Referee's report for "General Model for Variations of the Even Cycle Problem" by Kunkar, Macatangay and Pepich.

This is a well-written paper concerning the problem of finding even-length cycles in graphs and directed graphs, both of which may be 2-colored. There are two main components to the paper. The first shows that several variations of the even cycle problem can be transformed to problems about bipartite graphs or bipartite directed graphs. The second shows that one of these variations is NP-complete in two ways: first by showing that it is polynomial time equivalent to another variation that is known to be NP-complete, and second by giving a polynomial time reduction from the 3-satisfiability problem. I recommend publication, subject to addressing the following (mostly minor) suggestions/corrections.

- Add an "A" in the title: "A General Model...."

- Page 2: In the paragraph that begins "The classic even cycle problem..." the phrase "put forth" is used in consecutive sentences. Perhaps change the second instance to something else.

- Page 2: In the paragraph that begins "Another even cycle variation...," remove the period before "(i.e. color 1, color 2,...." Also, the last sentence states "A method for determining if such a cycle exists was solved by...." Reword this to reflect the fact that the problem is being solved, not the method.

- Page 2, third paragraph from the bottom: in the third line, change "direct path" to "directed path." In the fourth line, remove the comma after Gannon [1].

- Page 3, third line from the top: the two NP-complete problems should be AAC and ADC, and the unknown should be AD.

- Page 3, a general comment about section 2: Since you are transforming problems about graphs, directed graphs, and 2-colored graphs and directed graphs to problems about bipartite graphs and bipartite directed graphs, you might mention if any work as been done toward solving even cycle type problems on bipartite graphs, or if bipartite graphs have proved useful in any of the known algorithms that you reference.

- Page 3: Perhaps rewrite the first two paragraphs of section 2. I would start with the first two sentences of the second paragraph, to let the reader know what the section is about. You can also include the note in the first paragraph, although I would leave off the asterisks. Then introduce the definition of choice cycle (and note in the definition that B may also be a directed graph) and wrap it up with the last sentence in paragraph 2.

- Page 3, paragraph 3 of section 2: here you use (x_i, y_j) to represent a directed edge, but later you use $x_i \to y_j$. You should choose one or the other and be consistent throughout the paper.

- Page 3, sixth line from the bottom: move the period after "corresponding bipartite graph has directed cycles" to after " $\cdots \to x_{i_k} \to y_{i_1}$"

- Page 3, last sentence: mention that the two directed cycles in the bipartite graph are choice cycles, and briefly explain why. This also applies to the description of the AC problem on page 4.

- Page 4, 5th line from the bottom: change "Green in/ and Red out" to "Green in/Red out."

- Page 5, first line after Figure 3: change

$$1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$$

  to

$$y_1 \rightarrow x_2 \rightarrow y_4 \rightarrow x_3 \rightarrow y_1$$

- Page 5, paragraph beginning "4. Definition:"

  - Anti-directed cycle is already defined on Page 2, so you don't need the definition here.
  - Here and a few other places you refer to an "anti-directed graph" when you mean to be referring to an anti-directed cycle in a directed graph (the graph itself isn't anti-directed).
  - 5th line: the phrase "edges are translated using the same" needs fixing.
  - 5th line: change "through" to "though"

- Page 6, first line: As mentioned above, "anti-directed graph" needs fixing.

- Page 6: The last paragraph is a little awkward. Perhaps change the first sentence to "Therefore, each of these five variations of the even cycle problem can be translated into the problem of finding a choice cycle or choice directed cycle in a bipartite graph or directed graph." and eliminate the second sentence.

- Page 7: Give a brief description of the 3-SAT problem for those readers who may be unfamiliar with it.

- Page 7: in the proof, you use the abbreviation AAC to mean an alternating, anti-directed cycle, but you've previously used the same abbreviation to represent the problem of finding such a cycle. I would replace each instance of "AAC" in the proof with "alternating, anti-directed cycle"

- Page 7: Change semicolon after "two disjoint sets" to a colon.

- Page 7: The definition of the set X is awkward. Change it to $X = \{x_1, x_2, \ldots, x_{m+2}\}$ or $X = \{x_i : 1 \leq i \leq m + 2\}$. Also fix the spacing/line breaks in the following lines.

- Page 8, last paragraph: change the period after "third set of edges" to a comma, and left justify the text following the definition of Q (beginning "where if the variable...").

- Page 10, last paragraph: Change "$\Leftrightarrow$" to "if and only if"

- Page 11: Fix unnecessary line breaks.

- Proof of Theorem 2: You should mention briefly why your reduction from 3-SAT to AAC is a polynomial time reduction.