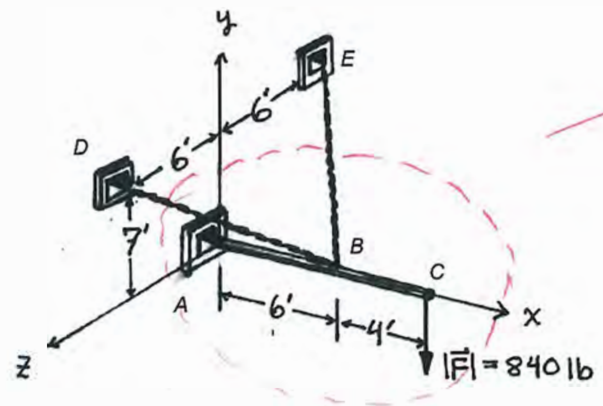
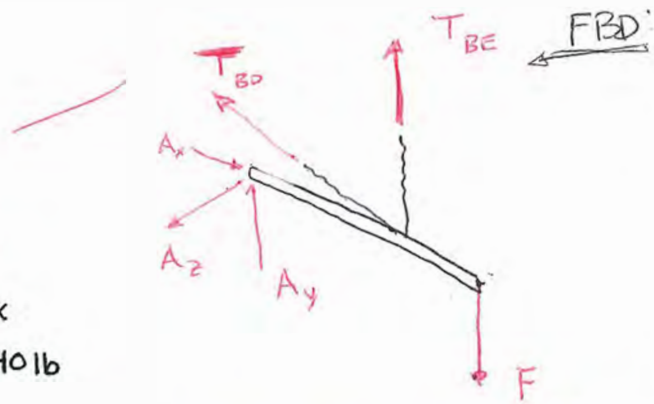
Example

The connections as A, D and E are ball and socket types. The rod AC can be modeled as weightless. Find the tension in each cable and the reaction at A.





$$\vec{T}_{BE} = T_{BE}\,\hat{e}_{BE} \qquad \hat{e}_{BE} = \frac{-6\hat{\imath} + 7\hat{\jmath} - 6\hat{k} \ \cancel{ft}}{\sqrt{6^2 + 7^2 + 6^2} \ \cancel{ft}} = -\frac{6}{11}\hat{\imath} + \frac{7}{11}\hat{\jmath} - \frac{6}{11}\hat{k}$$

$$\therefore \ \vec{T}_{BE} = T_{BE}\begin{bmatrix} -6/11 \\ 7/11 \\ -6/11 \end{bmatrix}$$

$$\vec{T}_{BD} = T_{BD}\,\hat{e}_{BD} \qquad \hat{e}_{BE} = \frac{-6\hat{\imath} + 7\hat{\jmath} + 6\hat{k} \ \cancel{ft}}{\sqrt{6^2 + 7^2 + 6^2} \ \cancel{ft}} = \begin{bmatrix} -6/11 \\ 7/11 \\ 6/11 \end{bmatrix}$$

$$\vec{T}_{BD} = T_{BD}\begin{bmatrix} -6/11 \\ 7/11 \\ 6/11 \end{bmatrix}$$

$$\vec{A} = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \qquad \vec{F} = F\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$\sum \vec{F} = \vec{0}$$

$$\vec{T}_{BE} + \vec{T}_{BD} + \vec{A} + \vec{F} = \vec{0}$$

$$T_{BE}\begin{bmatrix} -6/11 \\ 7/11 \\ -6/11 \end{bmatrix} + T_{BD}\begin{bmatrix} -6/11 \\ 7/11 \\ 6/11 \end{bmatrix} + \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} + \begin{bmatrix} 0 \\ -F \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

x-component:

$$-\frac{6}{11}\overset{?}{T_{BE}} - \frac{6}{11}\overset{?}{T_{BD}} + \overset{?}{A_x} = 0 \qquad (1)$$

y-component:

$$\frac{7}{11}T_{BE} + \frac{7}{11}T_{BD} + \overset{?}{A_y} - F = 0 \qquad (2)$$

z-component:

$$-\frac{6}{11}T_{BE} + \frac{6}{11}T_{BD} + \overset{?}{A_z} = 0 \qquad (3)$$

Three equations, five unknowns. Let's look @

$$\sum \vec{M}_B = 0$$

$$\vec{r}_{BA} \times \vec{A} + \vec{r}_{BC} \times \vec{F} = \vec{0}$$

$$\vec{r}_{BA} = -6\hat{\imath} \text{ ft}$$

$$\vec{r}_{BA} \times \vec{A} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -6 & 0 & 0 \\ A_x & A_y & A_z \end{vmatrix} = \begin{bmatrix} 0 \\ 6A_z \\ -6A_y \end{bmatrix}$$

$$\vec{r}_{BC} = 4\hat{\imath} \text{ ft}$$

$$\vec{r}_{BC} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 0 \\ 0 & -F & 0 \end{vmatrix} = \begin{bmatrix} 0 \\ 0 \\ -4F \end{bmatrix}$$

$\therefore \quad \sum \vec{M}_B = \vec{0}$

$$= \begin{bmatrix} 0 \\ 6A_z \\ -6A_y \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -4F \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

x-component

$0 = 0$

y-component

$6A_z = 0 \qquad A_z = 0 \qquad (4)$

z component

$-6A_y - 4F = 0 \qquad (5)$

Five equations with five unknowns. Solving...

$T_{BD} = 1100 \text{ lb}$

$T_{BE} = 1100 \text{ lb}$

$A_x = 1200 \text{ lb}$

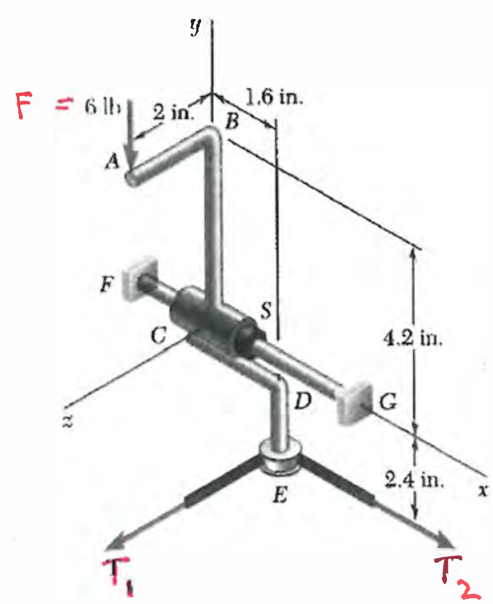
$A_y = -560 \text{ lb}$

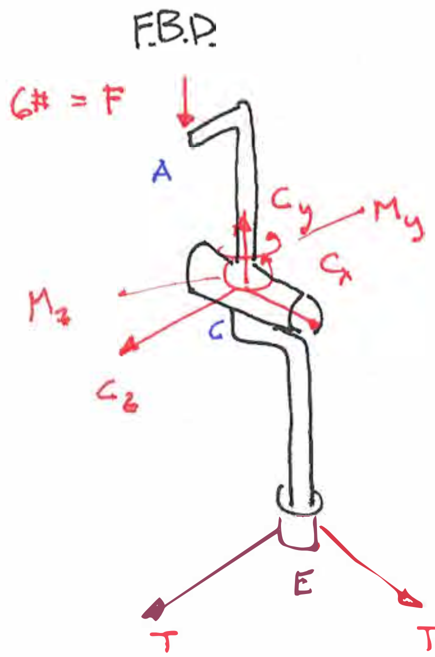
$A_z = 0$

Example[1]

THRUST BEARING

For the assembly shown, find the tension T and the reactions at C. The weight of the assembly is negligible.





$\Sigma \vec{F} = 0$

$\vec{F} = -F_y \hat{j}$

$\vec{C} = C_x \hat{i} + C_y \hat{j} + C_z \hat{k}$

$\vec{T}_1 = T \hat{k}$

$T_2 = T \hat{i}$

$\therefore \quad \vec{F} + \vec{C} + \vec{T}_1 + \vec{T}_2 = 0$

$\therefore \quad -F_y \hat{j} + C_x \hat{i} + C_y \hat{j} + C_z \hat{k} + T_2 \hat{k} + T \hat{i} = 0$

x COMP: $\quad C_x + T = 0 \quad (1)$

y COMP: $\quad -F_y + C_y = 0 \quad (2)$

z COMP: $\quad C_z + T = 0 \quad (3)$

4 UNKS, 3 EQNS

TAKE MOMENTS ABOUT O TO ELIMINATE $\vec{C}$

$$\Sigma \vec{M}_O = 0 = \vec{r}_{OA} \times \vec{F} + \vec{r}_{OE} \times \vec{T}_1 + \vec{r}_{OE} \times \vec{T}_2 + \vec{M}$$

$$0 = (4.2\hat{j} + 2\hat{k}) \times (-6\# \hat{j}) + (1.6\hat{i} - 2.4\hat{j}) \times (T\hat{k}) + (1.6\hat{i} - 2.4\hat{j}) \times (T\hat{i})$$

$$= 12\hat{i} - 1.6T\hat{j} - 2.4T\hat{i} + 2.4T\hat{k} + M_y\hat{j} + M_z\hat{k} = \vec{0} \qquad + \vec{M}$$

$$\vec{0} = (12 - 2.4T)\hat{i}$$

$$(-1.6T + M_y)\hat{j}$$

$$(2.4T + M_z)\hat{k}$$

[1] Taken from Beer and Johnson, *Vector Mechanics for Engineers*, 6th Ed.

$12 - 2.4T = 0$ (x-COMP) $T = 5$ lb

(y-COMP) $-1.6T + M_y = 0$ $M_y = 1.6T = 8$ lbin

z-COMP $2.4T + M_z = 0$ $M_z = -12$ lbin

FROM (1)

$C_x = -T = -5$ lb

FROM (2)

$C_y = F_y = 6$ lb

FROM (3)

$C_z = -T = -5$ lb