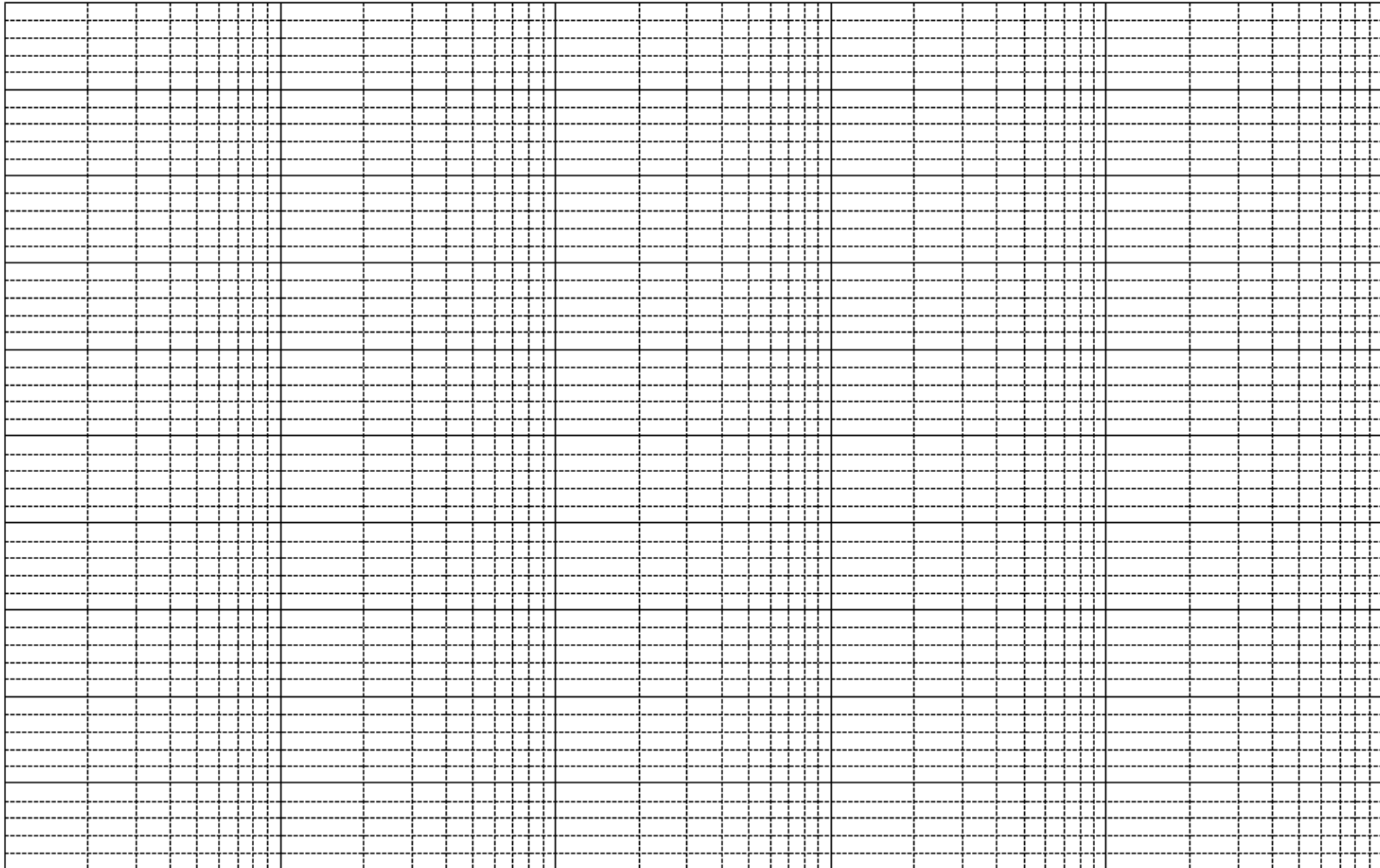
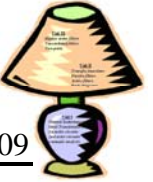
- Objectives:
- To create Bode diagrams for electric circuits and given transfer functions
 - To apply the steps to create Bode diagrams for first-order poles and zeros and poles and zeros at the origin
 - To be able to compute the transfer function of a circuit given the Bode diagram

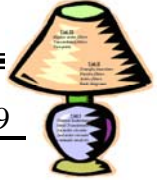
Example E.1. 2:

For the following circuit,

- a) Compute the transfer function, $H(s)$
- b) Construct the Bode amplitude plot for $H(s)$
- c) Construct the Bode phase plot for $H(s)$
- d) Suppose that $v_i(t) = 5\cos(500t + 15^\circ)$ V, use the Bode plot you constructed to predict the amplitude of $v_o(t)$ in the steady state.
- e) Suppose that $v_i(t) = 5\cos(500t + 15^\circ)$ V, use the Bode plot you constructed to predict the phase of $v_o(t)$ in the steady state.





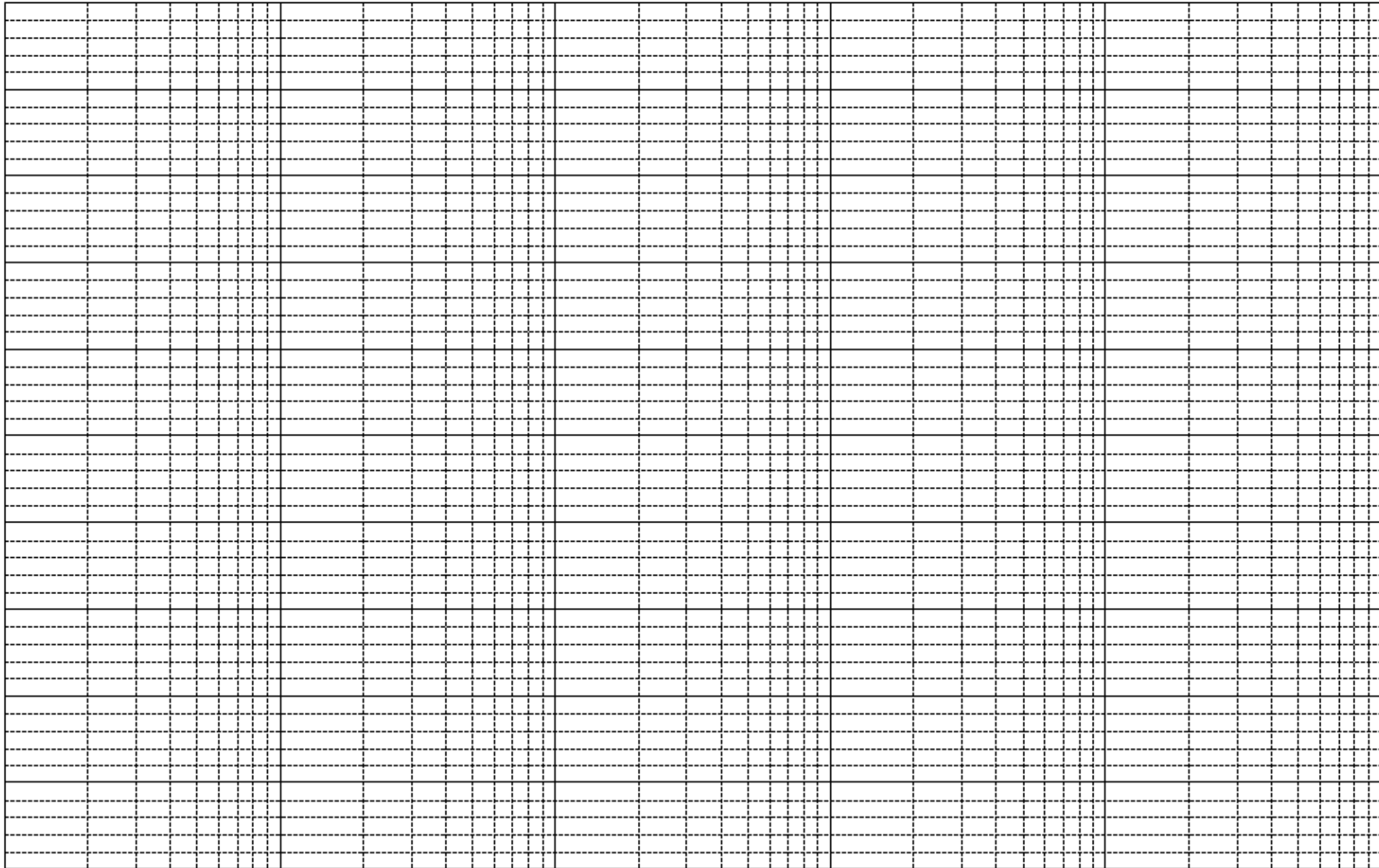
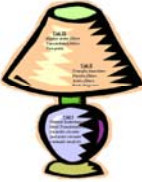


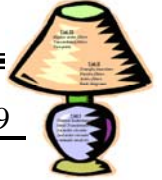
Example E.1.3:

For the following transfer function,

$$H(j\omega) = \frac{200j\omega}{(j\omega + 2)(j\omega + 10)}$$

- a) Construct the Bode amplitude plot for $H(s)$
- b) Construct the Bode phase plot for $H(s)$
- c) Suppose that $v_i(t) = 5\cos(5t)$ V, use the Bode amplitude plot you constructed to predict the amplitude of $v_o(t)$ in the steady state.
- d) Suppose that $v_i(t) = 5\cos(5t + 15^\circ)$ V, use the Bode phase plot you constructed to predict the phase of $v_o(t)$ in the steady state.





Lecture 7-1: Bode Diagrams

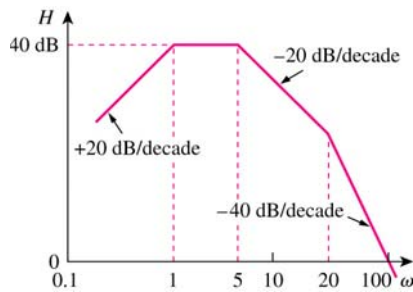
Reading: Appendix E.1 - 2

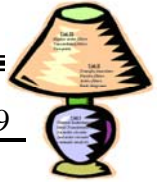
- Objectives:
- To create Bode diagrams for electric circuits and given transfer functions
 - To apply the steps to create Bode diagrams for first-order poles and zeros and poles and zeros at the origin
 - To be able to compute the transfer function of a circuit given the Bode diagram

Example E.1.4:

For the following Bode amplitude plot, compute the transfer function, $H(s)$ in the form

$$\frac{K(s + z_1)(s + z_2) \dots (s + z_n)}{(s + p_1)(s + p_2) \dots (s + p_n)}$$

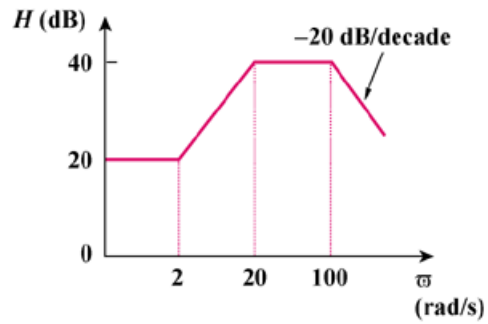


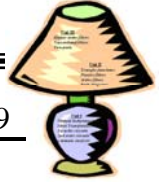


Example E.1.5:

For the following Bode amplitude plot, compute the transfer function, $H(s)$ in the form

$$\frac{K(s + z_1)(s + z_2) \dots (s + z_n)}{(s + p_1)(s + p_2) \dots (s + p_n)}$$





Lecture 7-2: Bode Diagrams

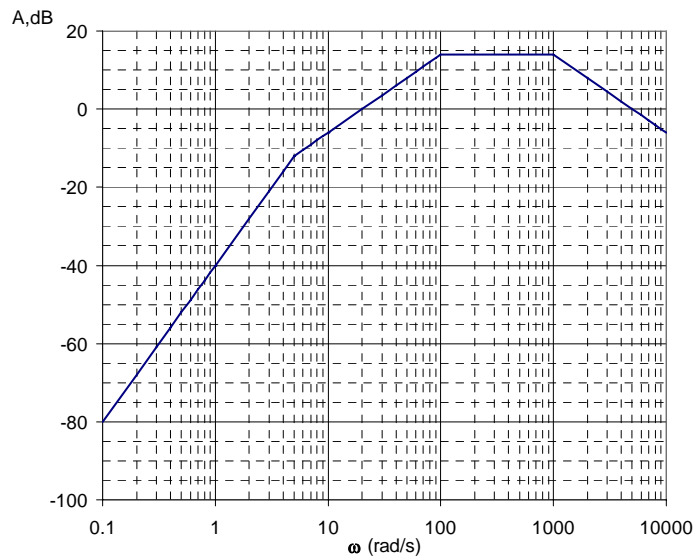
Reading: Appendix E.1 - 2

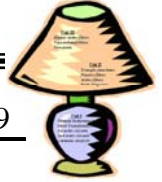
- Objectives:
- To create Bode diagrams for electric circuits and given transfer functions
 - To apply the steps to create Bode diagrams for first-order poles and zeros and poles and zeros at the origin
 - To be able to compute the transfer function of a circuit given the Bode diagram

Example E.1.6:

For the following Bode amplitude plot, compute the transfer function, $H(s)$ in the form

$$\frac{K(s + z_1)(s + z_2) \dots (s + z_n)}{(s + p_1)(s + p_2) \dots (s + p_n)}$$

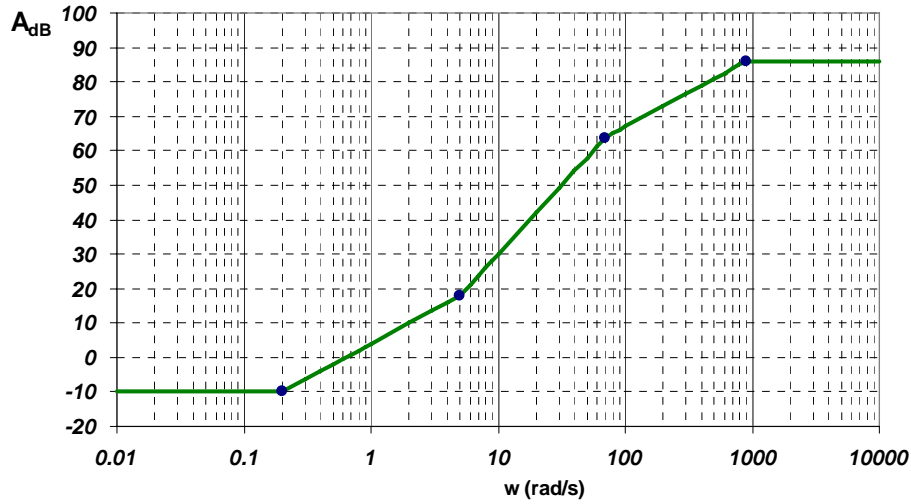


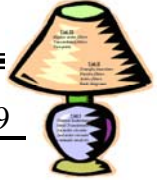


Example E.1.7:

For the following Bode amplitude plot, compute the transfer function, $H(s)$ in the form

$$\frac{K(s + z_1)(s + z_2) \dots (s + z_n)}{(s + p_1)(s + p_2) \dots (s + p_n)}$$





Lecture 7-3: Scaling
Reading: 15.2

- Objectives:
- To analyze passive and active filter circuits to determine the frequency response, gain, cut off frequencies, resonant frequency and bandwidth
 - To calculate the transfer function of passive and active filters
 - To design a passive or active filter given certain specifications
 - To use scaling to design both passive and active filters

15.2 Scaling

It is more convenient and simplifies the mathematics to design passive and active filter circuits with element values such as 1Ω, 1F, and 1H. **Scaling** can be used to transform the circuit and values for R, L, and C to more realistic values.

There are two types of scaling: **magnitude** and **frequency**

- In magnitude scaling, the scale factor, **k_m** , multiplies all of the impedances at a given frequency, the transfer function before and after scaling is the same.
- In frequency scaling, the scale factor, **k_f** , multiplies each impedance such that it is the same as it was at the original frequency.

Where **$k_m = R'/R$** and **$k_f = \omega'/\omega$**

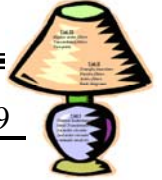
elements	magnitude, k_m scale factor	frequency, k_f scale factor
R	$R' = k_m R$	$R' = R$
L	$L' = k_m L$	$L' = L/k_f$
C	$C' = C/k_m$	$C' = C/k_f$

To simultaneously scale magnitude and frequency use the following. Note that the primed terms are the scaled values and the unprimed terms are the original values.

$$R' = k_m R$$

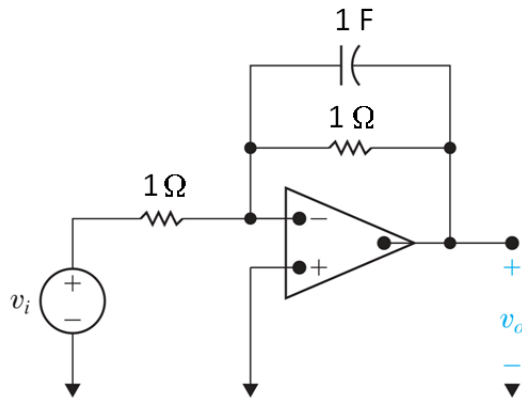
$$L' = k_m L / k_f$$

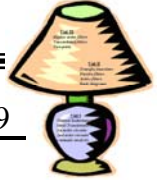
$$C' = C / (k_m k_f)$$



In-class activity 15.2.1 (Scaling, active lowpass filters):

Design a lowpass active filter with a dc gain of 4 and a corner frequency of 500 Hz. Use a 0.2 μF capacitor in your design.

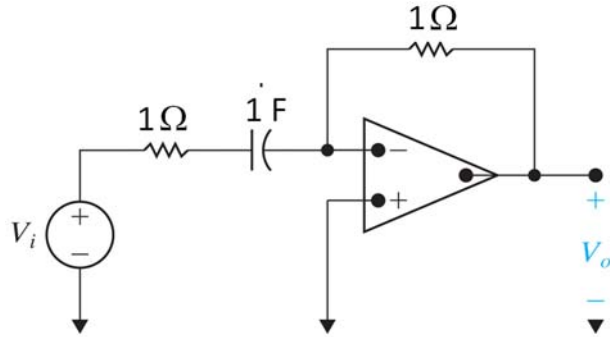


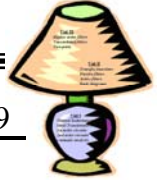


In-class activity 15.2.2 (Scaling, active highpass filters):

Design a highpass active filter with a high-frequency gain of 5 and a corner frequency of 2 kHz.

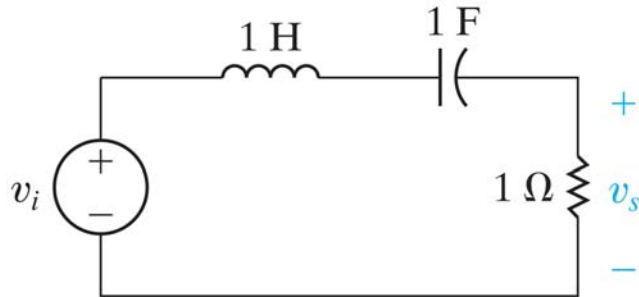
Use a $0.1 \mu\text{F}$ capacitor in your design.





In-class activity 15.2.3:

The following series RLC circuit has a center frequency of 1 rad/s, a bandwidth of 1 rad/s and a quality factor of 1. Use scaling to compute new values of R and L that yield a circuit with the same quality factor but with a center frequency of 10 kHz. Use a 0.01 μF capacitor in your design.

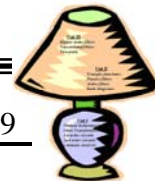


ECE200 Circuits & Systems

Unit III



- Higher order op amp filters
- Narrowband filters
- Two-port circuits



Lecture 8-1: Op Amp Bandpass and Bandreject Filters
Reading: 15.3

Objectives: To analyze op amp broadband bandpass and bandreject filters
To design op amp broadband bandpass and bandreject filters

15.3 Op Amp broadband bandpass and bandreject filters

The **bandpass filter** passes source voltages between the two cutoff frequencies (the passband) to the output and attenuates source voltages before they reach the output at frequencies outside the two cutoff frequencies (the stopband).

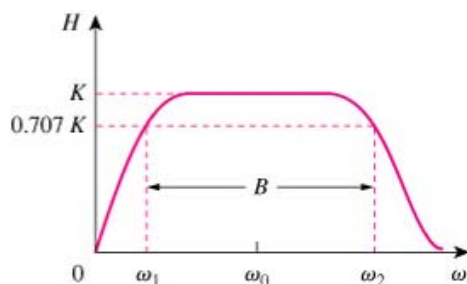
The **center frequency** or **resonant frequency**, ω_0 , is the frequency for which a circuit's transfer function is purely real. This was referred to as the natural response when analyzing second-order circuits. The center frequency is the geometric center of the passband $\omega_0 = \sqrt{\omega_{c1}\omega_{c2}}$

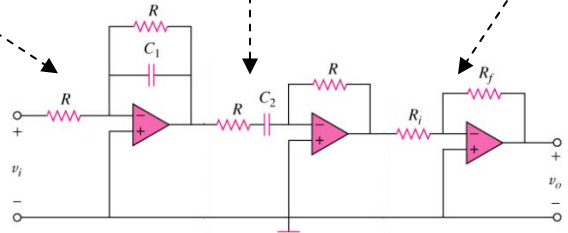
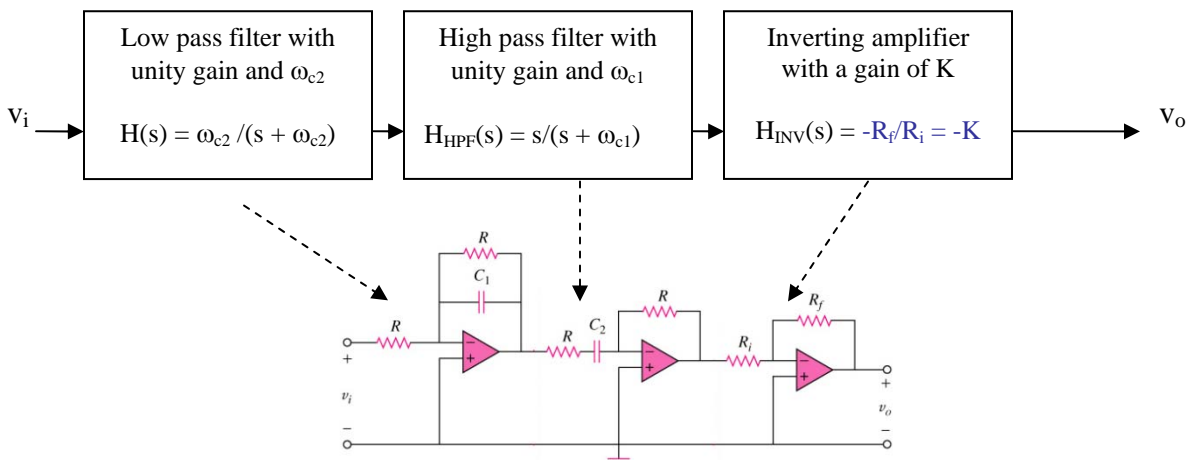
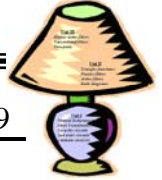
For bandpass filters, the magnitude of the transfer function is maximum at the resonant frequency $H_{\max} = |H(j\omega_0)|$

The **bandwidth, β** is the width of the passband for a band pass filter. ($\beta = \omega_{c2} - \omega_{c1}$)

The **quality factor, Q** is the ratio of the center frequency to the bandwidth. This gives a measure of the width of the passband, independent of its location on the frequency axis. ($Q = \omega_0/\beta$)

It is possible to create a **broadband** bandpass filter ($\omega_{c2} \geq 2\omega_{c1}$) by cascading a low-pass filter, high-pass filter, and inverting amplifier. The quality factor, Q , of a filter describes the width of the passband. For broadband band pass filter, $Q < 0.5$.





$$H(s) = [H_{L,PF}(s)](H_{HPF}(s))(H_{INV}(s))$$

$$H(s) = -K \frac{\omega_{c2} s}{s^2 + (\omega_{c1} + \omega_{c2})s + \omega_{c1}\omega_{c2}}$$

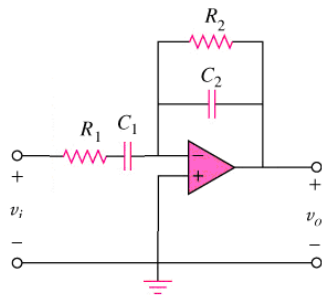
Alternate (Simplified) form of a Bandpass Filter

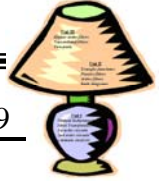
A passive high-pass filter cascaded with an active high pass filter

$$K = R_2/R_1$$

$$\omega_{c1} = 1/(R_1 C_1) \text{ (cutoff of HPF)}$$

$$\omega_{c2} = 1/(R_2 C_2) \text{ (cutoff of LPF)}$$



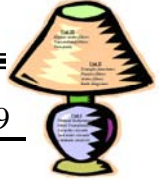


In-class activity 15.3.1 (active bandpass filters):

Design a bandpass filter for a graphic equalizer to provide an amplification of 2 within the band of frequencies between 100 and 10,000 Hz.

In-class activity 15.3.2 (active bandpass filters):

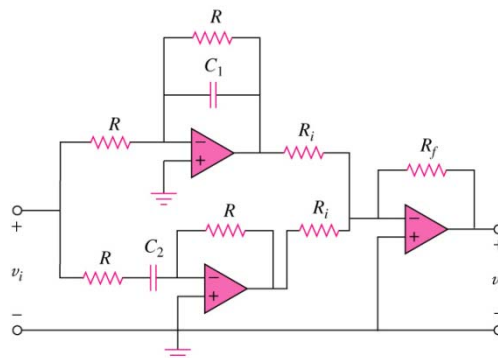
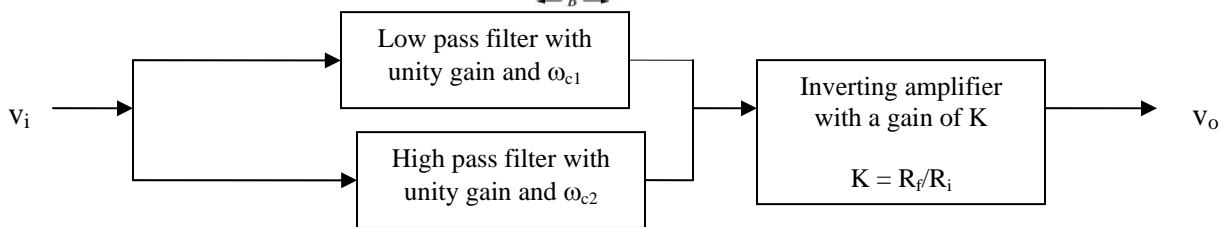
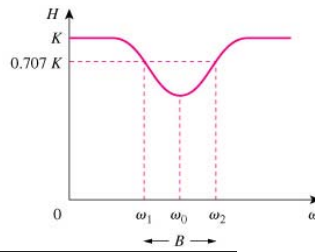
Design a bandpass filter to pass frequencies between 250 Hz and 3 kHz and with $K = 10$.



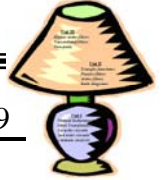
Bandreject Filters

- The **bandreject filter** passes source voltages outside a band between two cutoff frequencies (the passband) and attenuates source voltages before they reach the output at frequencies between the two cutoff frequencies (the stopband).
- The bandpass and bandreject filters perform complementary function in the frequency domain.
- Ideal bandpass and bandreject filters have two cutoff frequencies, ω_{c1} and ω_{c2} .
- At this frequency, the magnitude of the transfer function equals $(1/\sqrt{2}) H_{max}$

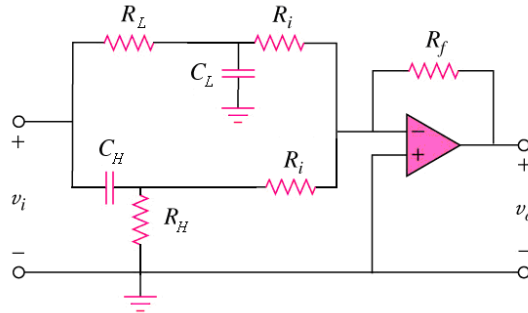
It is possible to create a **broadband** bandreject filter ($\omega_{c2} \geq 2\omega_{c1}$) by putting a low-pass and high-pass filter in parallel and putting them in series with an inverting amplifier.



$$H(s) = K \frac{s^2 + 2\omega_{c1}s + \omega_{c1}\omega_{c2}}{(s + \omega_{c1})(s + \omega_{c2})}$$



Alternate (Simplified) form of a Bandreject Filter

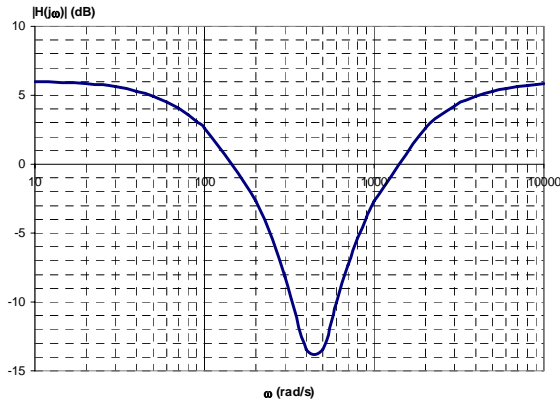


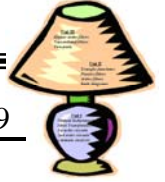
$$(R_H \gg R_i)$$

$$H(s) = K \frac{s^2 + 2\omega_{c1}s + \omega_{c1}\omega_{c2}}{(s + \omega_{c1})(s + \omega_{c2})}$$

In-class activity 15.2.1 (Scaling, active bandreject filters):

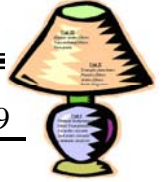
Design a circuit based on the parallel bandreject op amp filter. The bode magnitude response of this filter is shown in the following figure. Use 0.01 μ F capacitors in your design.





In-class activity 15.2.2 (Scaling, active bandreject filters):

Design a bandreject filter for $\omega_o = 20$ krad/s, $K = 5$, and $Q = 0.1$. Use 100 pF capacitors in your design. (Hint: Use $\beta = \omega_{c1} + \omega_{c2}$, $Q = \omega_o/\beta$, $\omega_o^2 = \omega_{c1}\omega_{c2}$)



Lecture 8-2: Higher order op amp filters

Reading: 15.4

Objectives: To define, design and perform calculations with higher order op amp filters
To determine the order of higher order filters using a template

15.4 Higher Order Op Amp Filters

A prototype low-pass operational amplifier filter has $R = 1\Omega$, $C = 1F$ and a cutoff frequency of 1 rad/s. In order to create a sharper transition from the pass band to the stop band, it is necessary to cascade filters. As more filters are cascaded, the slope of the transition increases by 20 dB/decade. The following figure shows that as each additional stage increases the order of the transfer function for the filter. The higher order cascaded **prototype low pass filter** has the following transfer function.

$$H(s) = \frac{(-1)^n}{(s + 1)^n}, \quad \omega_{cn} = \sqrt[n]{\sqrt{2} - 1}$$

Each stage has a cutoff frequency of 1 rad/s and as more stages are added to a filter, the overall filter cutoff frequency changes. The resultant filter can be frequency scaled in order to adjust it to the desired cutoff frequency.

$$k_f = \omega_c / \omega_{cn}$$

Since the gain of the higher-order filter is one, an inverting amplifier can be cascaded with the filter to adjust the gain or adjust the gain of each filter or one of the stage filters. An alternate method to achieve the gain is to change the input resistance, R_i on one of the stages to meet the gain specification.

The higher order cascaded **prototype high pass filter** has the following transfer function.

$$H(s) = \frac{(-s)^n}{(s + 1)^n}, \quad \omega_{cn} = \frac{1}{\sqrt[n]{\sqrt{2} - 1}}$$

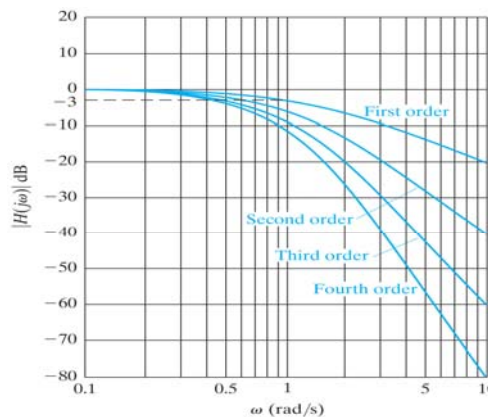
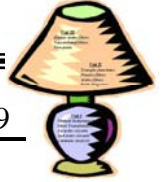


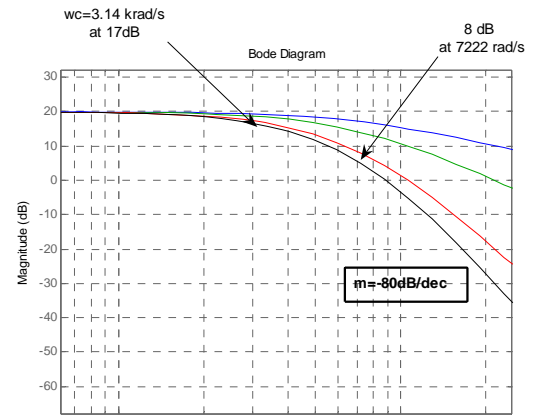
Figure: 15-16

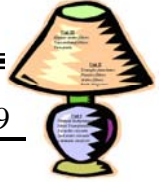
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In-Class Activity 15.4.1:

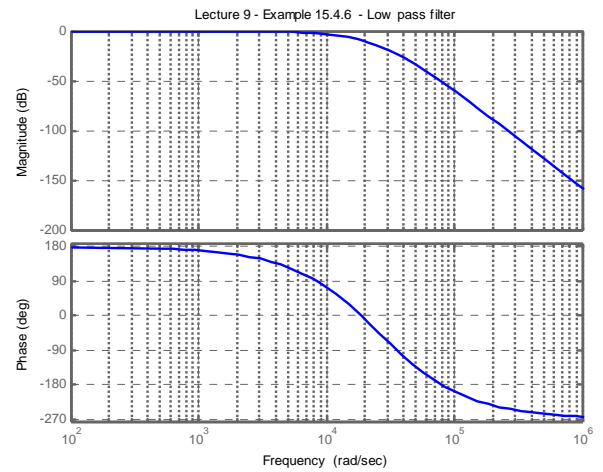
Design a fourth-order low-pass filter with a cutoff frequency of 500 Hz and a passband gain of 10. Use 0.01 μ F capacitors. Sketch the Bode magnitude plot for this filter.

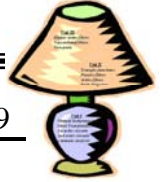




In-Class Activity 15.4.2:

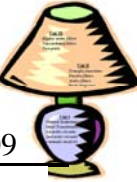
Using the template at the end of this lecture, design a low pass filter with a cut off frequency of 10 krad/s and a gain of no more than -60 dB at 100 krad/s. Use 0.01 μF capacitors in your design.





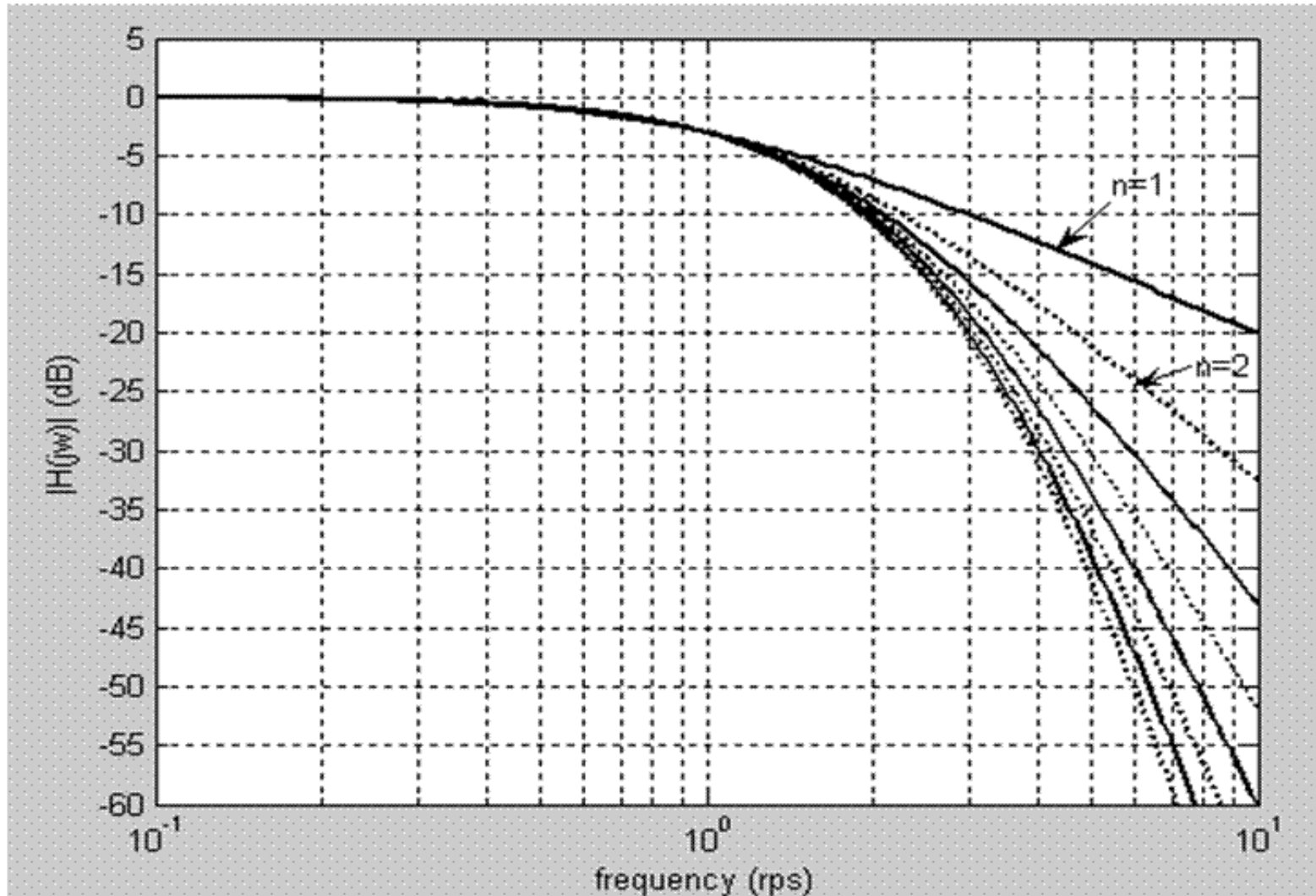
In-Class Activity 15.4.3:

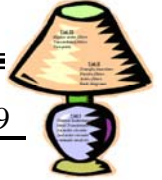
Using the template at the end of this lecture, design a high pass filter with a cut off frequency of 3 krad/s and a gain of no more than -15 dB at 1 krad/s. Use 0.1 μ F capacitors in your design.



#2: Cascade of First Order Lowpass

Cascade normalized to -3dB frequency of 1 rps





Lecture 9-1: Butterworth filters

Reading: 15.4

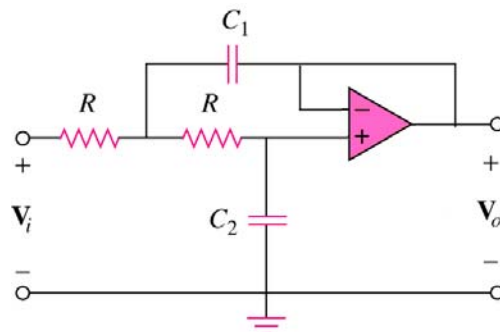
- Objectives: To introduce Butterworth filters (low-pass, high-pass, bandpass, bandreject)
 To define, design and perform calculations with Butterworth filters
 To determine the order of Butterworth order filters using a template or formula

An alternate method for designing higher order filters involves the use of **Butterworth filters**. The advantage of Butterworth filters is that there is a sharper transition between the passband and stopband. A unity-gain **Butterworth low-pass filter** has a transfer function whose magnitude is given by,

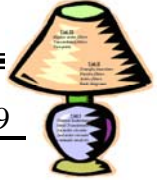
$$|H(j\omega)|_{dB} = 20\log_{10} \frac{1}{\sqrt{1 + (\omega/\omega_c)^{2n}}} = -10\log_{10}(1 + (\omega/\omega_c)^{2n})$$

Table 15.1 lists the Butterworth polynomials up to $n = 8$ which are used to design Butterworth filters scaled to $\omega_c = 1$ rad/s.

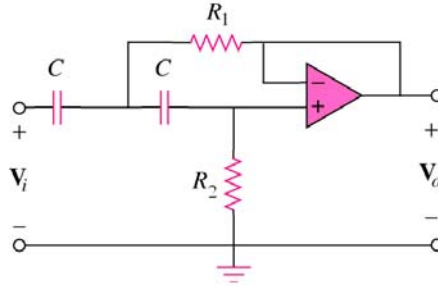
Note that most of the Butterworth filters are a product of first and second-order filters. The first order transfer function can be produced using a low-pass op amp filter. The following circuit provides the second-order transfer function for the low-pass Butterworth filter cascade.



	Prototype filter
General formulas, $\omega_c = 1$ rad/s	$\omega_c = 1$ rad/s, $R = 1\Omega$
$b_1 = 2/(RC_1)$	$b_1 = 2/C_1$
$R^2C_1C_2 = 1$	$C_1C_2 = 1$
$H(s) = \frac{1}{s^2 + b_1s + 1} = \frac{1/(R^2C_1C_2)}{s^2 + (2/RC_1)s + 1/(R^2C_1C_2)}$	$H(s) = \frac{1}{s^2 + b_1s + 1} = \frac{1/(C_1C_2)}{s^2 + (2/C_1)s + 1/(C_1C_2)}$



A second-order Butterworth high-pass filter circuit is shown in the following figure.



	Prototype filter
General formulas, $\omega_c = 1 \text{ rad/s}$	$\omega_c = 1 \text{ rad/s}, C = 1\text{F}$
$b_1 = 2/(R_2C)$	$b_1 = 2/R_2$
$R_1R_2C^2 = 1$	$R_1R_2 = 1$
$H(s) = \frac{s^2}{s^2 + b_1s + 1} = \frac{s^2}{s^2 + (2/R_2C)s + 1/(R_1R_2C^2)}$	$H(s) = \frac{s^2}{s^2 + b_1s + 1} = \frac{s^2}{s^2 + (2/R_2)s + 1/(R_1R_2)}$

$$|H(j\omega)|_{\text{dB}} = 20\log_{10} \frac{1}{\sqrt{1 + (\omega_c/\omega)^{2n}}} = -10\log_{10}(1 + (\omega_c/\omega)^{2n})$$

Figure 15.24 illustrates another method used to describe a higher-order filter based upon the four variables: $A_p, A_s, \omega_p, \omega_s$.

$$A_s = |H(j\omega_s)|_{\text{dB}} = 20\log_{10} \frac{1}{\sqrt{1 + \omega_s^{2n}}} = -10\log_{10}(1 + \omega_s^{2n})$$

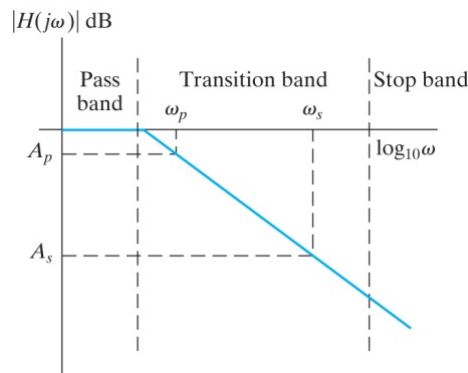
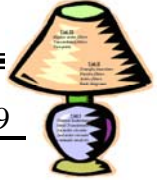


Figure: 15-24
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If we assume that ω_p is the cut off frequency then the order of the Butterworth filter is

$$n = \frac{-0.05A_s}{\log_{10}(\omega_s/\omega_c)} \quad (\text{Low Pass Filter}) \qquad n = \frac{-0.05A_s}{\log_{10}(\omega_c/\omega_s)} \quad (\text{High Pass Filter})$$



Advantages of the Butterworth filter versus the cascaded filter

- i. As the order of the Butterworth increases the cutoff frequency remains the same
- ii. A lower-order Butterworth filter achieves the same design specification as a higher order cascaded filter
- iii. It takes less operational amplifiers to implement a Butterworth filter with the same order as the cascaded filter

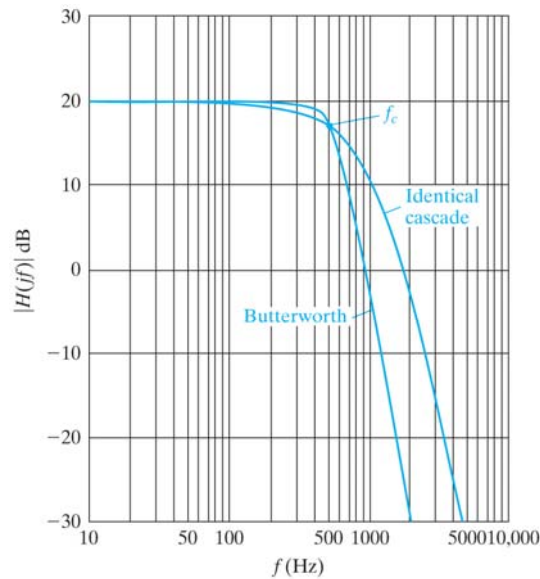
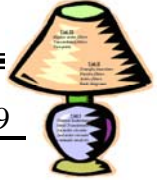


Figure: 15-23Ex15.9

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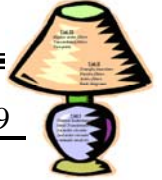
Example 15.4.1 (Butterworth order low-pass):

Determine the order of a low-pass Butterworth filter that has a cutoff frequency of 1 kHz and a gain of no more than -50 dB at 6 kHz. What is the actual gain in dB at 6 kHz?



Example 15.4.2 (Butterworth low-pass filter):

- a) Using $1\text{ k}\Omega$ resistors and ideal op amps, design a circuit that will implement the low pass Butterworth filter specified in Example 15.4.1. The gain in the passband is one.
- b) Draw the circuit diagram of the filter and label all component values.

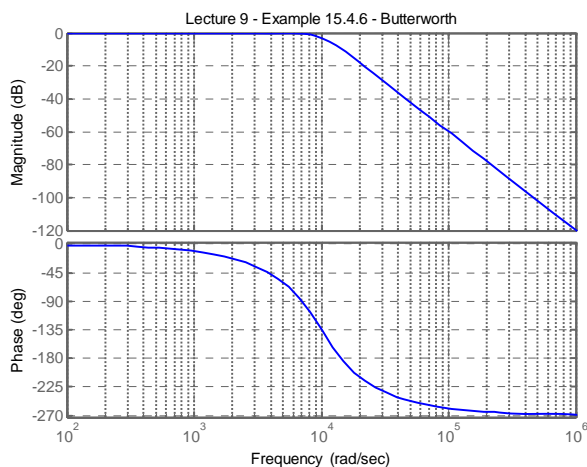


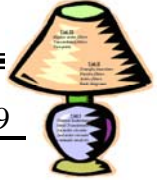
Example 15.4.3 (Butterworth order high-pass):

Determine the order of a high-pass Butterworth filter that has a cutoff frequency of 7 kHz and a gain of at least -30 dB at 2 kHz. What is the actual gain, in decibels, at 2 kHz?

Example 15.4.4 (Butterworth high-pass filter):

- Using 10 nF capacitors and ideal op amps, design a circuit that will implement the filter specified in Example 15.4.3. The gain in the passband is one.
- Draw the circuit diagram of the filter and label all component values.

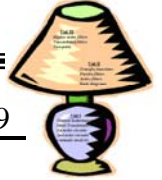




Example 15.4.5(broadband Butterworth bandpass filter):

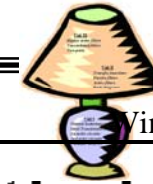
- a) Design a broadband Butterworth bandpass filter with a lower cutoff frequency of 500 Hz and an upper cutoff frequency of 4.5 kHz. The passband gain of the filter is 20 dB. The gain should be down at least 20 dB at 200 Hz and 11.25 kHz. Use 15 nF capacitors in the high-pass circuit and 10 k Ω resistors in the low-pass circuit.

- b) Draw the circuit diagram of the filter and label all component values.

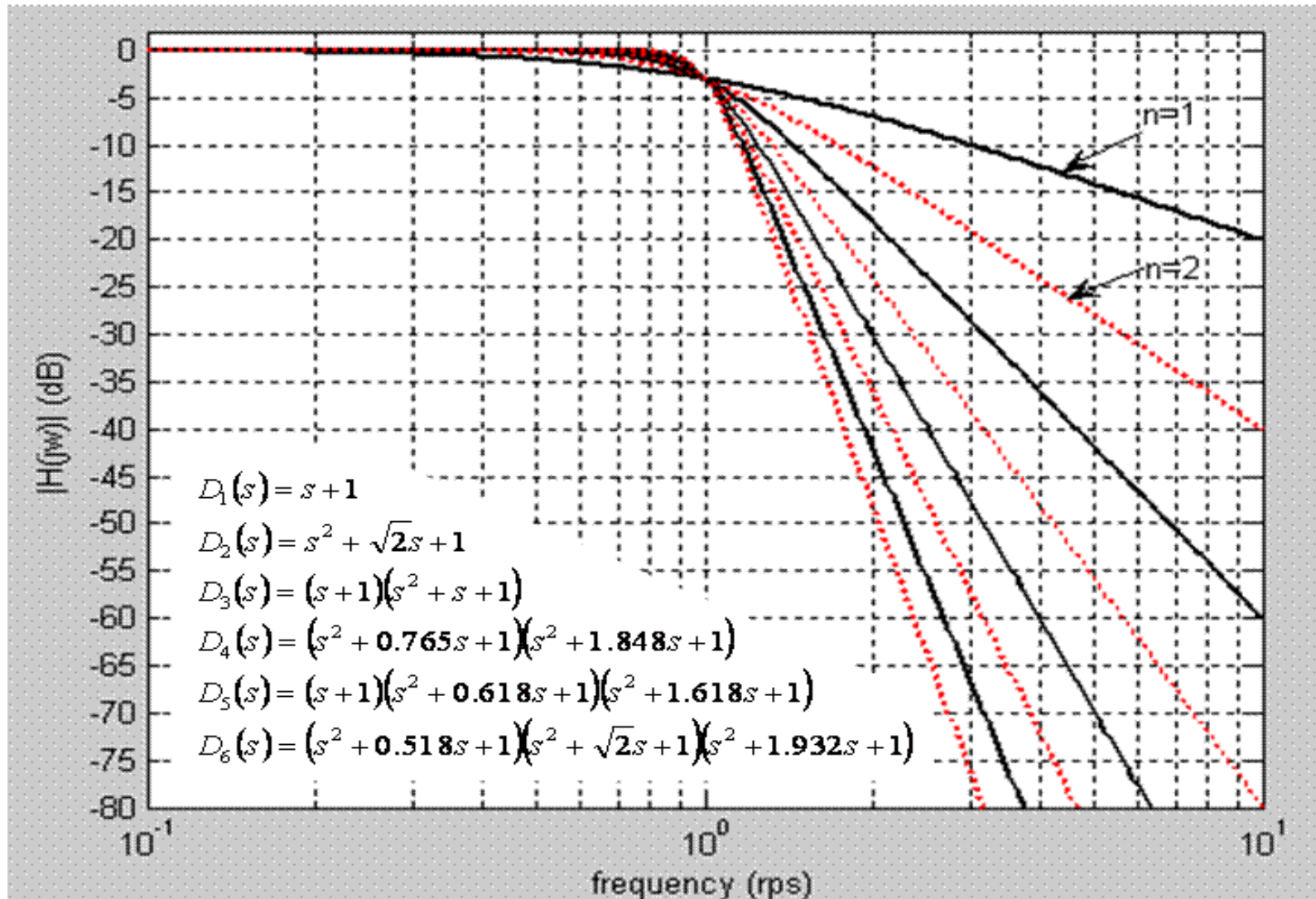


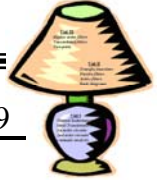
Example 15.4.6 (broadband Butterworth bandreject filter):

- a) Design a broadband Butterworth bandreject filter with a lower cutoff frequency of 1.0 kHz and an upper cutoff frequency of 10.0 kHz. The passband gain of the filter is 40 dB. The gain should be down at least 40 dB at 5.0 kHz and 6.0 kHz. Use 10 μF capacitors in the high-pass circuit and 1 k Ω resistors in the low-pass circuit.
- b) Draw the circuit diagram of the filter and label all component values.



#3: Normalized Butterworth LPF





Lecture 9-2: Narrowband bandpass and bandreject filters

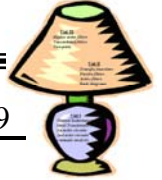
Reading: 15.5

Objectives: To analyze narrowband bandpass and bandreject filters to determine the cutoff frequencies, resonant frequency, gain, bandwidth and quality factor
To design narrowband filters to meet certain design specifications

15.5 Narrowband Bandpass and Bandreject Filters

Cascaded active filters with **high quality factors (narrowband filters)** require operational amplifiers. Unlike the broadband filters, these filters do not have discrete poles but complex poles. A narrowband or high Q filter must have $Q = \omega_0/\beta > 0.5$. The narrowband bandpass and band reject prototypes are provided on the following table.

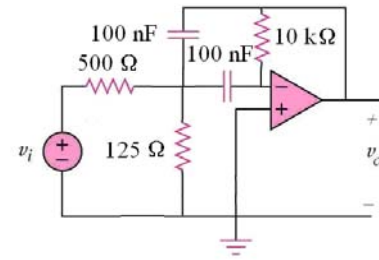
Bandpass	Bandreject (twin-T notch filter)
$H(s) = \frac{K\beta s}{s^2 + \beta s + \omega_0^2}$	$H(s) = \frac{(s^2 + \omega_0^2)}{s^2 + \beta s + \omega_0^2}$
$H(s) = \frac{-s/(R_1 C)}{s^2 + 2/(R_3 C)s + 1/(R_{eq} R_3 C^2)}$	$H(s) = \frac{s^2 + 1/(R^2 C^2)}{s^2 + \left(4(1-\sigma)/RC\right)s + 1/(R^2 C^2)}$
$\beta = 2/(R_3 C)$	$\beta = 4(1 - \sigma)/(RC)$
$\omega_0^2 = 1/(R_{eq} R_3 C^2)$	$\omega_0^2 = 1/(R^2 C^2)$
$R_{eq} = R_1 \parallel R_2$ $K\beta = -1/(R_1 C)$ $K = -R_3/(2R_1) = V_o/V_i$ (inverting amplifier)	$\sigma = 1 - \beta/(4\omega_0)$
prototype assuming $C = 1 \text{ F}$, $\omega_0 = 1 \text{ rad/s}$	
$R_1 = Q/ K $, $R_2 = Q/(2Q^2 - K)$, $R_3 = 2Q$	$R = 1 \Omega$, $\sigma = 1-1/(4Q)$

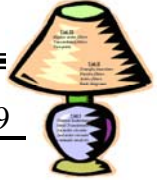


Example 15.5.1:

For the following circuit,

- a) determine the transfer function, $H(s)$
- b) what is the gain?
- c) what is the bandwidth?
- d) what is the resonant frequency?
- e) what is the quality factor?

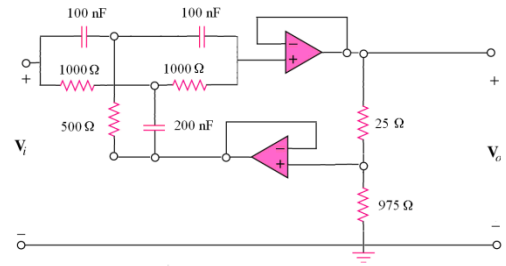


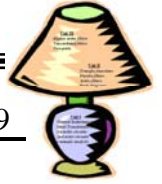


Example 15.5.2:

For the following circuit,

- determine the transfer function, $H(s)$
- what is the gain?
- what is the bandwidth?
- what is the resonant frequency?
- what is the quality factor?





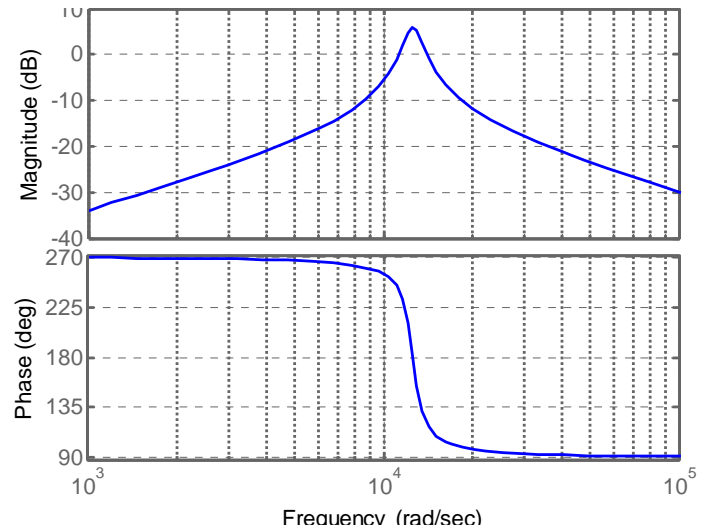
Lecture 9-3: Narrowband bandpass and bandreject filters

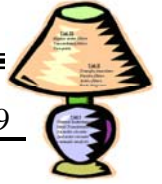
Reading: 15.5

Objectives: To analyze narrowband bandpass and bandreject filters to determine the cutoff frequencies, resonant frequency, gain, bandwidth and quality factor
To design narrowband filters to meet certain design specifications

Example 15.5.1(Narrowband bandpass):

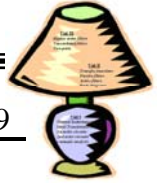
Design a bandpass filter which has a center frequency of 2 kHz, a quality factor of 8, and the pass band gain with a magnitude of 2. Use 0.01 μF capacitors in your design. Compute the transfer function of the circuit and sketch the Bode plot for the magnitude response.





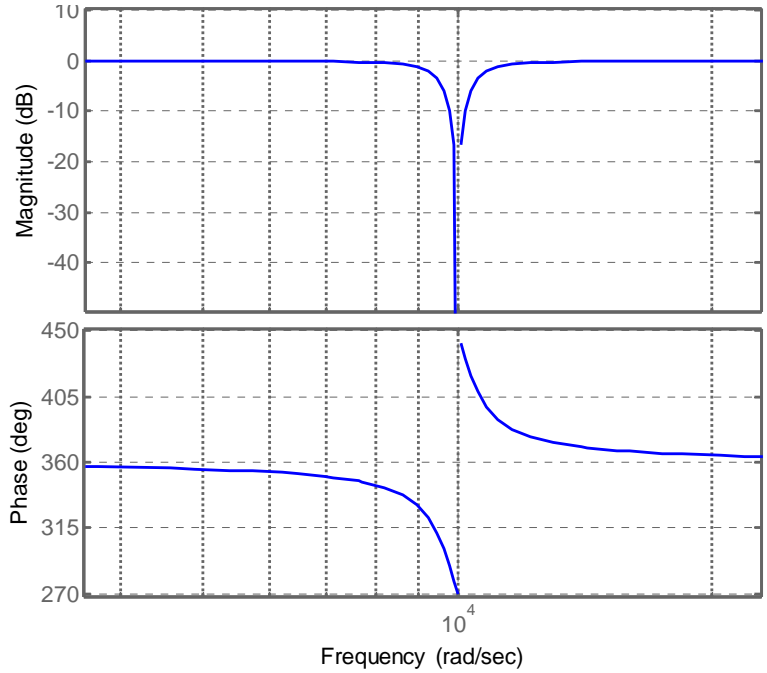
Example 15.5.2(Narrowband bandpass):

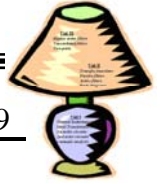
Design an active bandpass filter with $Q = 8$, $|K| = 5$, and $\omega_o = 1000$ rad/s. Use $1 \mu\text{F}$ capacitor and specify the value of the resistors.



Example 15.5.3(Narrowband bandreject):

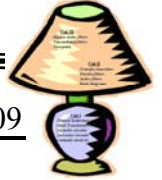
Design an active bandreject filter with a center frequency of 10 krad/s and a bandwidth of 1.250 krad/s. Use 0.01 μF capacitors in the design.





Example 15.5.4(Narrowband bandreject):

Design an active unity-gain bandreject filter with $\omega_o = 1000$ rad/s and $Q = 4$. Use $2 \mu\text{F}$ capacitors and specify the value of the resistors and σ .



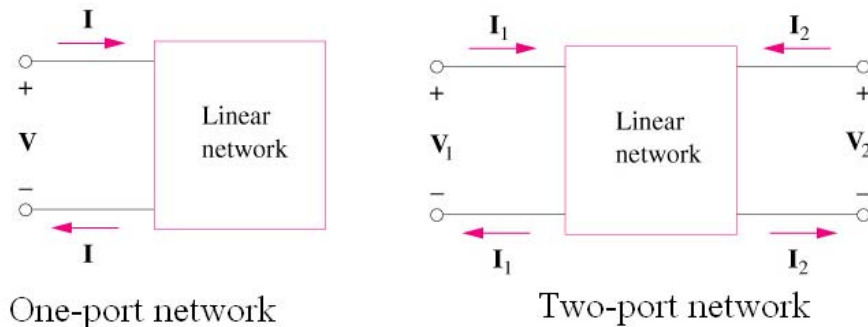
Lecture 10-1: The Terminal Equations and Two-Port Parameters

Reading: 18.1-2

- Objectives:
- To introduce two-port circuits
 - To define terminal equations and two-port parameters
 - To define the terms: symmetrical, reciprocal
 - To define the terms: immittance, transmission, and hybrid parameters

Terminal pairs on a circuit where signals are either fed in or extracted are referred to as the **ports** of the system.

A **two-port network** is an electrical network with two separate ports for input and output.



Restrictions:

- i. there can be no energy stored within the circuit
- ii. there can be no independent sources within the circuit
- iii. the current into the port must equal the current out of the port
- iv. all external connections must be made to either the input port or the output port

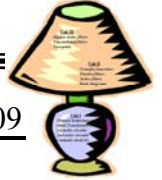
Similar to the fundamental principle of analysis for Thevenin and Norton equivalence and operational amplifiers only the terminal variables (I_1, I_2, V_1, V_2) are of interest not the currents and voltages inside the circuit.

The terms that relate the voltages and currents of a two-port network are the **parameters**.

The **open-circuit impedance parameters** or **z parameters** have the units of ohms. These are found by open-circuiting the input or output port.

$$z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} \quad z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} \quad z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} \quad z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0}$$

- z_{11} = open-circuit input impedance
- z_{12} = open-circuit transfer impedance from port 1 to port 2
- z_{21} = open-circuit transfer impedance from port 2 to port 1
- z_{22} = open-circuit output impedance



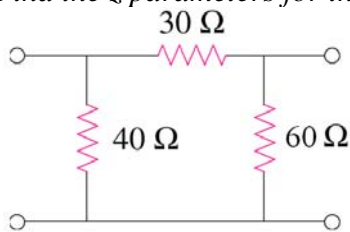
When $z_{11} = z_{22}$ the two-port network is said to be **symmetric**. When the two-port network is linear and has no dependent sources, the transfer impedances are equal ($z_{12} = z_{21}$) and the two-port network is said to be **reciprocal**. Any two-port network made entirely of resistors, capacitors, and inductors must be **reciprocal**.

The impedance parameters can also be described using the following matrix relationship.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

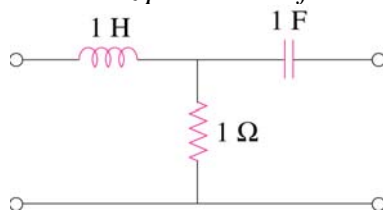
Example 18.1.1:

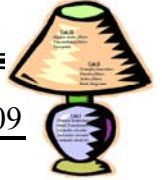
Find the z parameters for the following circuit.



Example 18.1.2:

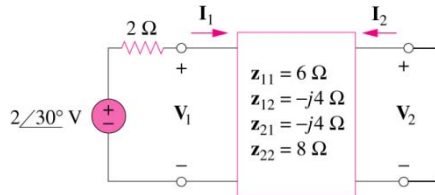
Find the z parameters for the following circuit.





Example 18.1.3

Find I_1 and I_2 in the following circuit.



Admittance parameters or y parameters have the units of Siemens. The values are determined by setting $V_1 = 0$ or $V_2 = 0$ (input and output short-circuited). These parameters are also called the short-circuit admittance parameters.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0}$$

$$y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0}$$

$$y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0}$$

$$y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0}$$

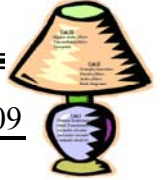
y_{11} = short-circuit input admittance

y_{12} = short-circuit transfer admittance from port 2 to port 1

y_{21} = short-circuit transfer admittance from port 1 to port 2

y_{22} = short-circuit output admittance

The impedance and admittance parameters are collectively referred to as the immittance parameters.

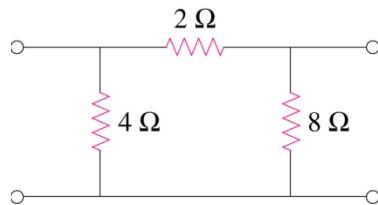


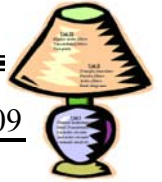
The convert between the immittance parameters, use the following

<u>y to z parameters</u>	<u>z to y parameters</u>
$z_{11} = \frac{y_{22}}{\Delta y}$	$y_{11} = \frac{z_{22}}{\Delta z}$
$z_{12} = \frac{-y_{12}}{\Delta y}$	$y_{12} = \frac{-z_{12}}{\Delta z}$
$z_{21} = \frac{-y_{21}}{\Delta y}$	$y_{21} = \frac{-z_{21}}{\Delta z}$
$z_{22} = \frac{y_{11}}{\Delta y}$	$y_{22} = \frac{z_{11}}{\Delta z}$
$\Delta y = y_{11}y_{22} - y_{12}y_{21}$	$\Delta z = z_{11}z_{22} - z_{12}z_{21}$

Example 18.1.4:

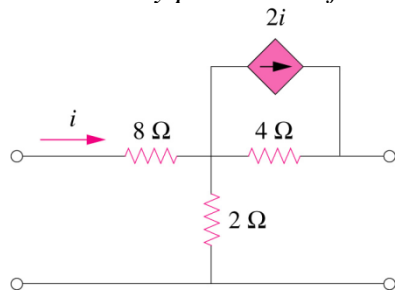
Obtain the y parameters for the following circuit.

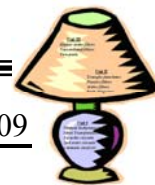




Example 18.1.5:

Obtain the y parameters for the following circuit.





Lecture 10-2: The Terminal Equations and Two-Port Parameters
Analysis of Terminated two-port circuits
Reading: 18.1-3

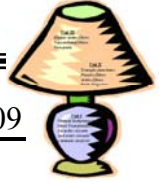
- Objectives:
- To introduce two-port circuits
 - To define terminal equations and two-port parameters
 - To define the terms: symmetrical, reciprocal
 - To define the terms: immittance, transmission, and hybrid parameters

The a and b parameters are called the transmission parameters because they describe the voltage and current at one end of the two-port network in terms of the voltage and current at the other end.

$V_1 = a_{11}V_2 - a_{12}I_2$ $I_1 = a_{21}V_2 - a_{22}I_2$	$V_2 = b_{11}V_1 - b_{12}I_1$ $I_2 = b_{21}V_1 - b_{22}I_1$
$a_{11} = \left. \frac{V_1}{V_2} \right _{I_2=0}$ open-circuit voltage ratio	$b_{11} = \left. \frac{V_2}{V_1} \right _{I_1=0}$ open-circuit voltage gain
$a_{12} = -\left. \frac{V_1}{I_2} \right _{V_2=0}$ negative short-circuit transfer impedance (Ω)	$b_{12} = -\left. \frac{V_2}{I_1} \right _{V_1=0}$ negative short-circuit transfer impedance (Ω)
$a_{21} = \left. \frac{I_1}{V_2} \right _{I_2=0}$ open-circuit transfer admittance (S)	$b_{21} = \left. \frac{I_2}{V_1} \right _{I_1=0}$ open-circuit transfer admittance (S)
$a_{22} = -\left. \frac{I_1}{I_2} \right _{V_2=0}$ negative short-circuit current ratio	$b_{22} = -\left. \frac{I_2}{I_1} \right _{V_1=0}$ negative short-circuit current gain

Example 18.3.1:

The following measurements pertain to a two-port circuit operating in the sinusoidal steady state. With port 2 open, a voltage equal to $150 \cos 4000t$ V is applied to port 1. The current into port 1 is $25 \cos(4000t - 45^\circ)$ A, and the port 2 voltage is $100 \cos(4000t + 15^\circ)$ V. With port 2 short-circuited, a voltage equal to $30 \cos 4000t$ V is applied to port 1. The current into port 1 is $1.5 \cos(4000t + 30^\circ)$ A, and the current into port 2 is $0.25 \cos(4000t + 150^\circ)$ A. Find the a parameters that can describe the sinusoidal steady-state behavior of the circuit.

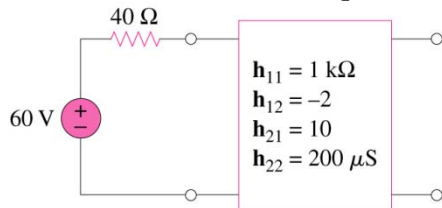


The h and g parameters relate cross variables, that is an input voltage and output current to an output voltage and input current. The h and g parameters are called hybrid parameters. The g parameters are also called inverse hybrid parameters.

$V_1 = h_{11}I_1 + h_{12}V_2$ $I_2 = h_{21}I_1 + h_{22}V_2$	$I_1 = g_{11}V_1 + g_{12}I_2$ $V_2 = g_{21}V_1 + g_{22}I_2$
$h_{11} = \left. \frac{V_1}{I_1} \right _{V_2=0}$ short-circuit input impedance (Ω)	$g_{11} = \left. \frac{I_1}{V_1} \right _{I_2=0}$ open-circuit input admittance (S)
$h_{12} = \left. \frac{V_1}{V_2} \right _{I_1=0}$ open-circuit reverse voltage gain	$g_{12} = \left. \frac{I_1}{I_2} \right _{V_1=0}$ short-circuit reverse current gain
$h_{21} = \left. \frac{I_2}{I_1} \right _{V_2=0}$ short-circuit forward current gain	$g_{21} = \left. \frac{V_2}{V_1} \right _{I_2=0}$ open-circuit forward voltage gain
$h_{22} = \left. \frac{I_2}{V_2} \right _{I_1=0}$ open-circuit output admittance (S)	$g_{22} = \left. \frac{V_2}{I_2} \right _{V_1=0}$ short-circuit output impedance (Ω)

Example 18.3.2:

Determine the Thevenin equivalent at the output port of the following circuit.



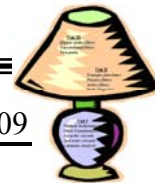


Table 18.1 in the textbook describes all of the relationships between the two-port parameters.

If a two-port circuit is **reciprocal** then the following relationships must exist.

- i. $Z_{12} = Z_{21}$
- ii. $y_{12} = y_{21}$
- iii. $a_{11}a_{22} - a_{12}a_{21} = \Delta a = 1$
- iv. $b_{11}b_{22} - b_{12}b_{21} = \Delta b = 1$
- v. $h_{12} = -h_{21}$
- vi. $g_{12} = -g_{21}$

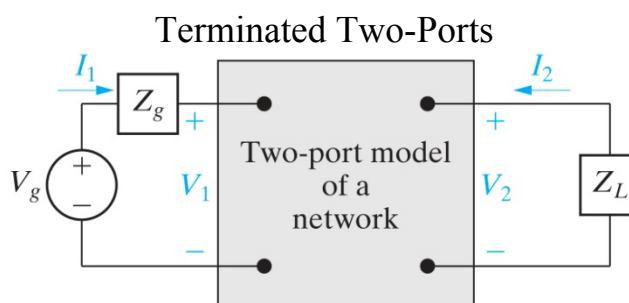
A two-port circuit is **reciprocal** if the interchange of an ideal voltage source at one port with an ideal ammeter at the other port produces the same ammeter reading. A two-port circuit is also reciprocal if the interchange of an ideal current source at one port with an ideal voltmeter at the other port produces the same voltmeter reading.

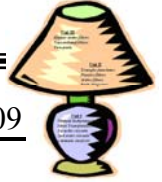
A reciprocal two-port circuit is **symmetric** if its ports can be interchanged without disturbing the values of the terminal currents and voltages. Figure 18.6 in the textbook shows four examples of symmetric two-port circuits. The following relationships exist among the port parameters.

- i. $Z_{11} = Z_{22}$
- ii. $y_{11} = y_{22}$
- iii. $a_{11} = a_{22}$
- iv. $b_{11} = b_{22}$
- v. $h_{11}h_{22} - h_{12}h_{21} = \Delta h = 1$
- vi. $g_{11}g_{22} - g_{12}g_{21} = \Delta g = 1$

18.3 Analysis of the Terminated Two-Port Circuit

In a typical application of a two-port network, the circuit is driven at port 1 and loaded at port 2. The following figure shows an example of a terminated two-port model.



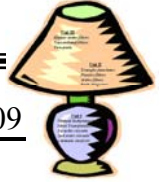


The six characteristics of the terminated two-port circuit define its terminal behavior:

- the input impedance $Z_{in} = V_1/I_1$ or the admittance $Y_{in} = I_1/V_1$
- the output current I_2
- the Thevenin voltage and impedance with respect to port 2
- the current gain I_2/I_1
- the voltage gain V_2/V_1
- the voltage gain V_2/V_g

Table 18.2 in the textbook includes all of the terminated two-port equations. The \underline{z} and \underline{y} parameters for a terminated two-port are given below. The following table illustrates the \underline{y} and \underline{z} parameters for the terminated two-port model.

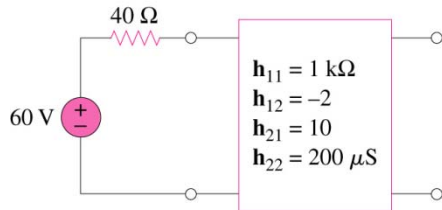
		z parameters	y parameters
input impedance/ admittance	Z_{in}/Y_{in}	$z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$	$y_{11} - \frac{y_{12}y_{21}Z_L}{1 + y_{22}Z_L}$
output current	I_2	$\frac{-z_{21}V_g}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$	$\frac{y_{21}V_g}{1 + y_{22}Z_L + y_{11}Z_g + \Delta y Z_g Z_L}$
Thevenin voltage	V_{th}	$\frac{z_{21}}{(z_{11} + Z_g)}V_g$	$\frac{-y_{21}}{(y_{22} + \Delta y Z_g)}V_g$
Thevenin impedance	Z_{th}	$z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g}$	$\frac{1 + y_{11}Z_g}{y_{22} + \Delta y Z_g}$
current gain	I_2/I_1	$\frac{-z_{21}}{z_{22} + Z_L}$	$\frac{y_{21}}{y_{11} + \Delta y Z_L}$
voltage gain	V_2/V_1	$\frac{z_{21}Z_L}{z_{11}Z_L + \Delta z}$	$\frac{-y_{21}Z_L}{1 + y_{22}Z_L}$
voltage gain	V_2/V_g	$\frac{z_{21}Z_L}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$	$\frac{y_{21}Z_L}{y_{12}y_{21}Z_g Z_L - (1 + y_{11}Z_g)(1 + y_{22}Z_L)}$
determinant of parameter matrix		$\Delta z = z_{11}z_{22} - z_{12}z_{21}$	$\Delta y = y_{11}y_{22} - y_{12}y_{21}$



Example 18.3.3:

For the following circuit, which is the same as Example 18.3.2,

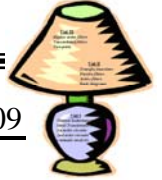
- determine the value of a load resistor placed across the output port for maximum power transfer
- For the selected resistor, what is the value of the power delivered to the load?



Conversion from h to y parameters

$$\Delta h = h_{11}h_{22} - h_{12}h_{21}$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{\Delta h} & \frac{h_{22}}{h_{11}} \end{bmatrix}$$



Lecture 10-3: Analysis of Terminated two-port circuits

Reading: 18.3

Objectives: To analyze terminated two-port circuits

Example 18.3.1:

The following two-port circuit is described in terms of the following y parameters.

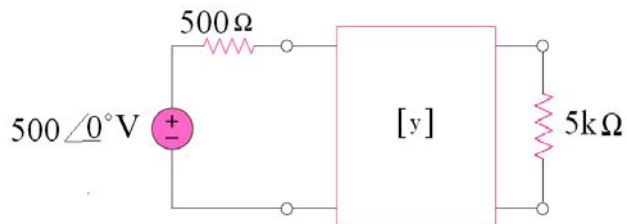
$$y_{11} = 0.15 \text{ S}$$

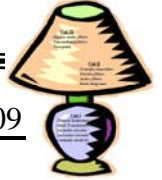
$$y_{21} = -0.25 \text{ S}$$

$$y_{12} = -0.05 \text{ S}$$

$$y_{22} = 0.25 \text{ S}$$

- a) Find the phasor voltage, V_2
- b) Find the average power deliver to the $5k\Omega$ load.
- c) Find the average power delivered to the input port.





Example 18.3.2:

The a parameters and source and load characteristics of a terminated two port network are:

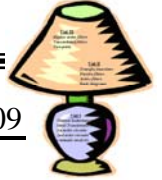
$$a_{11} = 0.5m \qquad a_{12} = 10\Omega \qquad a_{21} = 1\mu S \qquad a_{22} = -30m$$

$$V_g = 50\angle 0^\circ mV \qquad Z_g = 100\Omega \qquad Z_L = 5k\Omega$$

- a) Calculate the average power delivered to the load resistor
- b) Calculate the load resistance for maximum average power
- c) Calculate the maximum average power delivered to the resistor in b)

Conversions from \underline{a} to \underline{z} parameters ($\Delta a = a_{11}a_{22} - a_{12}a_{21}$)

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} \frac{a_{11}}{a_{21}} & \frac{\Delta a}{a_{21}} \\ \frac{1}{a_{21}} & \frac{a_{22}}{a_{21}} \end{bmatrix}$$



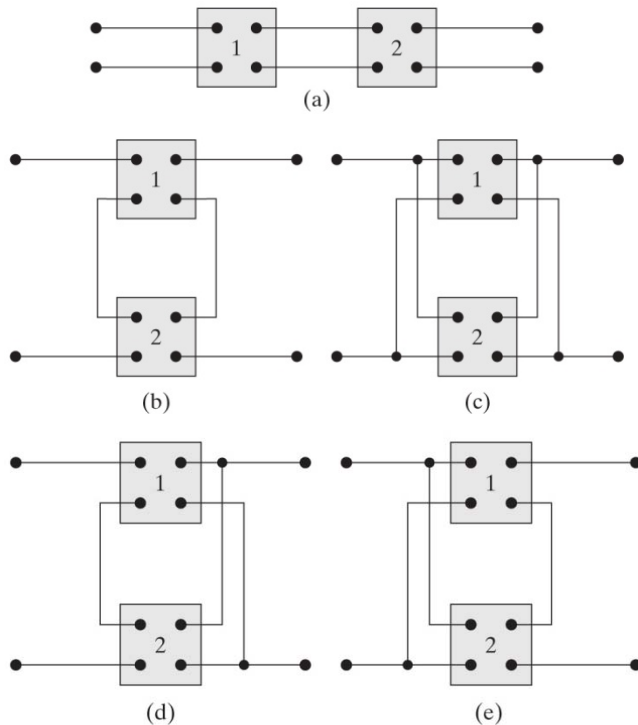
18.3 Interconnected Two-Port Circuits

Synthesizing a large, complex system is usually made easier by first designing subsections of the system. If the subsections are modeled by two-port circuits, synthesis involves the analysis of interconnected two-port circuits.

Two-port circuits may be interconnected five basic configurations:

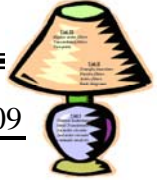
- a) cascade (port 2 feeds into port 1 of the second network)
- b) series-series (port 1a is in series with port 1b, port 2a is in series with port 2b)
- c) parallel-parallel (port 1a is in parallel with port 1b, port 2a is in parallel with port 2b)
- d) series-parallel (port 1a is in series with port 1b, port 2a is in parallel with port 2b)
- e) parallel-series (port 1a is in parallel with port 1b, port 2a is in series with port 2b)

These five basic configurations are shown in the following figure.



Using the corresponding two-port matrix representations, complicated networks can be analyzed by connecting elementary two ports.

- | | | |
|----------------------------------|-------------------|-----------------------|
| a) cascaded connection: | use a-parameters: | $a = [a_1][a_2]$ |
| b) series-series connection: | use z-parameters: | $z = [z_1] + [z_2]$ |
| c) parallel-parallel connection: | use y-parameters: | $[y] = [y_1] + [y_2]$ |
| d) series-parallel connection: | use h-parameters: | $[h] = [h_1] + [h_2]$ |
| e) parallel-series connection: | use g-parameters: | $[g] = [g_1] + [g_2]$ |

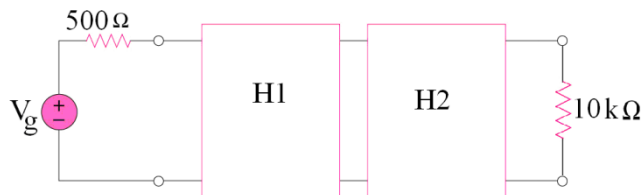


Example 18.3.1 (cascaded connection):

Two identical amplifiers are connected in cascade, as shown in the following Figure. Each amplifier is described in terms of its h parameters. The h parameters are:

$$h_{11} = 1 \text{ k}\Omega \qquad h_{12} = 0.15\text{m} \qquad h_{21} = 100 \qquad h_{22} = 100\mu\text{S}$$

Find the voltage gain V_2/V_g



Step 1. Convert the $[h]$ parameters to the $[a]$ parameters.

$$a_{11}' = -\frac{\Delta h}{h_{21}} \quad a_{12}' = -\frac{h_{11}}{h_{21}} \quad a_{21}' = -\frac{h_{22}}{h_{21}} \quad a_{22}' = -\frac{1}{h_{21}}$$

Step 2. Convert the $[a]$ parameters to the interconnected cascaded network,

$$[a] = [a'] [a'']$$

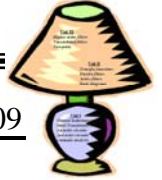
$$a_{11} = a_{11}' a_{11}'' + a_{12}' a_{21}''$$

$$a_{12} = a_{11}' a_{12}'' + a_{12}' a_{22}''$$

$$a_{21} = a_{21}' a_{11}'' + a_{22}' a_{21}''$$

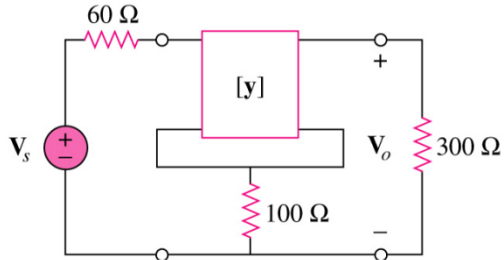
$$a_{22} = a_{21}' a_{12}'' + a_{22}' a_{22}''$$

Step 3. Find the voltage gain, $\frac{V_2}{V_g} = \frac{Z_L}{(a_{11} + a_{21} Z_g) Z_L + a_{12} + a_{22} Z_g}$



Example 18.3.2 (series-series connection):

In the following figure, let $y_{12} = y_{21} = 0$, $y_{11} = 2 \text{ mS}$, and $y_{22} = 10 \text{ mS}$. Find V_o/V_s .



ans: .09375

Step 1. Convert the $[y]$ parameters to $[z]$ parameters

$$z_{11}' = \frac{y_{22}}{\Delta y} \quad z_{12}' = -\frac{y_{12}}{\Delta y} \quad z_{21}' = -\frac{y_{21}}{\Delta y} \quad z_{22}' = \frac{y_{11}}{\Delta y}$$

$$\Delta y = y_{11}y_{22} - y_{12}y_{21}$$

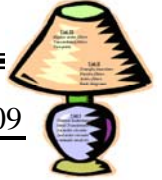
Step 2. The 100Ω resistor under the $[y]$ two-port has

$$z_{11}'' = z_{12}'' = z_{21}'' = z_{22}'' = 100\Omega$$

Step 3. Find the $[z]$ parameters for the overall interconnected series network

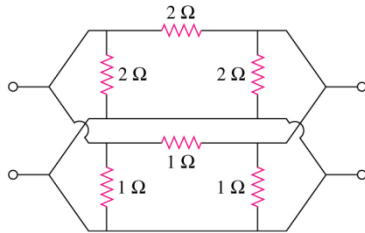
$$[z] = [z_a] + [z_b]$$

Step 4. Find the voltage gain, $\frac{V_o}{V_s} = \frac{z_{21}Z_L}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$



Example 18.3.3 (parallel-parallel connection):

Obtain the h parameters for the following network.



ans: 0.167Ω, 0.5, 0.5, 4.5S

- Step 1. Obtain the y parameters for the top network $[y_a]$
 Step 2. Obtain the y parameters for the bottom network $[y_b]$
 Step 3. Find the y parameters for the overall interconnected parallel network
 $[y] = [y_a] + [y_b]$
 Step 4. Convert the y parameters to h parameters
 $h_{11} = 1/y_{11}$ $h_{12} = -y_{12}/y_{11}$ $h_{21} = y_{21}/y_{11}$ $h_{22} = \Delta y/y_{11}$