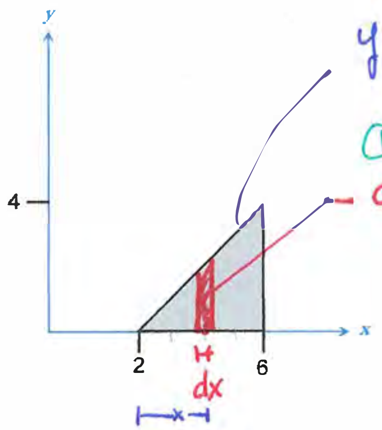
Example

Find the x-centroid for the shape below:



$y = ?\ mx+b = (1)(x) - 2 = x-2$

1 2

$dA = ? = y\,dx = (x-2)\,dx$

3

4

$$x_c = \frac{\int_A \tilde{x}\, dA}{\int_A dA} = \frac{\int_{x=2}^{6} x \overbrace{(x-2)\,dx}^{dA}}{\int_{x=2}^{6} (x-2)\,dx}$$

5

X = 2 .. 6 SWEEPS OUT ENTIRE AREA.

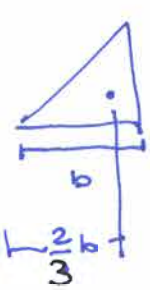
6

$$= \frac{\int (x^2-2x)\,dx}{\int (x-2)\,dx} = \frac{\left.\frac{x^3}{3} - x^2\right|_{x=2}^{6}}{\left.\frac{x^2}{2} - 2x\right|_{x=2}^{6}} = \frac{\left[\frac{216}{3} - 36\right] - \left[\frac{8}{3} - 4\right]}{\left[\frac{36}{2} - 12\right] - \left[\frac{4}{2} - 4\right]}$$

$$= \frac{36\,{}^{4}/_{3}}{8} = \boxed{4\,{}^{2}/_{3}}$$

AREA of TRIANGLE, SO OK.

$= 2 + \frac{2}{3} \times 4$

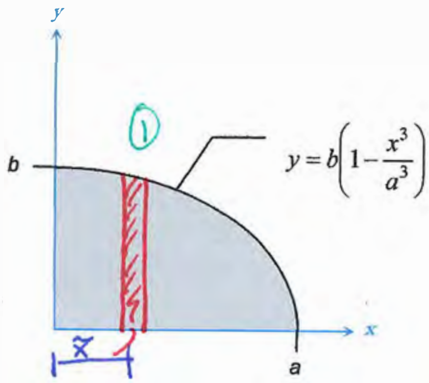


NOTE: FOR A RIGHT TRIANGLE, CENTROID IS $\frac{2}{3}$ DOWN FROM SKINNY END.

EASIER TO MAKE X THE VARIABLE of INTEGRATION & y IS GIVEN AS f(x).

Example

Find the x-centroid for the shape below. Do you prefer a horizontal or vertical strip for your elemental area? Why?



$dA = y\,dx$

2

3

$$dA = y\,dx = b\left(1 - \frac{x^3}{a^3}\right)dx$$

4

5

$$x_c = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_{x=0}^{a} x\, b\left(1 - \frac{x^3}{a^3}\right)dx}{\int_{x=0}^{a} b\left(1 - \frac{x^3}{a^3}\right)dx}$$

6

$$x_c = \frac{\cancel{b}\int_0^a \left(x - \frac{x^4}{a^3}\right)dx}{\cancel{b}\int_0^a \left(1 - \frac{x^3}{a^3}\right)dx}$$

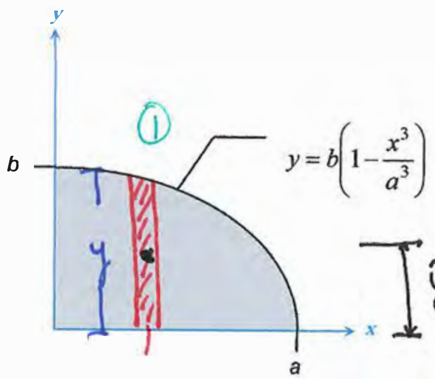
$$= \frac{\left.\frac{x^2}{2} - \frac{x^5}{5a^3}\right|_0^a}{\left.x - \frac{x^4}{4a^3}\right|_0^a} = \frac{\frac{a^2}{2} - \frac{a^{\cancel{5}\,2}}{5\cancel{a^3}}}{a - \frac{a^{\cancel{4}}}{4\cancel{a^3}}} = \frac{\frac{\cancel{3}a^2}{10}}{\frac{\cancel{3a}}{4}}$$

$$= \frac{2}{5}a$$

Example

Find the y-centroid for the shape below. Use the same vertical strip you did for the last example.

TRICKY!



$\tilde{y} = y/2$

2

$dA = y\,dx$

$= b\left(1 - \frac{x^3}{a^3}\right)dx$ 3

$\tilde{y} \neq y$! NEED y_c OF dA.

$$y_c = \frac{\int_A \tilde{y}\,dA}{\int_A dA}$$ ⟸ ALREADY DONE!

$dA = y\,dx$

4

$$y_c = \frac{\int_A \frac{y}{2}\,dA}{A} = \frac{\int_{x=0}^{a} \frac{y}{2}(y\,dx)}{A} = \frac{\int \frac{y^2}{2}\,dx}{A}$$ 5

$$= \frac{\frac{1}{2}\int_{x=0}^{a}\left[b\left(1 - \frac{x^3}{a^3}\right)\right]^2 dx}{A} = \frac{\frac{b^2}{2}\int_0^a 1 - \frac{2x^3}{a^3} + \frac{x^6}{a^6}\,dx}{A}$$ 6

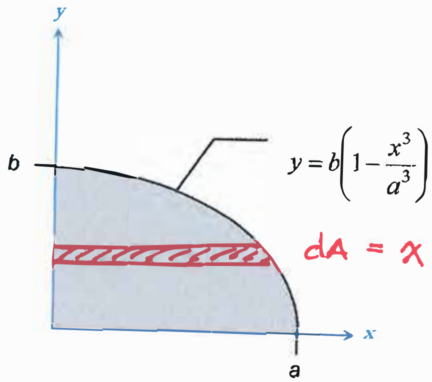
$$= \frac{\frac{b^2}{2}\left[x - \frac{2x^4}{4a^3} + \frac{x^7}{7a^6}\right]_0^a}{\frac{3ab}{4}} = \frac{\frac{b^2}{2}\left[a - \frac{1}{2}a + \frac{1}{7}a\right]}{\frac{3ab}{4}}$$

$$= \frac{\frac{b^{\cancel{2}}}{2}\,\frac{9}{14}\cancel{a}}{\frac{3\cancel{ab}}{4}} = \frac{\cancel{9}^{3}}{\cancel{28}_{7}}\,b\,\frac{4}{\cancel{3}} = \frac{3}{7}b$$

NOTE HERE THAT $\tilde{y}$ (CENTROID of dA) IS NOT y BUT RATHER $y/2$! THIS IS REALLY IMPORTANT!

Example

Find the y-centroid for the shape below. This time use a horizontal strip for the elemental area.



$$dA = x\,dy$$

$$\Rightarrow \quad y = b\left(1 - \frac{x^3}{a^3}\right)$$

SOLVE FOR x

$$\frac{y}{b} = 1 - \frac{x^3}{a^3} \qquad \frac{x^3}{a^3} = 1 - \frac{y}{b}$$

$$x = \left(a^3 - \frac{a^3 y}{b}\right)^{1/3}$$

$$= a\left(1 - \frac{y}{b}\right)^{1/3}$$

SO: $dA = x\,dy = (a)\left(1 - \frac{y}{b}\right)^{1/3} dy$

$$y_c = \frac{\int \tilde{y}\,dA}{A} = \frac{\int_0^b y\,x\,dy}{A}$$

A ← AGAIN, ALREADY DONE!

$$= \frac{\int_0^b y\,a\left(1 - \frac{y}{b}\right)^{1/3} dy}{A} = \frac{a\int_0^b y\left(1 - \frac{y}{b}\right)^{1/3} dy}{A}$$

GOOD ENOUGH! LET MAPLE SOLVE.