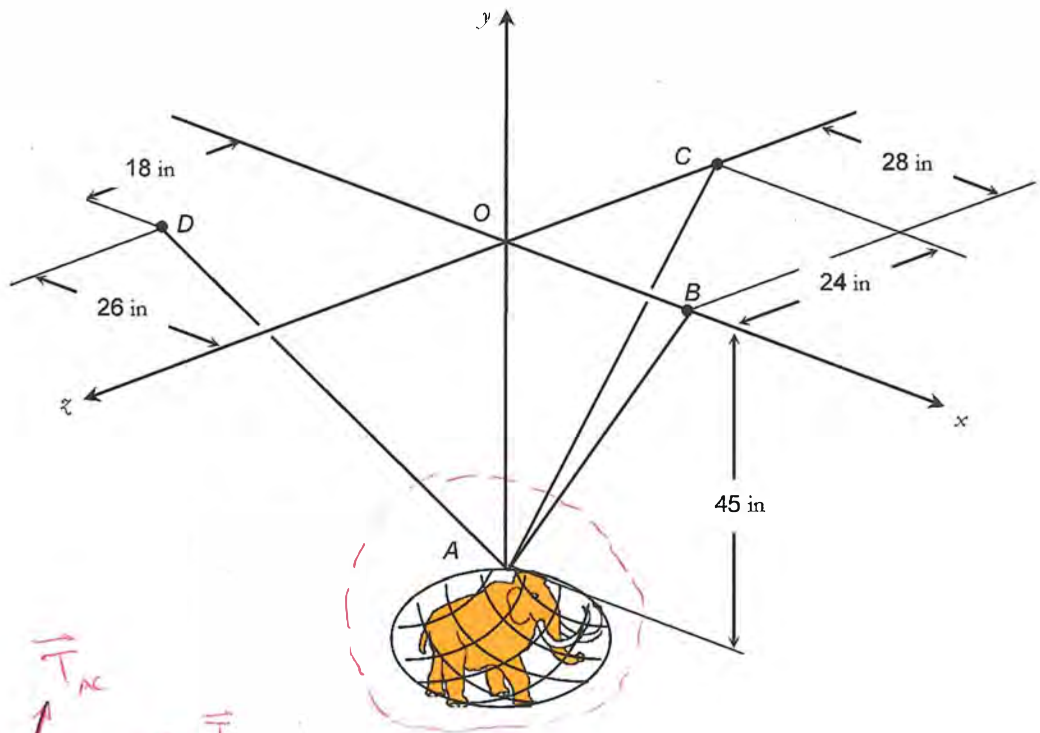
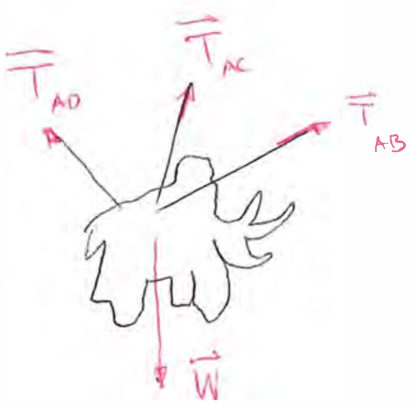
Example

A woolly mammoth has been caught up in the web of a giant alien spider. If the mammoth is suspended by three threads with the lengths/orientations shown in the figure, find the weight of the mammoth. The tension in thread *AB* is 1378 lb.





$$\vec{T}_{AB} = T_{AB}\,\hat{e}_{AB}$$

$$\hat{e}_{AB} = \frac{28''\,\hat{i} + 45''\,\hat{j} + 0\,\hat{k}}{\sqrt{28^2 + 45^2 + 0^2}\ \text{in}} = \frac{28}{53}\,\hat{i} + \frac{45}{53}\,\hat{j}$$

$$\therefore\ \vec{T}_{AB} = \left(1378\cdot\frac{28}{53}\,\hat{i} + 1378\cdot\frac{45}{53}\,\hat{j}\right)\ \text{lb}$$

$$= \begin{bmatrix} 1378\cdot\frac{28}{53} \\ 1378\cdot\frac{45}{53} \\ 0 \end{bmatrix}\ \text{lb}$$

$$\vec{T}_{AC} = T_{AC}\,\hat{e}_{AC}$$

$$\hat{e}_{AC} = \frac{0\hat{i} + 45\hat{j} - 24\hat{k}}{\sqrt{45^2 + 24^2}\ \text{in}} = \frac{45}{51}\hat{j} - \frac{24}{51}\hat{k}$$

$$\vec{T}_{AC} = \begin{bmatrix} 0 \\ \frac{45}{51} \cdot T_{AC} \\ -\frac{24}{51} \cdot T_{AC} \end{bmatrix}$$

$$\vec{T}_{AD} = T_{AD}\,\hat{e}_{AD} \qquad \hat{e}_{AD} = \frac{-26\hat{i} + 45\hat{j} + 18\hat{k}\ \text{in}}{\sqrt{26^2 + 45^2 + 18^2}\ \text{in}}$$

$$= \frac{-26}{55}\hat{i} + \frac{45}{55}\hat{j} + \frac{18}{55}\hat{k}$$

$$\vec{T}_{AD} = \begin{bmatrix} -\frac{26}{55} T_{AD} \\ \frac{45}{55} T_{AD} \\ \frac{18}{55} T_{AD} \end{bmatrix}$$

$$\vec{W} = -W\hat{j} = \begin{bmatrix} 0 \\ -W \\ 0 \end{bmatrix}$$

Equilibrium requires

Not (-)

$$\Sigma \vec{F} = \vec{0}$$

$$\vec{T}_{AB} + \vec{T}_{AC} + \vec{T}_{AD} + \vec{W} = \vec{0}$$

$$\begin{bmatrix} 1378 \cdot \frac{28}{53} \\ 1378 \cdot \frac{45}{53} \\ 0 \end{bmatrix} \text{lb} + \begin{bmatrix} 0 \\ \frac{45}{51} T_{AC} \\ \frac{-24}{51} T_{AC} \end{bmatrix} + \begin{bmatrix} \frac{-26}{55} T_{AD} \\ \frac{45}{55} T_{AD} \\ \frac{18}{55} T_{AD} \end{bmatrix} + \begin{bmatrix} 0 \\ -W \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\sum F_x = 0$ (x- component)

$$1378 \cdot \frac{28}{53}\text{ lb} + 0 - \frac{26}{55} T_{AD} + 0 = 0$$

$$\ldots \boxed{T_{AD} = 1538 \text{ lb}}$$

$\sum F_y = 0$

$$1378 \cdot \frac{45}{53} + \frac{45}{51} T_{AC} + \frac{45}{55} (T_{AD}) - W = 0$$

$$1378 \cdot \frac{45}{53} + \frac{45}{51} T_{AC} + \frac{45}{55} (1538 \text{ lb}) - W = 0 \qquad (1)$$

$\sum F_z = 0$

$$0 - \frac{24}{51} T_{AC} + \frac{18}{55} (T_{AD}) + 0 = 0$$

$$-\frac{24}{51} T_{AC} + \frac{18}{55} (1538 \text{ lb}) = 0$$

$$\ldots \boxed{T_{AC} = 1070 \text{ lb}}$$

Substitute into (1)

$$\ldots \boxed{W = 3370 \text{ lb}}$$