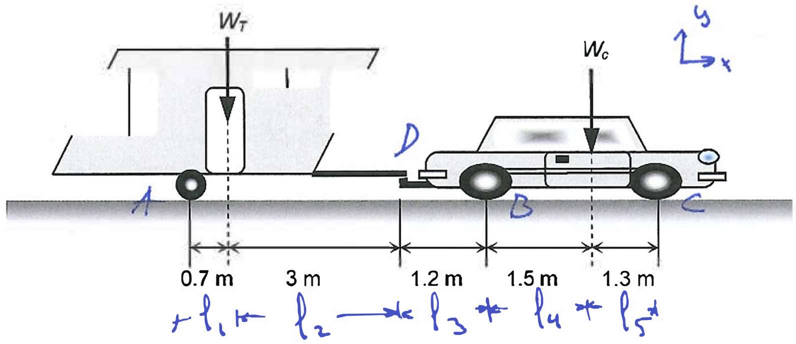
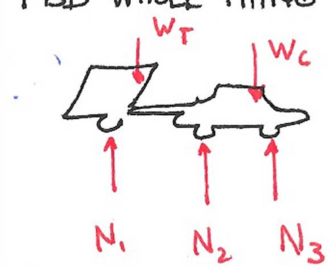
Example

The mass of the car in the figure is 1250 kg and the mass of the trailer is ~~100 kg~~ 1000 kg. The trailer hitch connecting the car to the trailer is a ball and socket. Find the reactions at the wheels.



FBD WHOLE THING



$\Sigma F_x = 0$ USELESS

$\Sigma F_y = 0$

$$N_1 + N_2 + N_3 - W_T - W_c = 0 \quad (1)$$

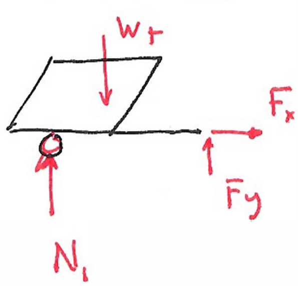
? ? ?

$$\circlearrowleft^{+} \ \Sigma M_A = 0$$

$$-\underbrace{l_1}_{0.7} W_T + \underbrace{(l_1 + l_2 + l_3)}_{4.9} N_B - \underbrace{(l_1 + l_2 + l_3 + l_4)}_{6.4} W_c + \underbrace{(l_1 + l_2 + l_3 + l_4 + l_5)}_{7.7} N_3 = 0 \quad (2)$$

2 EQNS, 3 UNKNOWNS! NEED A NEW FBD.

FBD TRAILER



$$\circlearrowleft^{+} \ \Sigma M_D = 0$$

$$-(l_1 + l_2) N_1 + l_2 W_T = 0 \quad (3)$$

SOLVE $\Rightarrow$

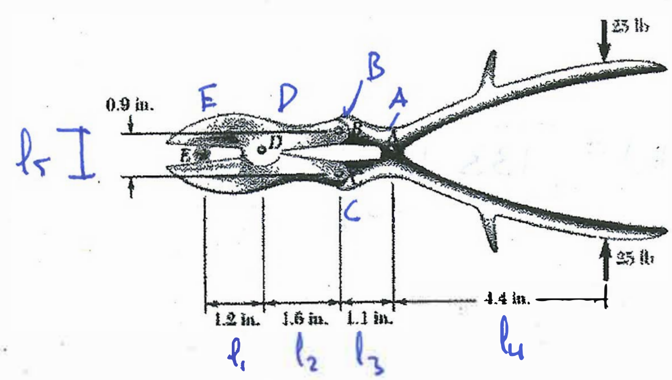
$N_1 = 7954$ N

$N_2 = 8345$ N

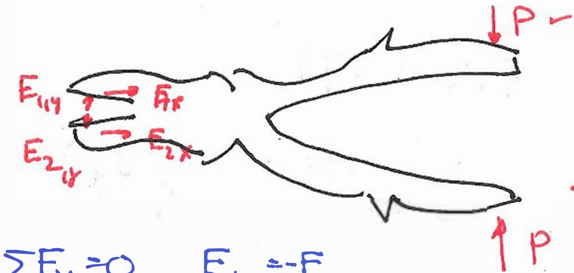
$N_3 = 5774$ N

Example

The device shown in the figure is called a bone rongeur and is used in surgical procedures to cut small bones. For the 25-lb forces applied to the instrument at the locations shown, find the force applied to the bone at *E*.



FBD WHOLE THING – BONE



$\Sigma F_x = 0 \quad E_{1x} = -E_{2x}$

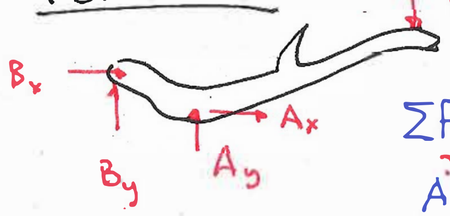
$\Sigma F_y = 0 \quad \cancel{P-P} + E_{1y} + E_{2y} = 0$

$E_{1y} = -E_{2y}$

$\circlearrowleft \Sigma M_E = 0 \quad -E_{1x}\cancel{d} + \cancel{d}E_{2x} = 0$

$E_{1x} = E_{2x} \quad \therefore \underline{=0}$

FBD BDP



$\Sigma F_x = 0$

$A_x + B_x = 0 \quad (1)$

$\Sigma F_y = 0$

$B_y + A_y - P = 0 \quad (2)$

$\Sigma M_A = 0$

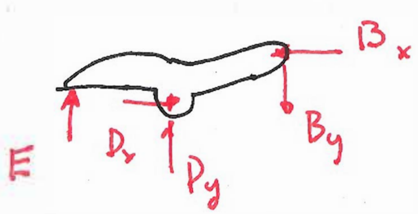
$-l_4 P - \frac{l_5}{2} B_x - l_3 B_y = 0 \quad (3)$

3 EQ 5 UNK

$A_x\ B_x\ B_y\ A_y$

E

FBD EDB



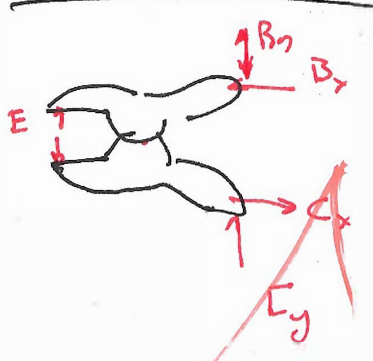
$\circlearrowleft \Sigma M_D = 0$

$-l_1 E + \frac{l_5}{2} B_x - l_2 B_y = 0 \quad (4)$ BLECH!

ADDED 1 EQN.

X EQ ADD MORE UNKNOWNS!

FBD EDB TOP & BOTTOM



$\Sigma F_x = 0 \quad -B_x + C_x = 0 \quad C_x = B_x$

$\Sigma M_D = 0 \quad \cancel{-l_1 E + l_1 E} + \frac{l_5}{2} B_x + \frac{l_5}{2} C_x = 0$

AH-HAH! $\quad C_x = -B_x \Rightarrow C_x = B_x = 0!!$

$B_x = 0 \quad (5)$

NOW I CAN SOLVE.

FROM (3)

$$B_y = -\frac{l_4}{l_3}P = -\frac{4.4}{1.1}(25\text{ lb}) = \underline{-100\text{ lb}}$$

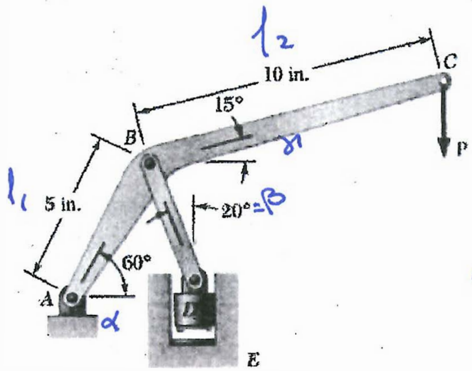
FROM (4)

$$F = -\frac{l_2}{l_1}B_y = -\frac{1.6}{1.2}(-100\text{ lb}) = \boxed{133\text{ lb}}$$

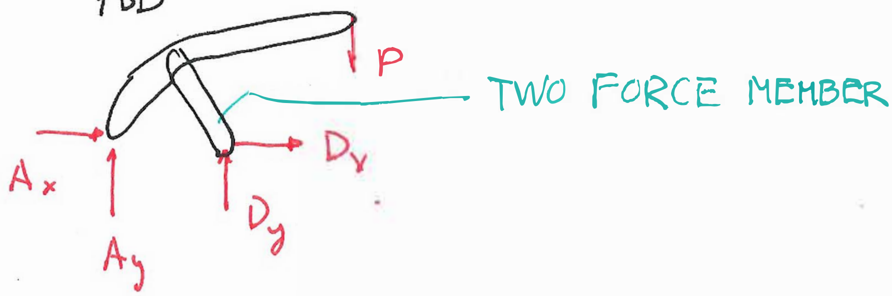
Example

ASSUME WEIGHTLESS MEMBERS

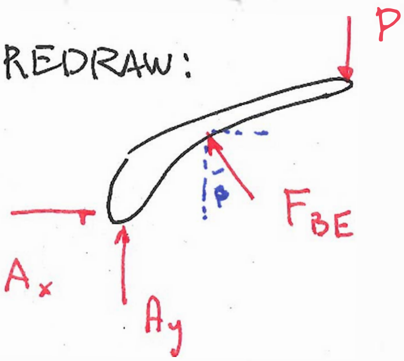
The figure shows a press used to emboss a seal at E. If the force $P = 60$ lb, find the reaction at A and the vertical component of the force exerted on the seal.



FBD



REDRAW:



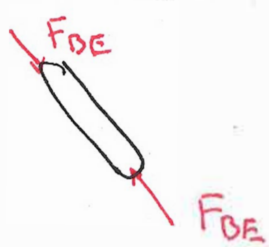
$$\circlearrowleft \sum M_A = 0$$

$$-(l_1 \cos\alpha + l_2 \cos\gamma)P$$

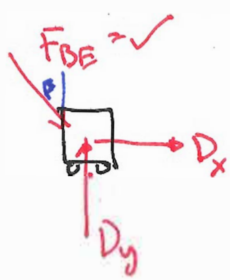
$$+ (l_1 \sin\alpha) F_{BE} \sin\beta + (l_1 \cos\alpha)(F_{DE} \cos\beta) = 0$$

$$\ldots \quad F_{BE} = 3.175P = \boxed{191 \text{ lb}}$$

FBD BD



FBD D



$$\sum F_y = 0$$

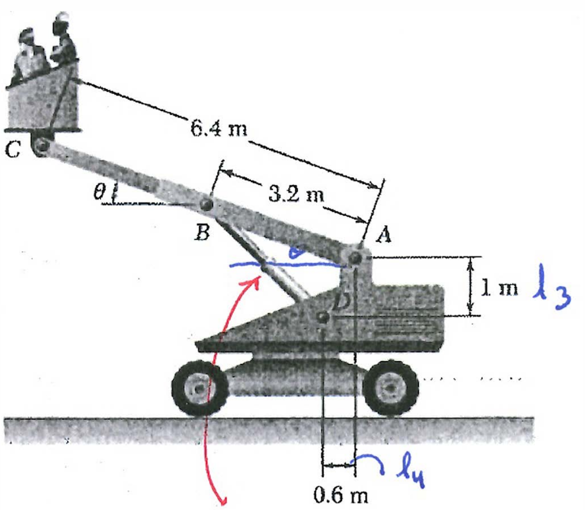
$$-F_{BE} \cos\beta + D_y = 0$$

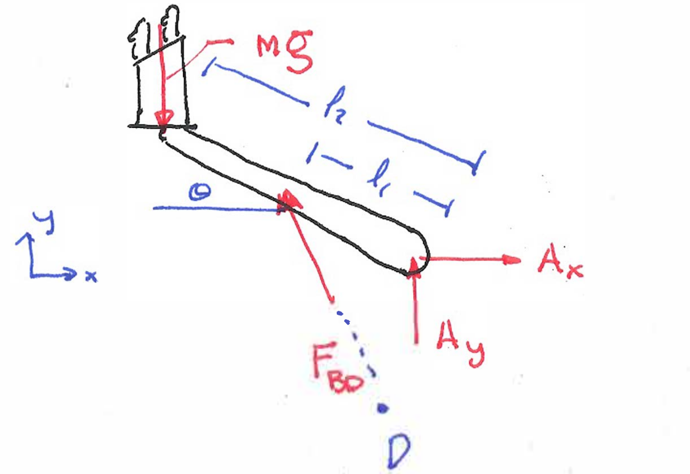
$$D_y = F_{BE} \cos\beta$$

$$= \ldots = \boxed{179 \text{ lb}}$$

Example

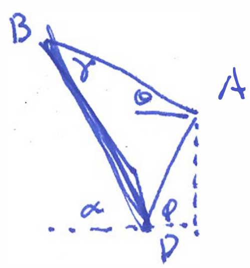
The telescoping arm ABC is used to elevate workers on a platform. The combined mass of the platform and the workers is 240 kg with a combined center of gravity at *C*. If the angle $\theta = 24°$, find the force exerted by the hydraulic cylinder *BD* on the arm and the reaction at *A*.





TWO-FORCE MEMBER!

GEOMETRY!



$$\phi = \tan^{-1}\left(\frac{l_3}{l_4}\right) = \tan^{-1}\left(\frac{1}{0.6}\right) = \underline{59.0°}$$

$$AD = \left(l_3^2 + l_4^2\right)^{1/2} = \left(1^2 + 0.6^2\right)^{1/2} = \underline{1.166} \text{ m}$$

LAW of COSINES

$$BD = \left(AB^2 + AD^2 - 2(AB)(AD)\cos(\phi + \theta)\right)^{1/2}$$

$$= \left[3.2^2 + (1.166)^2 - 2(3.2)(1.166) * \cos\left(59° + 24°\right)\right]^{1/2}$$

$$= \underline{3.2696 \text{ M}}$$

LAW of SINES

$$\frac{BD}{\sin(\theta + \phi)} = \frac{AD}{\sin\gamma}$$

$$\sin\gamma = \frac{AD}{BD}\sin(\theta + \phi)$$

$$= \frac{1.166}{3.2696}\sin(59° + 24°) = 0.3540$$

$$\gamma = \underline{20.73°}$$

$$\alpha = 44.73$$

$(+)\ \sum M_{A} = 0$

$(l_2 \cos\theta) mg - l_1 F_{BD} \sin\gamma = 0$

$$F_{BD} = \frac{l_2 \cos\theta \, mg}{l_1 \sin\gamma} = \frac{(6.4\,m)(\cos 24^\circ)(240\,kg)(9.81\,m/s^2)}{(3.2\,m)\sin 20.73^\circ}$$

$$= \boxed{12{,}150\ N}$$

$\sum F_x = 0$

$F_{BD} \cos\alpha + A_x = 0$

$A_x = F_{BD} \cos\alpha$

$$= \boxed{8632\ N}$$

$\sum F_y = 0$

$A_y - mg + F_{BD} \sin\alpha = 0$

$A_y = mg - F_{BD} \sin\alpha$

$= (240\,kg)(9.81\,\frac{m}{s^2}) - (12{,}150\,N)(\sin 44.73)$

$$= \boxed{6{,}196\ N}$$