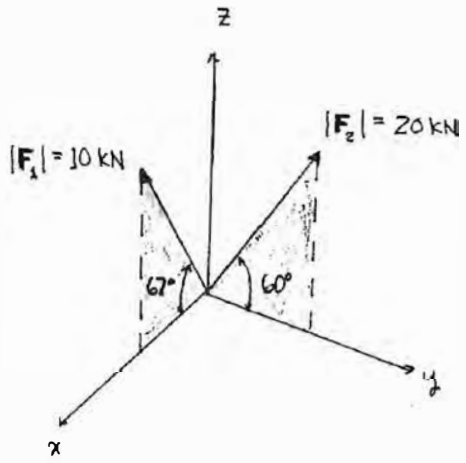
Example

For the forces shown in the figure, find

a) the magnitude of the resultant R, and
b) the angles 0_x, 0_y, and 0_z between the line of action of the resultant and the coordinate axes.



a). $\vec{F}_1 = 10\text{kN} \cdot \cos 67^\circ \hat{i} + 0\hat{j} + 10 \cdot \sin 67^\circ \hat{k}$

$= 3.907\,\hat{i} + 9.205\,\hat{k}$

$\vec{F}_2 = 0\hat{i} + 20 \cdot \cos 60^\circ \hat{j} + 20 \sin 60^\circ \hat{k}$

$= 10\hat{j} + 17.321\,\hat{k}$

$\vec{R} = 3.907\,\hat{i} + 10\hat{j} + 26.526$

$|\vec{R}| = (R_x^2 + R_y^2 + R_z^2)^{1/2} = (3.907^2 + 10^2 + 26.526^2)^{1/2}$

$= \boxed{28.6\text{KN}}$

b) $\theta_x = \cos^{-1}(R_x/R) = \cos^{-1}(3.907/28.6) = 82.1^\circ$

$\theta_y = \cos^{-1}(R_y/R) = \cos^{-1}(10/28.6) = 69.5^\circ$

$\theta_z = \cos^{-1}(R_z/R) = \cos^{-1}(26.5/28.6) = 22.1^\circ$

WHAT IS $\hat{e}_R = ?$

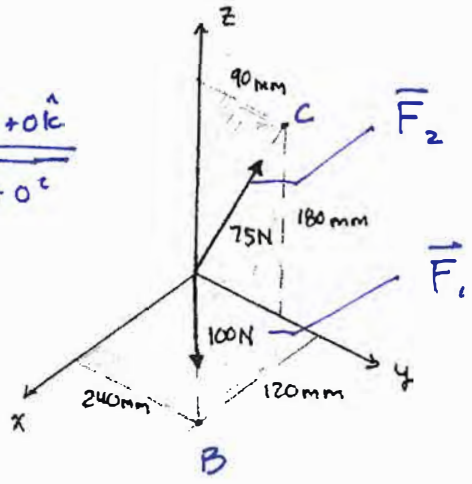
Example

Find the magnitude and the direction of the resultant of the two forces shown.

$$\vec{F}_1 = F_1 \cdot \hat{e}_{F_1} = F_1 e_{OB}$$

$$\hat{e}_{OB} = \frac{(120-0)\hat{i} + (240-0)\hat{j} + 0\hat{k}}{\sqrt{(120-0)^2 + (240-0)^2 + 0^2}}$$

$$= 0.45\hat{i} + .89\hat{j} + 0\hat{k}$$



$\therefore$

$$\vec{F}_1 = 45\hat{i} + 89\hat{j}$$

YOU DO $\vec{F}_2$

$$\vec{F}_2 = F_2 \hat{e}_{F_2} = F_2 \hat{e}_{OC}$$

$$\hat{e}_{OC} = \frac{0\hat{i} + 90\hat{j} + 180\hat{k}}{\sqrt{0^2 + 90^2 + 180^2}}$$

$$= 0\hat{i} + 0.45\hat{j} + 0.89\hat{k}$$

$$\vec{F}_2 = 75\,N \cdot \hat{e}_{OC}$$

$$= 0\hat{i} + 34\hat{j} + 67\hat{k}$$

$$\vec{R} = \vec{F}_1 + \vec{F}_2 = (45+0)\hat{i} + (89+34)\hat{j} + (0+67)\hat{k}$$

$$= 45\hat{i} + 123\hat{j} + 67\hat{k}$$

$$|\vec{R}| = (45^2 + 123^2 + 67^2)^{1/2} = \boxed{147.\ N}$$

$$\hat{e}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{45}{147}\hat{i} + \frac{123}{147}\hat{j} + \frac{67}{147}\hat{k}$$

$$= \boxed{0.306\hat{i} + 0.837\hat{j} + 0.456\hat{k}}$$

EXTRA!

FIND COMPONENT of $\vec{F}_2$ IN DIRECTION of $\vec{F}_1$.

$$F_{2\,in\,1} = \vec{F}_2 \cdot \hat{e}_{F_1} = \vec{F}_2 \cdot \hat{e}_{OB}$$

$$= (0\hat{i} + 34\hat{j} + 67\hat{k}) \cdot (0.45\hat{i} + 0.89\hat{j} + 0\hat{k})$$

$$= 0 \quad (34 \times 0.89)$$

$$= \quad 30.3$$