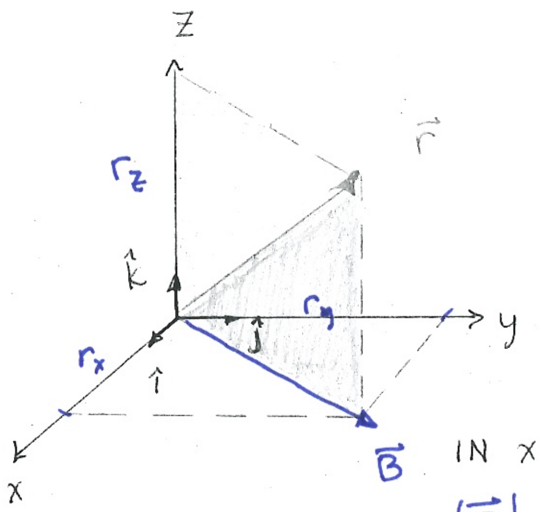
VECTORS.. IN... SPACE



IN COMPONENT FORM

$$\vec{r} = r_x \hat{\imath} + r_y \hat{\jmath} + r_z \hat{k}$$

MAGNITUDE?

IN x-y PLANE

$$|\vec{B}| = \sqrt{r_x^2 + r_y^2}$$

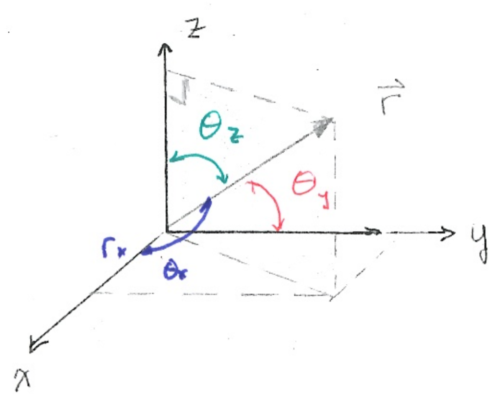
IN SHADED PLANE

$$r = \sqrt{B^2 + r_z^2}$$

HENCE

$$\boxed{|\vec{r}| = \left(r_x^2 + r_y^2 + r_z^2\right)^{1/2}}$$

DIRECTION?



$$\frac{r_x}{r} = \cos\theta_x \rightarrow r_x = r\cos\theta_x$$

$$\frac{r_y}{r} = \cos\theta_y$$

$$\frac{r_z}{r} = \cos\theta_z$$

$$\vec{r} = r\cos\theta_x \; \hat{\imath} + r_y \cos\theta_y \; \hat{\jmath} + r_z \cos\theta_z \; \hat{k}$$

$$= r\left[\cos\theta_x \; \hat{\imath} + \cos\theta_y \; \hat{\jmath} + \cos\theta_z \; \hat{k}\right]$$

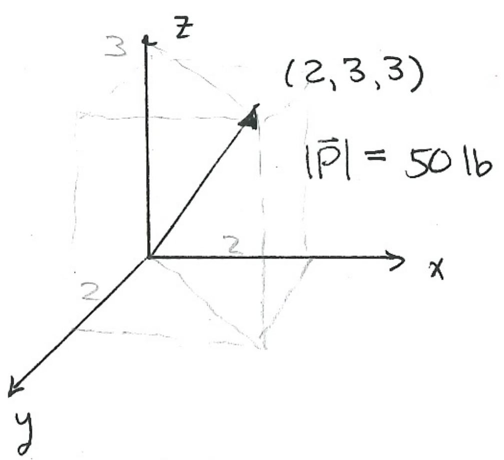
DIRECTION COSINES

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z \stackrel{?}{=} 1 \,!$$ (DIMENSIONLESS)

UNIT VECTOR IN DIRECTION OF $\vec{F}$

$$\hat{e}_{\vec{F}} = \cos\theta_x \,\hat{\imath} + \cos\theta_y \,\hat{\jmath} + \cos\theta_z \,\hat{k}$$

DEFINING A DIRECTION IN 3-D SPACE



1) FIND COOR. OF HEAD

2, 3, 3

2) FIND COOR. OF TAIL.

0, 0, 0

3) SUBTRACT TAIL FROM HEAD. (POSITION VECTOR)

$$\vec{r} = (2-0)\,\hat{\imath} + (3-0)\,\hat{\jmath} + (3-0)\,\hat{k}$$

$$2\hat{\imath} + 3\hat{\jmath} + 3\hat{k}$$

4) FIND UNIT VECTOR IN DIRECTION OF $\vec{P}$

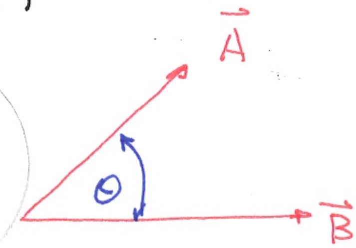
$$\boxed{\hat{e}_{\vec{P}} = \frac{\vec{r}_P}{|\vec{r}_P|}} = \frac{2\hat{\imath} + 3\hat{\jmath} + 3\hat{k}}{\sqrt{2^2 + 3^2 + 3^2}}$$

$$= 0.43\,\hat{\imath} + 0.64\,\hat{\jmath} + 0.64\,\hat{k}$$

$$\vec{P} = P\hat{e}_P = 50\text{ lb}\;\hat{e}_P = 21.5\,\hat{\imath} + 31.5\,\hat{\jmath} + 31.5\,\hat{k} \text{ lb}$$

DOT PRODUCT (AKA SCALAR PRDT)

$$\boxed{\vec{A}\cdot\vec{B} = AB\cos\theta}$$



$(\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A})$

$$\vec{A}\cdot\vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k})\cdot(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$

$$= A_xB_x\,\cancel{\hat{\imath}\cdot\hat{\imath}}^{\,1} + A_xB_y\,\cancel{\hat{\imath}\cdot\hat{\jmath}}^{\,0} + \cancel{\dots}$$

$$+ \cancel{\dots} + A_yB_y\,\cancel{\hat{\jmath}\cdot\hat{\jmath}}^{\,1} + \cancel{A_yB_z\,\hat{\jmath}\cdot\hat{k}}^{\,0}$$

$$+ \cancel{\dots} + \cancel{\dots} + A_zB_z\,\cancel{\hat{k}\cdot\hat{k}}^{\,1}$$

$\hat{\imath}\cdot\hat{\imath} = ?$ 1   0   0

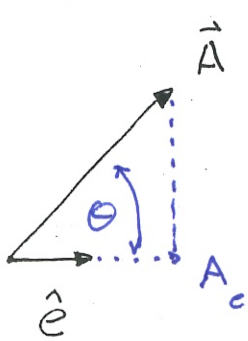
$\hat{\imath}\cdot\hat{\jmath} = ?$ 0 $\hat{\jmath}\cdot\hat{\jmath} = ?$ 1   0

$\hat{\imath}\cdot\hat{k} = ?$ 0   0   1

SO:

$$\boxed{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z}$$

INTERPRETATION & ULTILITY:



(UNIT VECTOR)

$\vec{A}\cdot\hat{e} = (A)(1)\cos\theta$

COMPONENT of $\vec{A}$ IN DIRECTION of $\hat{e}$

ALSO USE TO FIND ANGLE BETWEEN VECTORS:

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} \rightarrow \theta$$