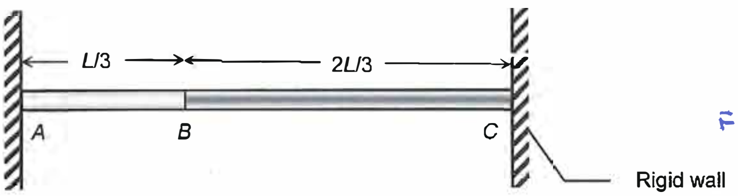
Example

Two bars, both with cross sectional areas A, are attached to rigid walls. Bar AB is made of aluminum, whereas bar BC is made of steel. At room temperature the bars are stress-free. In service the temperature of the system rises by an amount ΔT.

Assuming $E_{st} = 3E_{Al}$ and $\alpha_{st} = ½\, \alpha_{Al}$, does point B move when heated by ΔT? If so, in which direction and how far?

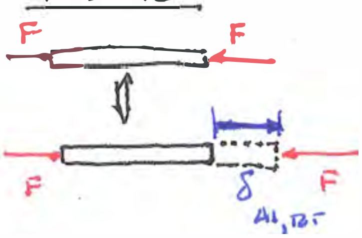


FBD WHOLE THING (AFTER HEATED)

$\Sigma F_x = 0$

$F_1 = F_2 = F$

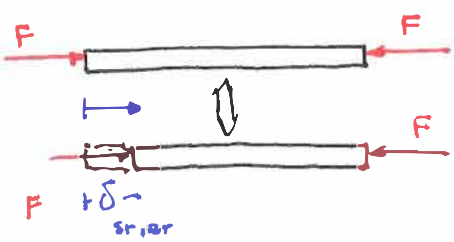
FBD AB



$$\delta_{AL,TOT} = \delta_{AL,T} + \delta_{AL,MECH}$$

$$= \alpha_{AL} \Delta T L_{AL} + \frac{-F L_{AL}}{E_{AL} A} = \alpha_{AL} \Delta T \left(\frac{L}{3}\right) - \frac{F\left(\frac{L}{3}\right)}{E_{AL} A} \qquad (1)$$

FBD BC



$$\delta_{ST,TOT} = \delta_{ST,T} + \delta_{ST,MECH}$$

$$\alpha_{ST} \Delta T L_{ST} - \frac{F L_{ST}}{E_{ST} A} = \alpha_{ST} \Delta T \left(\frac{2L}{3}\right) - \frac{F\left(\frac{2L}{3}\right)}{E_{ST} A} \qquad (2)$$

GEOMETRIC CONSTRAINT

$$\delta_{AL,TOT} = -\delta_{ST,TOT} \implies (1) = (2)$$

$$\alpha_{Al} \Delta T \frac{L}{3} - \frac{F L}{3 E_{AL} A} = -\left[\alpha_{st} \Delta T \frac{2L}{3} - \frac{2 F L}{3 E_{ST} A}\right]$$

$$\left[\frac{2}{3 E_{ST} A} + \frac{1}{3 E_{AL} A}\right] F = \left[\frac{\alpha_{AL}}{3} + \frac{2}{3}\alpha_{ST}\right] \Delta T$$

$$\left[\frac{2}{3E_{AL}A} + \frac{1}{E_{AL}A}\right]F = \left[\alpha_{AL} + \cancel{2}\left(\frac{1}{\cancel{2}}\alpha_{AL}\right)\right]\Delta T$$

$$\frac{5}{3E_{AL}A}F = 2\alpha_{AL}\,\Delta T$$

$$F = \frac{6\,\alpha_{AL}\,\Delta T\,E_{AL}A}{5}$$

FROM (1)

$$\delta_{AL} = \alpha_{AL}\,\Delta T\,\frac{L}{3} - \left(\frac{\overset{2}{\cancel{6}}\,\alpha_{AL}\,\Delta T\,\cancel{E_{AL}A}}{5}\right)\left(\frac{L}{\cancel{3}}\,\frac{\quad}{\cancel{E_{AL}A}}\right)$$

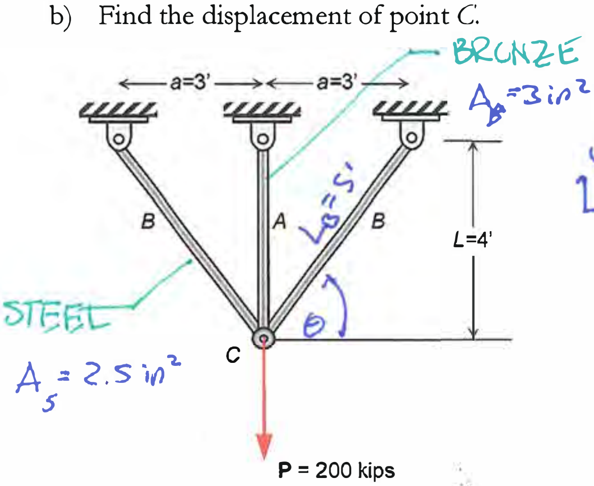
$$= -\frac{1}{15}\,\alpha_{AL}\,\Delta T\,L =$$

↑ MOVES TO <u>LEFT</u> NOT RIGHT AS ASSUMED.

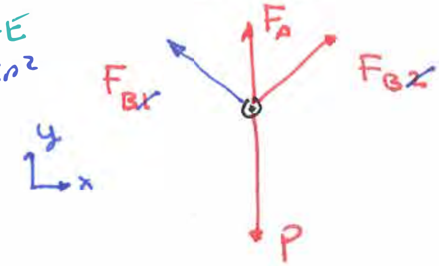
Example

The structure shown in the figure consists of one cold-rolled bronze (E_b = 15×10³ ksi, α_b = 9.4×10⁻⁶/°F) bar A two 0.2% carbon hardened steel (E_s = 30×10³ ksi, α_s = 6.6×10⁻⁶/°F) bars B. A load P=200 kips is applied to point C while bar A experiences a temperature decrease ΔT_b = 50°F and both bars B experience a temperature increase ΔT_s = 30°F.

a) Find the stress is each bar.
b) Find the displacement of point C.



FBD WHOLE THING



$\Sigma F_x = 0$

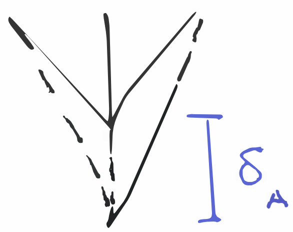
$F_{B1}\cos\theta = F_{B2}\cos\theta$

$F_{B_1} = F_{B2} = F_B$

$\Sigma F_y = 0$

$F_A + 2F_B \sin\theta = P \quad (1)$

GEOMETRIC CONSTRAINT

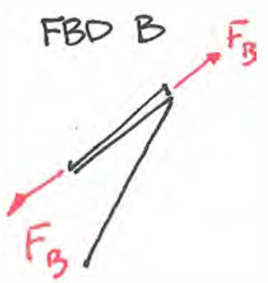


$\delta_{A,TOT} = \delta_{A,MECH} - \delta_{A,T}$

$= \dfrac{F_A L}{E_b A_A} - \alpha_b \Delta T_b L \quad (2)$

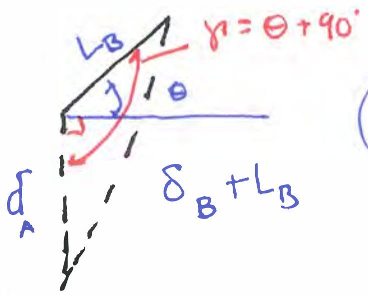
FBD A

FBD B



$\delta_{B,TOT} = \delta_{B,MECH} + \delta_{B,T}$

$= \dfrac{F_B L_B}{E_s A_s} + \alpha_s \Delta T_s L_B \quad (3)$



LAW of COSINES

$(L_B + \delta_B)^2 = \delta_A^2 + L_B^2 - 2\delta_A L_B \cos\gamma \quad (4)$

(OR CAN APPROXIMATE

$\delta_B \approx \frac{4}{5}\delta_A$)

FOUR EQNS &
" UNKS !
SOLVE 'EM!!

File:LE35_example_2_soln.EES

Given info

$A_b = 3$

$A_s = 2.5$

$P = 200$

$L = 4 \cdot 12$

$L_B = 5 \cdot 12$

$E_b = 15000$

$\alpha_b = 9.4 \cdot 10^{-6}$

$E_s = 30000$

$\alpha_s = 6.6 \cdot 10^{-6}$

$\Delta_{T,b} = 50$

$\Delta_{T,s} = 30$

Equilibrium of entire structure

$F_A + 2 \cdot F_B \cdot 4 / 5 = P$

Deformation equations

$$\delta_A = \frac{F_A \cdot L}{E_b \cdot A_b} - \alpha_b \cdot \Delta_{T,b} \cdot L$$

$$\delta_B = \frac{F_B \cdot L}{E_s \cdot A_s} + \alpha_s \cdot \Delta_{T,s} \cdot L_B$$

Geometric constraints

$\delta_B = 4 / 5 \cdot \delta_A$

$$[L_B + \delta_B]^2 = \delta_A^2 + L_B^2 - 2 \cdot \delta_{A,true} \cdot L_B \cdot \cos[\theta + 90]$$

$\theta = \arctan[4 / 3]$

File:LE35_example_2_soln.EES

SOLUTION

Unit Settings: [kJ]/[C]/[kPa]/[kg]/[degrees]

α_b = 0.0000094 [1/F]

α_s = 0.0000066 [1/F]

A_b = 3 [in^2]

A_s = 2.5 [in^2]

δ_A = 70.996E-3 [in]

$\delta_{A,true}$ = 70.977E-3 [in]

δ_B = 0.0568 [in]

$\Delta_{T,b}$ = 50 [F]

$\Delta_{T,s}$ = 30 [F]

E_b = 15000 [ksi]

E_s = 30000 [ksi]

F_A = 87.71 [kip]

F_B = 70.18 [kip]

L = 48 [in]

L_B = 60 [in]

P = 200 [kip]

θ = 53.13 [deg]

4 potential unit problems were detected.