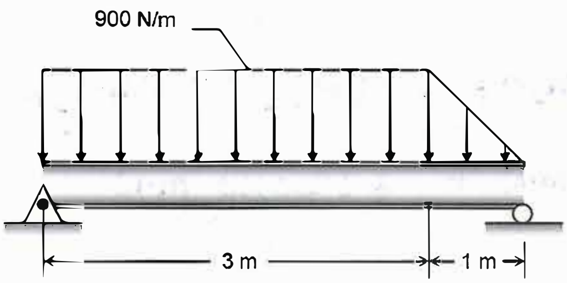
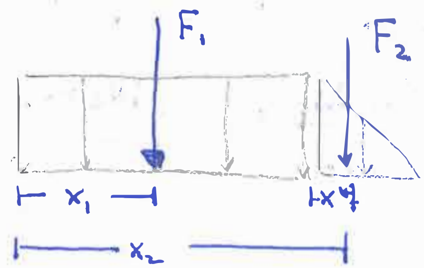
Example

For the simply supported beam below, replace the distributed load with a single force and give its location.





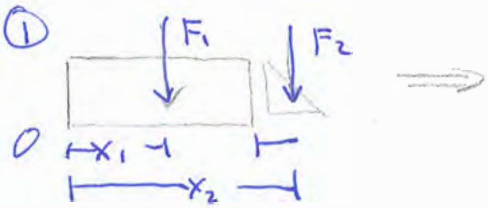
$F_1 = (900\ \frac{N}{m})(3m) = 2700\ N$

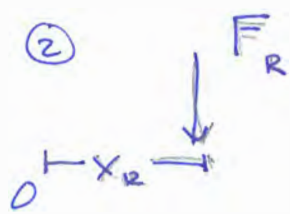
X_1 = CENTROID of RECTANGLE = $\frac{1}{2}\,3m = 1.5m$

F_2 = AREA of $\triangle$ = $\frac{1}{2}(900\ \frac{N}{m})(1m) = 450\ N$

X^* = CENTROID of $\triangle$ = $\frac{1}{3}(1) = \frac{1}{3}m$

$\Rightarrow X_2 = 3 + \frac{1}{3} = 3\frac{1}{3}\ m$

① 

② 

$F_R = F_1 + F_2 = 2700 + 450 = \boxed{3150\ N}$

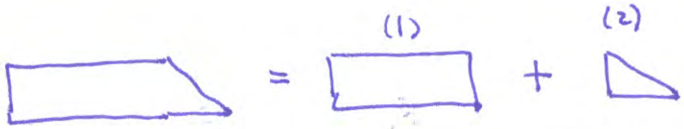
$\Sigma M_{O①} = \Sigma M_{O②}$

$X_1F_1 + X_2F_2 = X_R F_R \qquad X_R = \frac{X_1F_1 + X_2F_2}{F_R}$

$$X_R = \frac{(1.5m)(2700N) + (3\frac{1}{3}m)(450N)}{3150\ N} = \boxed{1.76\ m}$$

METHOD 2:

$$F_R = AREA_{\square} + AREA_{\triangle} = (900)(3) + \tfrac{1}{2}(900)(1)$$
$$= \boxed{3150\ N}$$

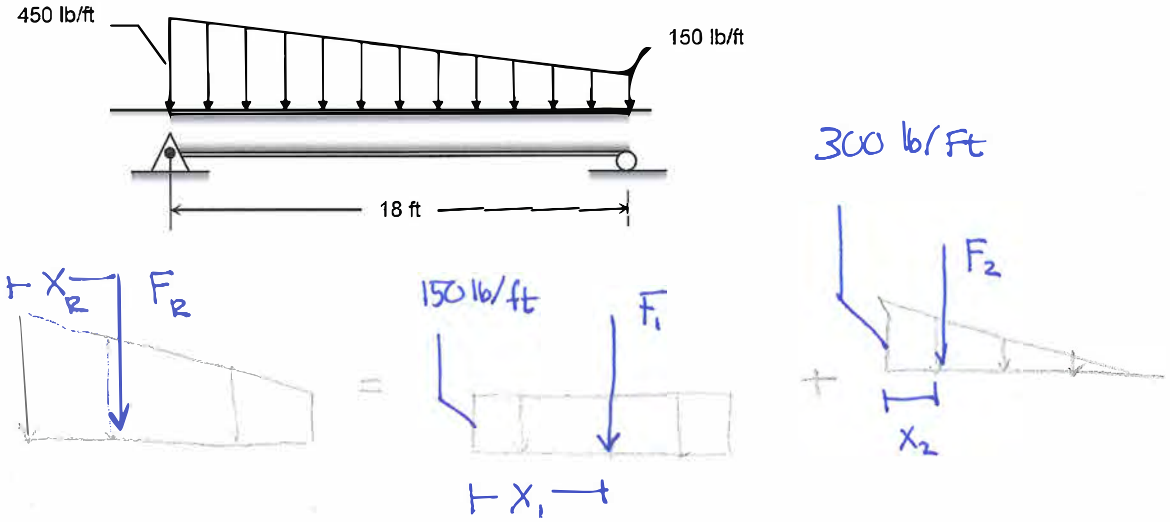


$$X_R = \frac{\Sigma x_i A_i}{\Sigma A_i} = \frac{(1\tfrac{1}{2}\,m)(3 \times 900\,N)m + (3\tfrac{1}{3}\,m)(\tfrac{1}{2} \cdot 900\,N \cdot 1)m}{(3 \times 900)\,m \cdot N + \tfrac{1}{2}(900)(1)\,m \cdot N}$$

$$= \boxed{1.76\ m}$$

Example

For the simply supported beam below, replace the distributed load with a single force and give its location.



$$F_1 = (150\,\text{lb/ft})(18\,\text{ft}) = 2700\,\text{lb}$$

$$x_1 = \frac{1}{2}(18\,\text{ft}) = 9\,\text{ft}$$

$$F_R = F_1 + F_2$$
$$= 5400\,\text{lb}$$

$$F_2 = \frac{1}{2}(300\,\text{ft/ft})(18\,\text{ft}) = 2700\,\text{lb}$$

$$x_2 = \frac{1}{3}(18\,\text{ft}) = 6\,\text{ft}$$

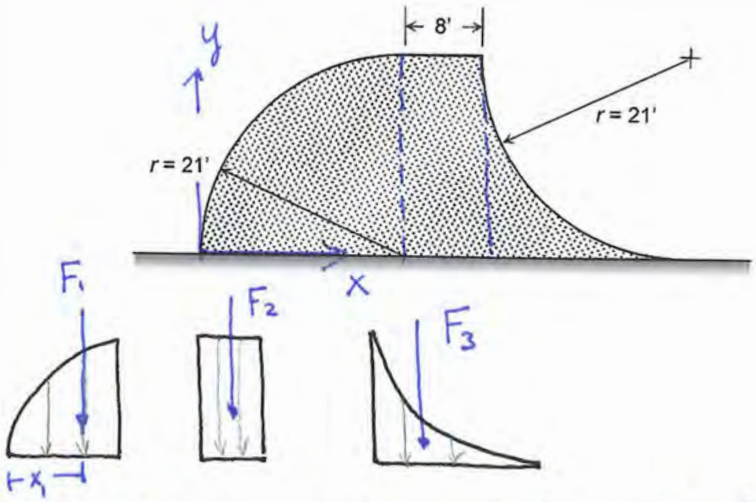
$$x_R F_R = x_1 F_1 + x_2 F_2$$

$$x_R = \frac{x_1 F_1 + x_2 F_2}{F_R} = \frac{(9')(2700\,\text{lb}) + (6')(2700\,\text{lb})}{5400\,\text{lb}}$$

$$= 7.5\,\text{ft}$$

Example

The concrete structure in the figure is suggested as a design for a dam. For a one-foot thickness, find the resultant weight of the dam and give its location. The specific weight of concrete can be taken to be $\gamma_c = 150\ \text{lb/ft}^3$.



$W = \gamma_c \cdot 1\,\text{ft}$

$F_1 = (\gamma_c \cdot 1\,\text{ft}) \times A_{\triangle} = 150\,\frac{\text{lb}}{\text{ft}^3} \cdot 1\,\text{ft} \cdot \frac{1}{4}\pi(21)^2\,\text{ft}^2 = 51{,}954\ \text{lb}$

$x_1 = r - \frac{4r}{3\pi} = 21 - \frac{4(21)}{3\pi} = 12.09\ \text{ft}$

$F_2 = (\gamma_c \cdot 1\,\text{ft}) A_{\sqcap} = 150\,\frac{\text{lb}}{\text{ft}^3} \cdot 1\,\text{ft} \cdot (8\,\text{ft} \times 21\,\text{ft}) = 25{,}200\ \text{lb}$

$X_2 = r + \frac{1}{2} 8\,\text{ft} = 21 + 4 = 25\ \text{ft}$

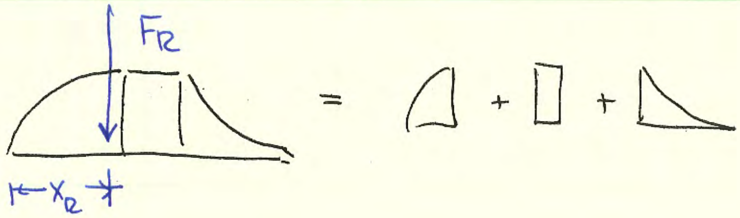
$F_3 = (\gamma_c \cdot 1\,\text{ft}) * A_{\triangle} = 150\,\frac{\text{lb}}{\text{ft}^3} \cdot 1\,\text{ft} \cdot \left[(21)(21)\,\text{ft}^2 - \frac{1}{4}\pi(21)^2\,\text{ft}\right]$

$= 14{,}196\ \text{lb}$

$$d = \frac{\sum x_i A_i}{\sum A_i} = \frac{\left(\frac{21\,\text{ft}}{2}\right)(21\,\text{ft})^2 - \left[21\,\text{ft} - \frac{4(21)}{3\pi}\,\text{ft}\right]\left(\frac{1}{4}\pi(21)^2\right)}{21^2\,\text{ft}^2 - \frac{1}{4}\pi(21)^2\,\text{ft}^2}$$

$= 4.691\ \text{ft}$

$x_3 = 21' + 8' + 4.691\ \text{ft} = 33.691\ \text{ft}$



$$x_R F_R = x_1 F_1 + x_2 F_2 + x_3 F_3$$

$$F_R = F_1 + F_2 + F_3$$

$$= 51{,}954 + 25{,}200 \text{ lb} + 14{,}196 \text{ lb} = \boxed{91{,}350 \text{ lb}}$$

$$x_R = \frac{x_1 F_1 + x_2 F_2 + x_3 F_3}{F_R}$$

$$= \frac{(12.09')(51{,}954 \text{ lb}) + (25')(25{,}200 \text{ lb}) + (33.69')(14{,}196 \text{ lb})}{91{,}350 \text{ lb}}$$

$$= \boxed{19.0 \text{ ft}}$$