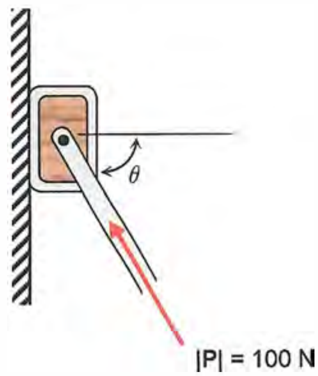
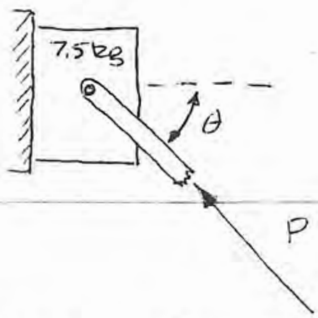
Example

A 7.5-kg mass is subject to a force **P** as shown in the figure. The coefficients of static and kinetic friction between the mass and the wall are $\mu_s = 0.45$ and $\mu_k = 0.35$, respectively. Find the range of angles for θ for which the mass is in equilibrium.



Given:



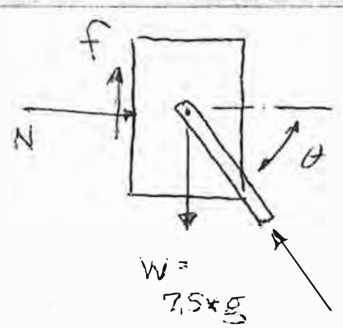
$P = 100\,N$

$\mu_s = 0.45$
$\mu_k = 0.35$

Find: RANGE of θ FOR WH/ EQUIL. IS MAINTAINED

Sol'n:

FBD: 1:



$\xrightarrow{+} \sum F_x = 0$

$N - (100)\cos\theta = 0$

$N = (100)(\cos\theta)$ (1) $N = P\cos\theta$ (1)

$+\uparrow \sum F_y = 0$

$f - W + 100\sin\theta = 0$ (P) $f - W + P\sin\theta = 0$

AT IMPENDING MOTION $f = \mu_s N$. SO:

$\mu_s N - (7.5)(9.81) + 100\sin\theta = 0$ $\mu_s N - mg + P\sin = 0$

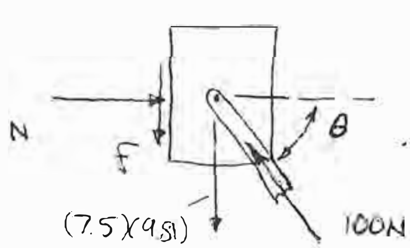
USING (1)

$\mu_s(100\cos\theta) - (7.5)(9.81) + 100\sin\theta = 0$
$\mu_s P\cos\theta - mg + P\sin\theta = 0$
$(0.45)(100)\cos\theta - (7.5)(9.81) + 100\sin\theta = 0$ (2)

Solving (2) for θ: $\theta = 17.91°$

(NOTE: THIS IS TRANSCENDANTAL & MUST BE SOLVED NUMERICALLY - HELLO MAPLE!)

FBD 2:   FBD 1 ASSUMED IMPENDING MOTION WAS *DOWN*.
FBD 2 ASSUMES " " IS *UP*.



$\xrightarrow{+} \sum F_x = 0$ $N - (100)\cos\theta = 0$

$N = (100)(\cos\theta)$ (3)

$N = P\cos\theta$

$\uparrow \sum F_y = 0$

$-f - w + p\sin\theta = 0$

$-f - W + 100\sin\theta = 0$

AT IMPENDING MOTION $f = \mu_s N$ SO:

$-\mu_s N - mg + P\sin\theta = 0$

$-\mu_s N - (7.5)(9.81) + 100\sin\theta = 0$

USING (3):

$-\mu_s \, p\cos\theta - mg + P\sin\theta = 0$

$-(0.45)(100\cdot\cos\theta) - (7.5)(9.81) + 100\sin\theta = 0 \quad (4)$

SOLVING 4 FOR θ:

(AGAIN, A NUMERICAL SOL'N IS NECESSARY)

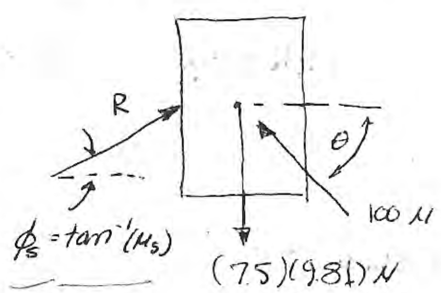
$\theta = 66.37°$

SO, FOR EQUIL. $\boxed{17.91° \leq \theta \leq 66.37°}$

Comments

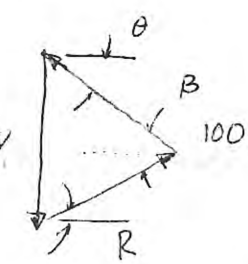
ALTERATE SOL'N:

FBD 1: IMPENDING MOTION *DOWN*



(WHY?)

SINCE THERE ARE JUST 3 FORCES, DRAW A FORCE TRIANGLE:



$\phi_s = \tan^{-1}\mu_s = 24.23°$

FROM ALTERNATE INTERIOR ANGLES

$\beta = \phi_s + \theta = 24.23 + \theta \quad (5)$

FROM LAW of SINES

$$\frac{73.575}{\sin\beta} = \frac{100}{\sin(90 - \phi_s)}$$

$\sin\beta = 0.73575 \times \sin(90 - 24.23) = 0.6709$

$\beta = \sin^{-1}(0.6709) = 42.14°$

$\theta = \acute{\beta} - \acute{\phi}_s$ (FROM 5)

$= \underline{17.91^\circ}$

SAME ANS, & NO NUMERICAL SOL'N NEEDED.

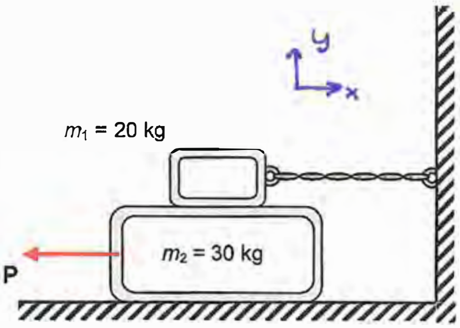
USING THIS TECHNIQUE USUALLY HELPS WHEN ONLY 3 FORCES ARE PRESENT. WHY DO YOU THINK THAT IS?

FINDING THE UPPER LIMIT ON θ WOULD PROCEED THE SAME WAY...

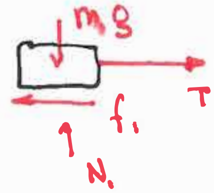
Example

The coefficients of static and kinetic friction between all surfaces in the figure are $\mu_s = 0.40$ and $\mu_k = 0.35$, respectively.

a) Find the smallest force P that is required to move the 30-kg block.
b) Repeat a) if the cable is removed.
c) What if the friction force between the blocks for part b)?



a).



$\sum F_y = 0 \quad -m_1 g + N_1 = 0$

$N_1 = m_1 g = \ldots = \underline{196\,N}$

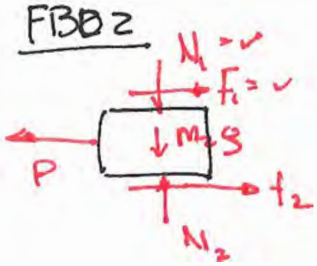
$\sum F_x = 0$

$-f_1 + T = 0$

IMPENDING MOTION

$f_1 = \mu_s N_1 = (0.40)(196\,N) = \underline{78.5\,N}$

FBD 2



$\sum F_y = 0$

$-N_1 + N_2 - m_2 g = 0$

$N_2 = N_1 + m_2 g = \ldots = \underline{491\,N}$

$\sum F_x = 0$

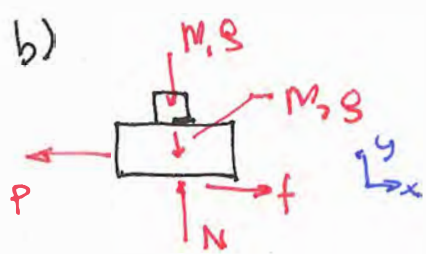
$-P + f_1 + f_2 = 0 \qquad P = f_1 + f_2$

IMPENDING MOTION

$f_2 = \mu_s N_2 = (0.40)(491\,N) = \underline{196\,N}$

$P = 78.5\,N + \qquad = \boxed{275\,N}$

b)



$\sum F_y = 0$

$-m_1 g - m_2 g + N = 0$

$N = \ldots = 491\,N$

$\sum F_x = 0$

$-P + f = 0$

$P = f = \mu_s N = (0.4)(491\,N)$

IMPENDING MOTION

$= \boxed{196\,N}$