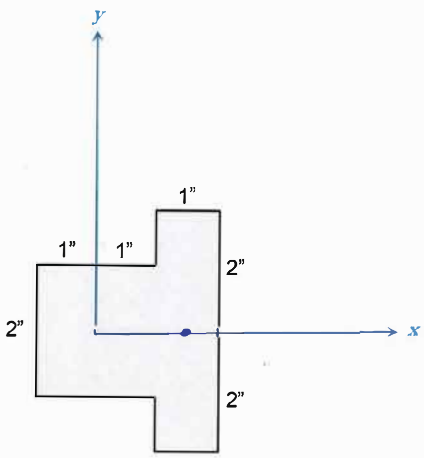
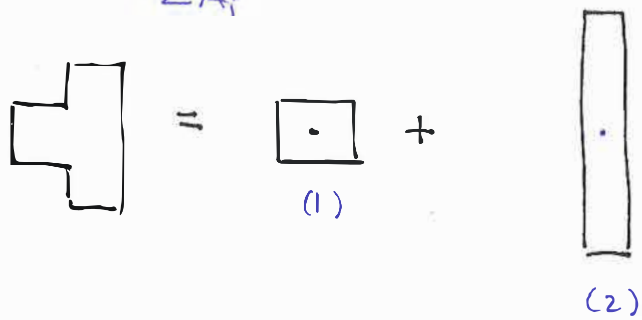
Example

Find the x- and y-centroids for the composite shape below.



$$X_c = \frac{\sum X_{c_i} A_i}{\sum A_i}$$



PART	X_c	A (in²)	XA (in³)
1	0	4	0
2	$1\frac{1}{2}$"	4	6
Σ		8	6

$A_1 = (2)(2) = 4\ in^2$

$X_{c,2} = 1 + \frac{1}{2} = 1\frac{1}{2}$"

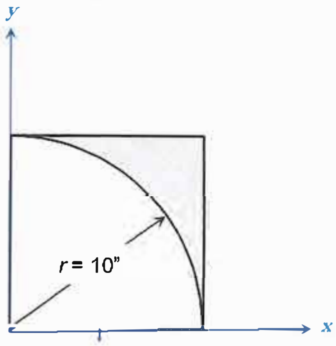
$A_2 = (4)(1) = 4 in^2$

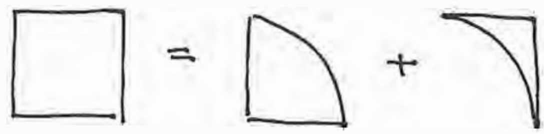
$\Rightarrow X_c = \frac{6}{8} = \boxed{\frac{3}{4}"}$

$y_c = ? = 0!$

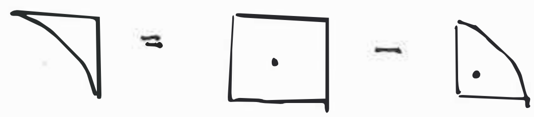
Example

Find the x- and y-centroids for the shape given by the shaded area in the figure.





OR



(1) (2)

$X_{c,1} = 5''$

$y_{c,1} = 5''$

$A_1 = (10)(10) = 100 \text{ in}^2$

$X_{c,2} = \frac{4}{3}\frac{r}{\pi} = \frac{4}{3} \cdot \frac{10''}{\pi}$ (FROM TABLE)

$y_{c,2} = \frac{4}{3}\frac{r}{\pi} = \frac{4}{3} \cdot \frac{10''}{\pi}$

$A_2 = \frac{1}{4}\pi r^2 = \frac{\pi}{4} \cdot 10^2 = 25\pi$

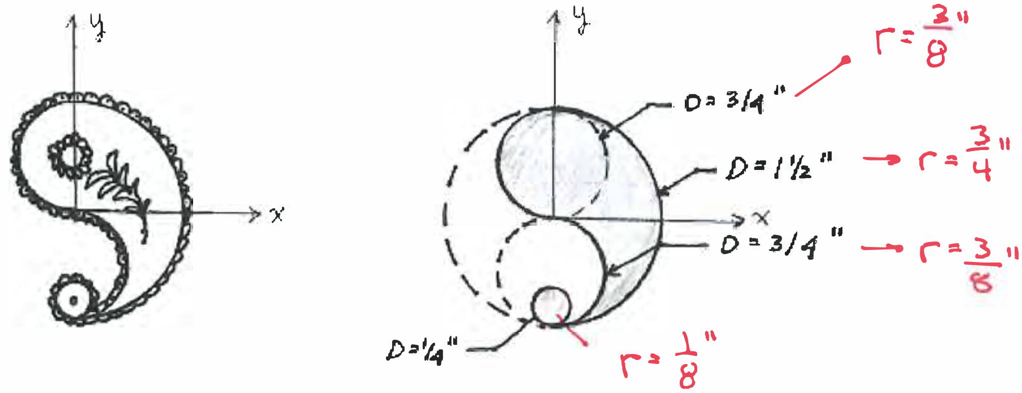
PART	X_{ci} (in)	y_{ci} (in)	A_i (in²)	X_iA_i in³	y_iA_i
(1)	5	5	100	500	500
(2)	$\frac{40}{3\pi}$	$\frac{40}{3\pi}$	(-25π) !!	$-\frac{1000}{3}$	$-\frac{1000}{3}$
Σ	9.244	9.244	21.46	166.7	166.7

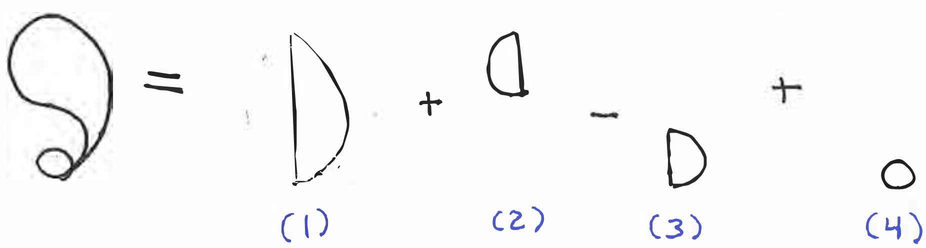
$$X_c = \frac{\Sigma X_{ci}A_i}{\Sigma A_i} = \frac{166.7 \text{ in}^3}{21.46 \text{ in}^2} = \boxed{7.77 \text{ in}}$$

$$y_c = \frac{\Sigma y_{ci}A_i}{\Sigma A_i} = \frac{166.7 \text{ in}^3}{21.46 \text{ in}^2} = \boxed{7.77 \text{ in}}$$

Example

Find the x- and y-centroids for a paisley by approximating it as the shape given in the right of the figure.





PART	x_{ci}	y_{ci}	A_i	$x_{ci}A_i$	$y_{ci}A_i$
(1)	$\frac{\cancel{4}\cdot\cancel{3}}{\cancel{3}\pi\cancel{4}}$	0	$\frac{\pi}{2}\left(\frac{3}{4}\right)^2$	$\frac{1}{2}\left(\frac{3}{4}\right)^2$	0
(2)	$-\frac{\cancel{4}\cdot\cancel{3}}{\cancel{3}\pi\,\cancel{8}\,2}$	$\frac{3}{8}$	$\frac{\pi}{2}\left(\frac{3}{8}\right)^2$	$-\frac{1}{4}\left(\frac{3}{8}\right)^2$	$\frac{\pi}{2}\left(\frac{3}{8}\right)^3$
(3)	$\frac{1}{2\pi}$	$-\frac{3}{8}$	$-\frac{\pi}{2}\left(\frac{3}{8}\right)^2$	$-\frac{1}{4}\left(\frac{3}{8}\right)^2$	$+\frac{\pi}{2}\left(\frac{3}{8}\right)^3$
(4)	0	$-\frac{5}{8}$	$\pi\left(\frac{1}{8}\right)^2$	0	$-\frac{5}{8}\pi\left(\frac{1}{8}\right)^2$
Σ	0.3183"	$-\frac{5}{8}$	0.9327	0.2109	0.1350

$$x_c = \frac{\Sigma x_c A_{ci}}{\Sigma A_i} = \frac{0.2109}{0.9327} = 0.226 \text{ in}$$

$$y_c = \frac{\Sigma y_{ci} A_i}{\Sigma A_i} = \frac{0.1350}{0.9327} = 0.145 \text{ in}$$