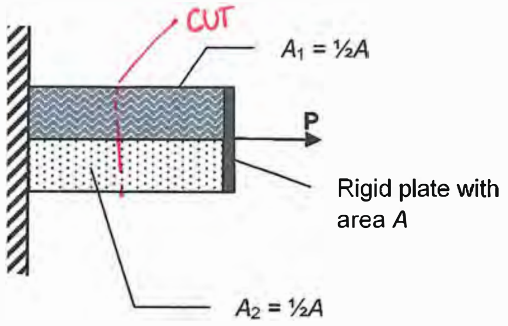
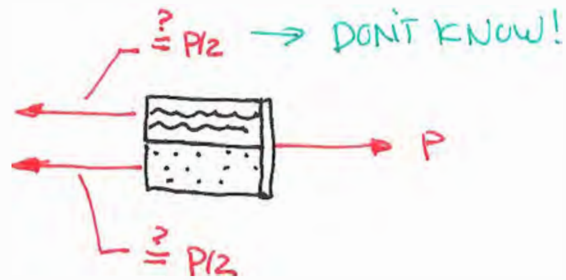
## Example

Two deformable bodies are subjected to an axial load of $P$ as shown in the figure. Draw a free body diagram that would help you to determine the load (axial force) in each material.



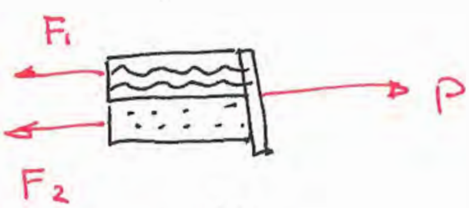




STATICALLY INDETERMINATE

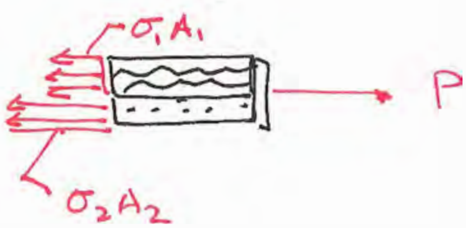
- STATICS BY ITSELF CAN'T GIVE YOU FORCES.
- ALSO NEED DEFORMATION INFORMATION

  → LOOK FOR GEOMETRIC CONSTRAINTS.

CORRECT FBD(S)




OR




EQUIL. GIVES

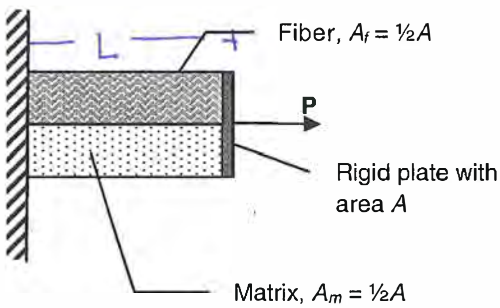
$\xrightarrow{+} \sum F_x = 0$

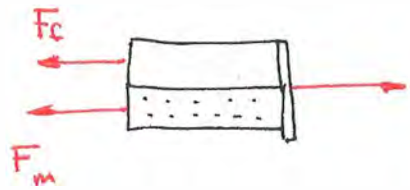
$P - \sigma_1 A_1 - \sigma_2 A_2 = 0$

NEED STRESS/STRAIN RELATION(S) & GEOMETRIC CONSTRAINT(S)

## Example

A composite structure made of fiber ($E_f$ = 231 GPa) and a matrix ($E_m$ = 3.4 GPa) is subjected to an axial load of $P$ as shown in the figure. Find the load carried by the fiber, the load carried by the matrix, and the total deformation of the composite.




$\rightarrow x$

$\Sigma F_x = 0$

$$P = \overset{?}{F_f} + \overset{?}{F_m} \quad (1)$$

$$\overset{?}{\delta_f} = \frac{F_f L}{E_f A_f} = \frac{F_f L}{E_f A/2} \quad (2)$$

LIKEWISE

$$\overset{?}{\delta_m} = \frac{F_m L}{E_m A/2} \quad (3)$$

GEOMETRIC CONSTRAINT

$$\delta_m = \delta_f \quad (4)$$

4 EQNS, 4 UNKNOWNS. WOO-HOO!

(2) = (3)

$$\frac{F_f \cancel{L}}{E_f \cancel{A/2}} = \frac{F_m \cancel{L}}{E_m \cancel{A/2}} \qquad F_f = \frac{E_f}{E_m} F_m$$

SUB INTO (1)

$$P = \frac{E_f}{E_m} F_m + F_m$$

$$\boxed{F_m = P\left[\frac{E_m}{E_f + E_m}\right]}$$

$$\boxed{F_f = P\left[\frac{E_f}{E_f + E_m}\right]}$$

FROM (3)

$$\delta = \frac{F_m L}{E_m A/2} = \frac{P\left[\frac{\cancel{E_m}}{E_f + E_m}\right] L}{\cancel{E_m} A/2} = \frac{PL}{\left[\frac{E_f + E_m}{2}\right] A} = \frac{PL}{E_{eff} A}$$

where $E_{eff} = \dfrac{E_m + E_f}{2}$

USING VALUES of E.

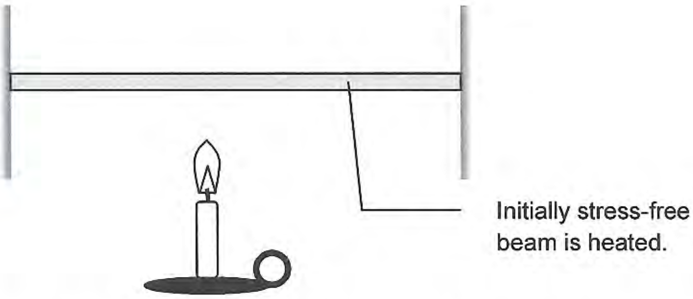
$$F_m = \ldots = 0.015P$$

$$F_f = \ldots = 0.985P$$

$$E_{Eff} = 117.2 \text{ GPa}$$

## Example

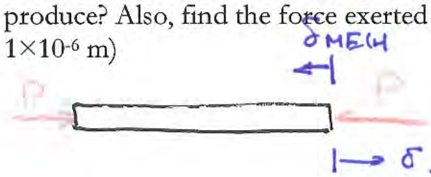
A thin rod suspended between two fixed supports is initially in a stress free state. The rod is then uniformly heated resulting in a temperature change of the rod of $\Delta T$. Because of the heating, the rod wants to expand. However, the fixed supports prevent this from happening resulting in a compressive stress in the rod.




a. Find an expression for the resulting stress in the rod in terms of Young's modulus $E$, the thermal expansion coefficient $\alpha$, and the temperature change $\Delta T$. Assume that the thermal expansion coefficient is constant.

b. If the rod is made of $SiO_2$ with $E$ = 69 GPa and $\alpha = 0.55 \times 10^{-6}$ /°C, what stress will a 10°C temperature change produce? Also, find the force exerted on a rod with a square cross section with side length $a$ = 10 μm. (1 μm = $1\times10^{-6}$ m)

a.




$$\delta_{TOT} = \delta_{MECH} - \delta_T = \ldots 0!$$

$$\delta_{MECH} = \delta_T \Rightarrow \varepsilon_T = \alpha\Delta T = \frac{\delta_T}{L}$$

$$\delta_T = \alpha\Delta T \cdot L$$

$$\delta_{MECH} = \alpha\Delta T \cdot L$$

HOOKE'S LAW

$$\sigma = E\varepsilon_{MECH} = E\left(\frac{\delta_{MECH}}{L}\right) = E\left(\frac{\alpha\Delta T\cdot \cancel{L}}{\cancel{L}}\right) = \boxed{E(\alpha\Delta T)}$$

$$\sigma = E\varepsilon_T$$

b. $$\sigma = E\alpha\Delta T = 69\times10^9\,\text{Pa}\cdot 0.55\times10^{-6}/{}^\circ\text{C} \times 10^\circ\text{C}$$

$$= 380\ \text{kPa}$$

NOTE THAT IT IS COMPRESSIVE, BUT ASSUMED IT WAS TOO!

SEE WHY A SIGN CONVENTION IS IMPORTANT?

$$\sigma = \frac{P}{A}$$

$$P = |\sigma| A = \sigma a^2 = +380\ \text{kPa}\cdot(10\times10^{-6})^2\,\text{m}^2 = \boxed{38.0\ \mu\text{N}}$$