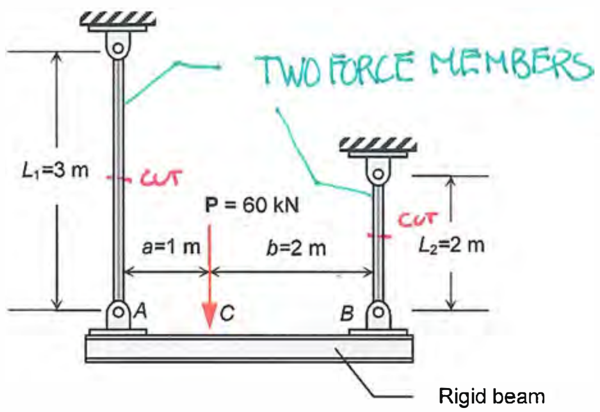
Example

A rigid beam is supported by two vertical rods. Rod A has a diameter of d_A = 25 mm and rod B has a diameter of d_B = 10.2 mm. Both rods are made of steel (E=210 GPa). For the 60 kN force applied as shown,

a) find the reactions at A and B, and
b) the displacements of each rod.



a) FBD

$\Sigma F_x = 0$

$0 = 0$

$\Sigma F_y = 0$

$-P + F_A + F_B = 0 \quad (1)$

$\circlearrowleft + \; \Sigma M_A = 0$

$-aP + (a+b) F_B = 0 \quad (2)$

$F_B = \dfrac{a}{a+b} P = \dfrac{1\,m}{3\,m}(60\,kN)$

$= \boxed{20\,kN}$

FROM (1)

$F_A = P - F_B = (60-20)\,kN = \boxed{40\,kN}$

b) FBD ROD A W/ CUT:

$\uparrow \Sigma F_y = 0$

$\sigma_A A_A = F_A$

$\sigma_A = \dfrac{F_A}{A_A} \quad (3)$

HOOKE'S LAW:

$\sigma_1 = \varepsilon_A E$

$\sigma_1 = \left(\dfrac{\delta_1}{L_1}\right) E \quad (4)$

(3) & (4)

$\dfrac{\delta_1}{L_1} E = \dfrac{F_A}{A_A}$

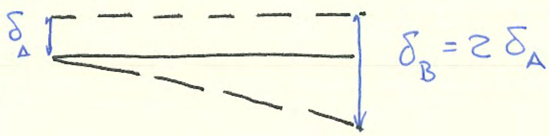
$\delta_1 = \dfrac{F_A L_1}{E A_A}$

$\delta_A = \dfrac{F_A L_1}{(E)\frac{\pi d_A^2}{4}} = \dfrac{(40 \times 10^3 N)(3m)}{(210 \times 10^9)\frac{N}{m^2} \frac{\pi}{4}(0.025)^2} = \boxed{1.164\,mm}$

SIMILARLY...

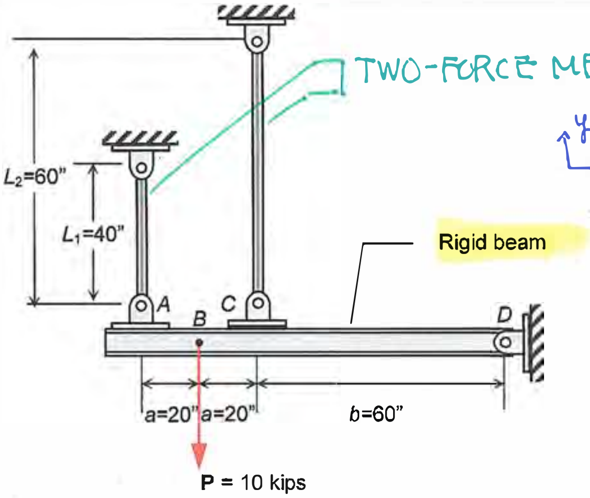
$\delta_B = \dfrac{F_B L_2}{E \frac{\pi d_B^2}{4}} = \dfrac{(20 \times 10^3 N)(2m)}{(210 \times 10^9 \frac{N}{m^2})\frac{\pi}{4}(0.0102)^2} = \boxed{2.331\,mm}$

NOTE HOW IT DEFLECTS



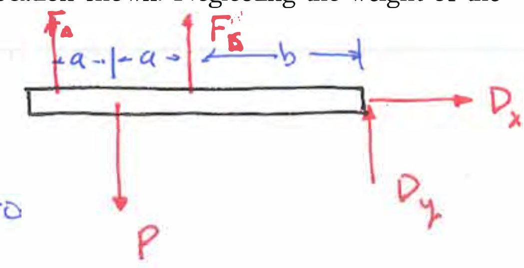
Example

Two steel (E=30×10³ ksi) rods both with cross sectional area A=1.0 in² are used to support a rigid beam connected to a wall via a smooth pin. A 10 kip point load is applied to the beam at the location shown. Neglecting the weight of the beam, find the tension in each rod.



TWO-FORCE MEMBERS

FBD



$\Sigma F_x = 0$

$D_x = 0$

$\Sigma F_y = 0$

$F_A + F_B + D_y - P = 0 \quad (1)$

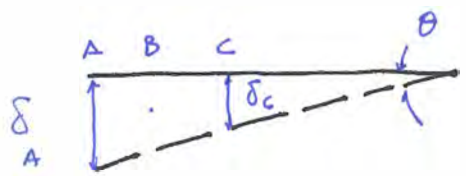
$\Sigma M_D = 0$

$-(2a+b)F_A + (a+b)P - bF_B = 0 \quad (2)$

TWO EQNS, 3 UNKNOWNS!
FBDs of RODS DON'T HELP $\Longrightarrow$ STATICALLY INDETERMINATE.

MUST LOOK @ GEOMETRY of DEFORMATION.

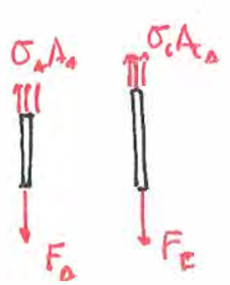
BECAUSE BEAM IS RIGID:



FROM SIMILAR TRIANGLES

$$\frac{\delta_A}{2a+b} = \frac{\delta_C}{b} \quad (3)$$

ADDED TWO UNKNOWNS! LOOK AT RODS A & C



$$\delta_A = \frac{F_A L_1}{E_A A_A} \quad (4)$$

$$\delta_C = \frac{F_C L_2}{E_C A_C} \quad (5)$$

TWO MORE EQNS, NO NEW UNKNOWNS!

(4) & (5) INTO (3)

$$\frac{F_A L_1}{\cancel{EA}}\left(\frac{1}{2a+b}\right) = \frac{F_C L_2}{\cancel{EA}}\left(\frac{1}{b}\right) \qquad F_C = \frac{b}{2a+b}\left(\frac{L_1}{L_2}\right) F_A \quad (6)$$

SUB INTO (2)

$$-(2a+b)F_A + (a+b)P - b\,\frac{b}{2a+b}\left(\frac{L_1}{L_2}\right)F_A = 0$$

$$F_A = \frac{(a+b)}{(2a+b) + \frac{b^2}{(2a+b)}\left(\frac{L_1}{L_2}\right)} P$$

$$= \frac{20'' + 60''}{(40'' + 60'') + \frac{(60'')^2}{(40''+60'')}\left(\frac{40''}{60''}\right)} (10 \text{ kips})$$

$$= \boxed{6.451 \text{ kips}}$$

FROM (6)

$$F_c = \frac{60''}{40''+60''}\left(\frac{40''}{60''}\right)(6.451 \text{ kip}) = \boxed{2.581 \text{ kips}}$$

NEW! FIND STRESS IN EA. ROD & θ.

$$\sigma_A = F_A/A = 6.45 \text{ kips}/1 \text{ in}^2 = 6.45 \text{ ksi}$$

$$\sigma_B = F_B/A = \ldots = 2.58 \text{ ksi}$$

$$\theta \approx \tan^{-1}\left(\frac{\delta_A}{2a+b}\right) = \tan^{-1}\left(\frac{F_A L_1}{EA(2a+b)}\right)$$

$$= \tan^{-1}\left(\frac{(6.451 \text{ kip})(40'')}{30\times10^3 \frac{\text{kip}}{\text{in}^2}(1 \text{ in}^2)(40''+60'')}\right) = \boxed{0.00493°}$$

$$\tan^{-1}\left(\frac{\delta_a}{a}\right) = \theta = \tan^{-1}\left(\frac{0.0129\,m}{2\,m}\right) = \boxed{0.36^\circ} \quad (a)$$

NOTE: $\varepsilon_A = \frac{\delta_A}{L_1} = \frac{0.0129\,m}{5\,m} = 0.002 = 0.2\%!$

BIG STRAIN! REMEMBER YIELD? DEFINED @ 0.02%.

$$\sigma_A = \frac{F_A}{A_A} = \frac{36\,kN}{(200)\times10^{-6}\,m^2} = \boxed{180\,MPa}$$

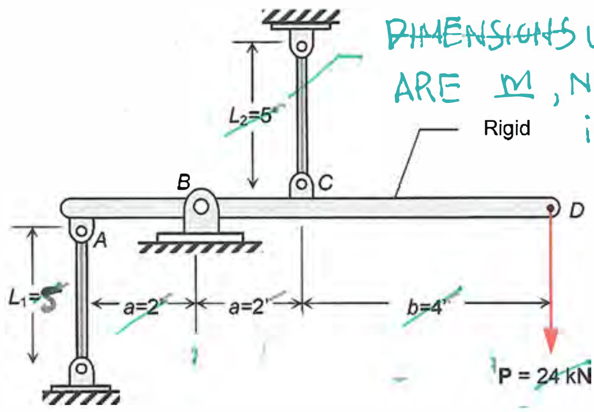
CHECK IF IT'S CLOSE TO σ_{yield}

$$\sigma_B = \frac{F_B}{A_B} = \dots = \boxed{180\,MPa}$$

Example

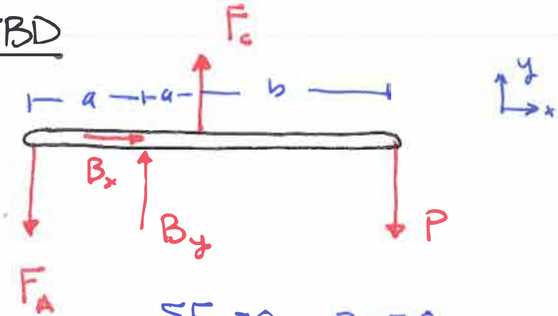
A rigid, weightless beam is supported by a smooth pin at *B*. Two aluminum (*E*=70 GPa) rods, both with cross sectional area *A*=200 mm², also support the rod at pins *A* and *C*. For the 24 kN load at *D*,

a) find the rotation angle of the rod,
b) the force in each rod, and
c) the stress in each rod.



DIMENSIONS UNITS ARE M, NOT in.

FBD



$\Sigma F_x = 0 \qquad B_x = 0$

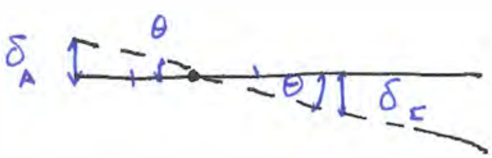
$\Sigma F_y = 0 \qquad -F_A + B_y + F_c - P = 0 \quad (1)$

$\Sigma M_B = 0$

$aF_A + aF_c - (a+b)P = 0 \quad (2)$

$a(F_A + F_c) - (a+b)P = 0$

GEOMETRY & DEFORMATION



FROM SIMILAR TRIANGLES

$\delta_A = \delta_c \quad (3) \qquad \theta = \tan^{-1}\left(\frac{\delta_A}{a}\right)$

STRESS/STRAIN

$\delta_A = \frac{F_A L_1}{E_A A_A} \quad (4) \qquad \delta_c = \frac{F_c L_2}{E_B A_B} \quad (5)$

FIVE EQNS & FIVE UNKS CAN SOLVE!

(3), (4) & (5) GIVE

$\frac{F_A \cancel{L_1}}{\cancel{E_A} \cancel{A_A}} = \frac{F_c \cancel{L_2}}{\cancel{E_B} \cancel{A_B}} \qquad F_A = F_c$

SUB INTO (2)

$2aF_A = (a+b)P = 0$

$F_A = \frac{(a+b)P}{2a} = \frac{(6\,m)(24\,kN)}{(4\,m)} = \boxed{36\,kN} \quad (b)$

$F_c = F_A = \boxed{36.\,kN} \quad (b)$

FROM (4)

$\delta_A = \frac{(36\,kN)(5\,m)}{(70\,GPa)(200)\times10^{-6}\,m^2} \left\langle \frac{Pa}{A}\ m \right\rangle = 0.0129 \quad m$

$= 12.9\,mm$