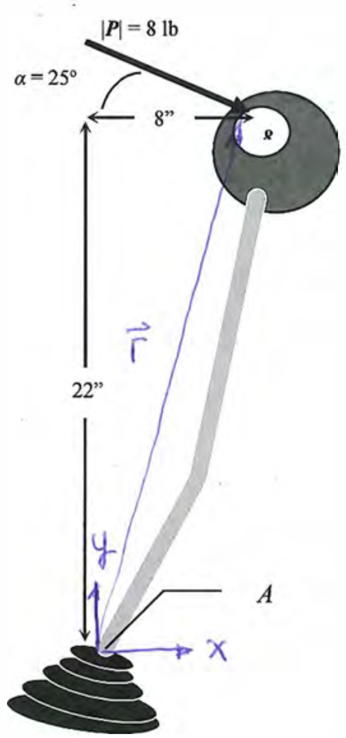
Example

A force of 8 lbs is applied to the gearshift as shown in the figure.

a) Calculate the moment due to the applied force about ~~pint~~ point A using the cross product $\mathbf{r}\times\mathbf{P}$.
b) Calculate the moment about point A by multiplying "perpendicular distance times force."
c) Calculate the moment by breaking $\mathbf{P}$ into components.
d) Which way was easiest, at least in this example?



(a) $\vec{r} = 8\hat{\imath} + 22\hat{\jmath}$ in

$\vec{P} = 8\cos\alpha - 8\sin\alpha$ lb

$= 8\cos 25° - 8\sin 25°$ lb

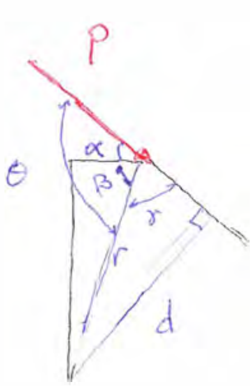
$= 7.25\,\hat{\imath} - 3.381\,\hat{\jmath}$ lb

$$\vec{M}_A = \vec{r}\times\vec{P} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 8 & 22 & 0 \\ 7.25 & -3.381 & 0 \end{vmatrix}$$

$$= [22(0) - 0(-3.381)]\,\hat{\imath} - [(8)(0) - (0)(7.25)]\,\hat{\jmath} + [(8)(-3.381) - (22)(7.25)]\,\hat{k} = \begin{bmatrix} 0 \\ 0 \\ -187 \text{ in}\cdot\text{lb} \end{bmatrix}$$

$$= \boxed{-187\,\hat{k} \text{ in-lb}}$$

(b)



$M_A = (d)(P)$

$d = r\sin\gamma = r\sin\theta$

$\theta = \alpha + \beta$ ✓ $\qquad \beta = \tan^{-1}\left(\frac{22"}{8"}\right) = 70°$

$= 25° + 70° = 95°$

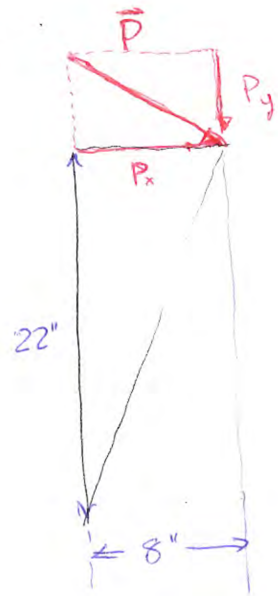
$r = \sqrt{8^2 + 22^2} = 23.4"$

$d = r \cdot \sin\theta = (23.4") \sin(95°) = 23.31$ in

$M_A = (d)(P) = (23.31 \text{ in})(8 \text{ lb}) = 187$ in-lb

$\vec{M}_A = 187$ in-lb ↻ or $\boxed{-187 \text{ in-lb } \hat{k}}$ [Direction from right-hand rule].

(c)



Moment is due to *two* pieces; the moment due to P_x & the moment due to P_y added with the correct sign(s).

Due to P_x

$(+)\ -(22")(P_x) = -(22")(P)(\cos\alpha)$

$= -(22")(8 \text{ lb})(\cos 25°)$

Due to P_y

$(+)\ -(8")(P_y) = -(8")(P)(\sin\alpha)$

$= -(8")(8 \text{ lb})(\sin 25°)$

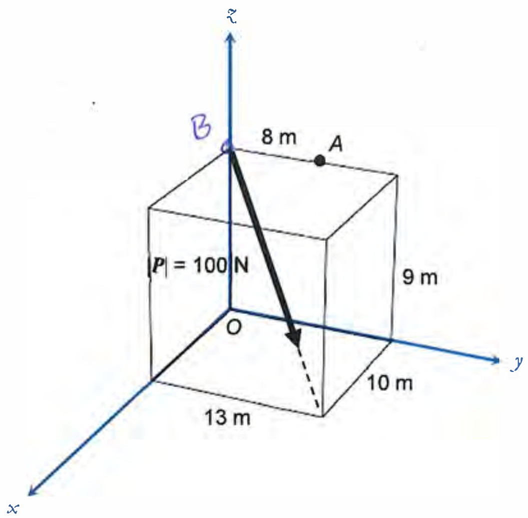
$\therefore (+)\ M_A = -(22")(8 \text{ lb})(\cos 25°) - (8")(8 \text{ lb})(\sin 25°)$

$= \boxed{-187 \text{ in-lb}}$

Example

For the force shown,

(a) find the moment of force **P** about the origin, and
(b) about point A.



(a) We can pick any vector that starts @ O and ends somewhere along the line of action of $\bar{P}$. Pick an easy one!

$$\vec{M}_o = \vec{r}_{OB} \times \vec{P}$$

$$\vec{r}_{OB} = (9\,m)\hat{k}$$

What about $\vec{P}$?

$$\bar{P} = |\bar{P}| \cdot \hat{e}_p = P \cdot \frac{10\hat{\imath} + 13\hat{\jmath} - 9\hat{k}}{\sqrt{10^2 + 13^2 + 9^2}} \frac{\cancel{m}}{\cancel{m}} = 100\,N\,(0.535\hat{\imath} + 0.695\hat{\jmath} - 0.491\hat{k})$$

$$\vec{M}_o = \vec{r}_{OB} \times \vec{P} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & 9 \\ 53.5 & 69.5 & -49.1 \end{vmatrix} = \begin{array}{l} [(0)(-49.1) - (9)(69.5)]\,\hat{\imath} \\ -[(0)(-49.1) - (9)(53.5)]\,\hat{\jmath} \\ +[(0)(69.5) - (0)(53.5)]\,\hat{k} \end{array}$$

$$= \begin{array}{l} -626\,\hat{\imath} \\ +482\,\hat{\jmath} \\ -0\,\hat{k} \end{array} = \boxed{-626\hat{\imath} + 482\hat{\jmath}\ N\cdot m}$$

(b) About point A:

$$\vec{r}_{OA} = -8\hat{j} \ \text{m}$$

$$\vec{M}_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -8 & 0 \\ 53.5 & 69.5 & -48.1 \end{vmatrix} = \begin{aligned} &[(-8)(-48.1) - (0)(69.5)]\,\hat{i} \\ &-[(0)(-48.1) - (0)(53.5)]\,\hat{j} \\ &+[(0)(69.5) - (-8)(53.5)]\,\hat{k} \end{aligned}$$

$$= \boxed{385\,\hat{i} + 428\,\hat{k} \ \ \text{N}\cdot\text{m}}$$