

## Responses to comments made by the reviewer.

### Structural

- I agreed with the reviewer's comments about the need for a proper introduction. A new Section 1 was created which contains the former first paragraph of Section 4 (properly reworded). It briefly mentions the alternate completions and then gives a general summary or outline of what is contained the subsequent sections.

In addition, I deleted what was the first few sentences of the paper as suggested.

### Other comments

- I agreed with the unlabeled suggestions the reviewer gave and made the appropriate changes throughout the paper. However instead of  $|\cdot| : \mathbb{Q} \rightarrow \mathbb{R}$ , I inserted  $|\cdot| : \mathbb{Q} \rightarrow \mathbb{Q}$  as this is more representative of the image space.
  - I agreed with the following comments the reviewer gave and made the appropriate changes: (pg2, l2); (p2, l10); (p2l11); (p2, ln12); (p2, Theorem 1); (p2, ln-11,-10); (p2,l-7); (p2,l-3,l-2); (p3, Definition 2); (p4, l2); (p4, Theorem 2); (p4, l9); (p4, l13); (p4, l-7); (p5, l3-5); (p5, l3,6,13); (p5, Definition 3); (p5,l-9); (pg5,l-8); (p5,l-5); (p6, l10); (p6, Theorem 4), (p6, Theorem 5); (p7,l-4); (p8, l6).
- (p2,l5) I agreed and removed the word “naively” and stated “Since the geometric series test still applies to the  $p$ -adic absolute value and to convergence in  $\mathbb{Q}_p$ , then it gives the following.”
- (p2,l18) I agreed and inserted “in fact, it is a subring of  $\mathbb{Q}$ ”.
- (p4,l9) I chose not to eliminate the  $\epsilon_i$  and  $\delta_i$  notation as I felt using  $v(\eta_i)$  and  $v(q_i)$  was both visually and conceptually distracting.
- (p4, l10) I agreed with the reviewer's point that (i) was not obvious from the statement of Theorem 1. I inserted the suggested remark following the proof of Theorem 1:

Note in the proof of Theorem 1 that if  $q \neq 0$ , then in fact  $p \nmid \bar{q}$ , and therefore  $v(q) = v(\sigma) - v(\tau) \leq 0$ ; that is  $|q|_p \geq 1$ .

Then in Theorem 2, for part (i), I stated

By Theorem 1 and the note afterwards,  $v(q_i) = v(\eta_{i-2}) - v(\eta_{i-1})$ . Thus  $\delta_i = \epsilon_{i-2} - \epsilon_{i-1}$ .

I did not feel like the reviewer's suggested inequalities were needed to justify this point further.

However, in the process of examining the reviewer's suggestion, I realized that there was a misstatement in the proof of Theorem 1. Originally it stated

where  $\bar{q} \equiv st^{-1} \pmod{p^{v(\tau)-v(\sigma)+1}}$  and  $\bar{q} \in \{\frac{1-p}{2}, \dots, \frac{p-1}{2}\}$ . Since  $v(\sigma) \leq v(\tau)$ , then  $|q|_\infty \leq |\bar{q}|_\infty < \frac{p}{2}$ .

This was corrected to

where  $\bar{q} \in \mathbb{Z}$  such that  $\bar{q} \equiv st^{-1} \pmod{p^{v(\tau)-v(\sigma)+1}}$  and  $|\bar{q}|_\infty < p^{v(\tau)-v(\sigma)+1}/2$ . Then  $|q|_\infty < \frac{p}{2}$ .

With these corrections made and the remark added after Theorem 1, both Theorem 1 and Theorem 2 should now stand on more solid ground.

- (p5, Def 3) I agreed and inserted “ $f \in \mathbb{Q}$ ”. I also agreed with the suggested rephrasing and inserted it.
- (p5,l10,11) I agreed and stated more explicitly the properties that the rational gcd satisfies. I did not include a proof as I feel the reader should be able to see that it is true with little inspection. I agreed with the other comment about the fractions needing to be in lowest terms.
- (p7, Sec 3 end) I agreed with the reviewer's suggestion and added a corollary to explicitly state the conditions under which the  $p$ -adic Euclidean algorithm terminates.
- (p8, bib) I appreciated the reviewer informing me of this text, and after inspection have decided to include it in the bibliography and a citation for it in Section 2.
- I agreed with the reviewer's comments about comma usage and tried to be more careful throughout the paper, taking into consideration the reviewer's common situations where they should and should not be used.