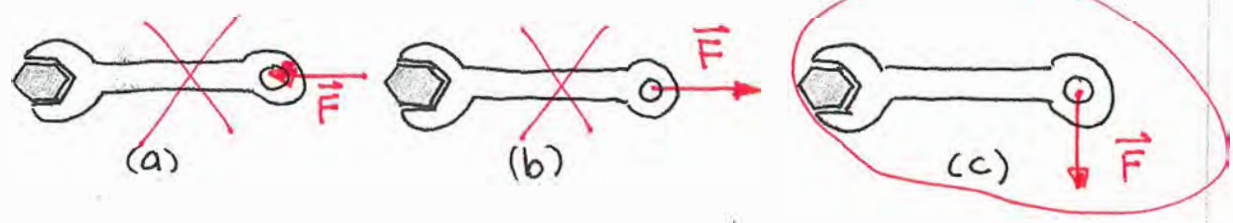
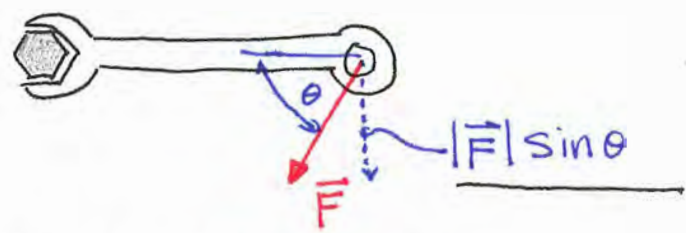


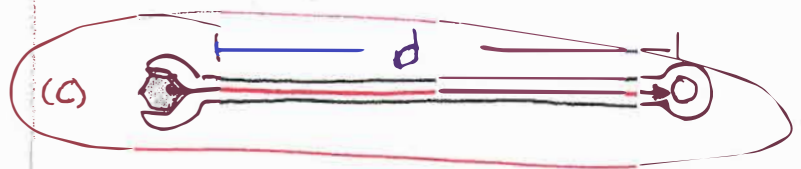
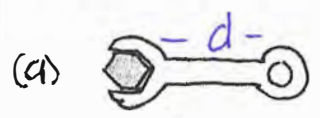
### HOW TO USE A WRENCH:



ONLY THE PART of THE FORCE THAT IS PERPENDICULAR TO THE WRENCH IS USEFUL.



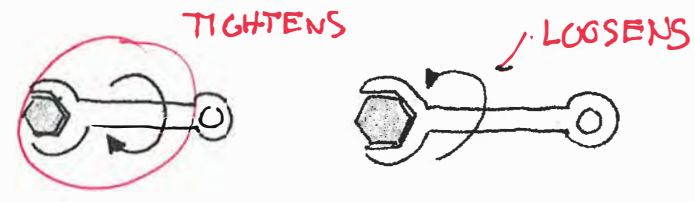
### NOW, PICK YOUR FAVORITE WRENCH:



AND SO THE USEFUL QUANTITY IS

$d|\vec{F}|\sin\theta$

WHICH WAY TIGHTENS BOLT? WHICH WAY LOOSENS IT?



MAGNITUDE & DIRECTION ARE IMPORTANT.

SOUNDS LIKE A VECTOR.

# Formal Definition OF A MOMENT



$$\vec{M}_o \equiv \vec{r} \times \vec{F}$$

POSITION VECTOR FROM O TO F

ABOUT POINT O

FINGERS:  $\vec{r}$  INTO  $\vec{F}$   
 THUMB:  $\vec{M}_o$  DIR.

- $|\vec{M}_o| = |\vec{r}| |\vec{F}| \sin \theta$
- DIRECTION IS  $\perp$  TO  $\vec{r}$  &  $\vec{F}$  USING RIGHT HAND RULE.



OR USE

$$\vec{M}_o = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

MOST USEFUL IN 3-D

PROPERTIES: NOT COMMUTATIVE

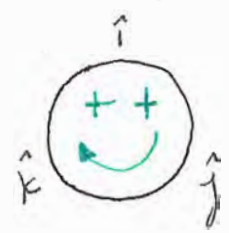
- $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$
- $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
- IF  $\vec{B} = \vec{B}_1 + \vec{B}_2$

$$\vec{A} \times \vec{B} = \vec{A} \times (\vec{B}_1 + \vec{B}_2) = \vec{A} \times \vec{B}_1 + \vec{A} \times \vec{B}_2$$

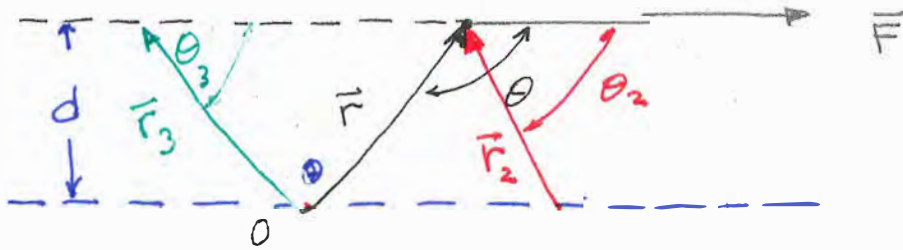
LET'S US FIND

$\vec{M}_o$  BY BREAKING  $\vec{F}$  INTO COMPONENTS.

$$\begin{aligned} \hat{i} \times \hat{i} &= 0 & \hat{i} \times \hat{j} &= \hat{k} & \hat{i} \times \hat{k} &= -\hat{j} \\ \hat{j} \times \hat{i} &= -\hat{k} & \hat{j} \times \hat{j} &= 0 & \hat{j} \times \hat{k} &= \hat{i} \\ \hat{k} \times \hat{i} &= \hat{j} & \hat{k} \times \hat{j} &= -\hat{i} & \hat{k} \times \hat{k} &= 0 \end{aligned}$$



CONSIDER THE MOMENT ABOUT O DUE TO FORCE  $\vec{F}$ .



WHAT IS  $|\vec{M}_O| = ? |\vec{r}| |\vec{F}| \sin \theta$

NOW FIND  $|\vec{r}_2 \times \vec{F}| = |\vec{r}_2| |\vec{F}| \sin \theta_2 = |\vec{r}_3| |\vec{F}| \sin \theta_3$

$$|\vec{r}| \sin \theta = |\vec{r}_2| \sin \theta_2 = \underline{d}$$

COOL THING NUMBER 1: CAN FIND  $\vec{M}_O$  BY USING ANY  $\vec{r}$  FROM O TO ANY PLACE ALONG LINE OF ACTION OF  $\vec{F}$ .

COOL THING NUMBER 2: INFORMAL DEFINITION OF A MOMENT.

A MOMENT ABOUT A POINT IS A "FORCE TIMES A PERPENDICULAR DISTANCE."