

Referee's comments on

Proof of a special case of the Yelton-Gaines Conjecture on Isomorphic Dessins

by

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The term dessin d'enfant refers to a bipartite graph that is embedded in a compact, oriented Riemann surface. Every dessin is determined by an ordered pair of permutations from a symmetric group.

A conjecture, first formulated in 2007 by Yelton, refined in 2008 by one of the present authors states: Let (ρ_0, ρ_1) and (ρ'_0, ρ'_1) be two ordered pairs of permutations in S_n for which there is a group isomorphism $\phi : \langle \rho_0, \rho_1 \rangle \rightarrow \langle \rho'_0, \rho'_1 \rangle$ between the corresponding subgroups of S_n which is a local conjugation in S_n and that maps ρ_0 to ρ'_0 and ρ_1 to ρ'_1 . If at least one of the four permutations is a product of t -cycles for some divisor t of n , then there is a permutation $\tau \in S_n$ that conjugates ρ_0 to ρ'_0 and ρ_1 to ρ'_1 .

The conclusion of the conjecture can be restated to say that the two dessins defined by the ordered pairs of permutations are isomorphic.

In the paper, the special case when n is even and all four permutations considered are involutions without a fixed point is considered.

This case is a natural starting point in the attempt to prove the conjecture in the full generality. The involutions without fixed points make use of the bipartite nature of the underlying graphs. Nevertheless, the proof of the conjecture for this elementary case requires several technical lemmas and theorems and is challenging enough.

Overall, this neat short paper is written nicely, all arguments and proofs are stated and explained clearly and with the right amount of detail. The whole problem is nicely set into the context. Therefore, **I recommend the paper for publication in the RHIT Math Journal.**

As for possible improvements, I would like the authors to include more details on the background directly in to the text – instead of saying "see [XY] reference for details". All of the references listed in the paper are unpublished and not easily available. In particular, more details should be provided on how the Gassmann's equivalent dessins arise from the action of group elements on the cosets G/H and G/H' and the consequent pair of permutations.

Perhaps the word "Proof" in the title could be omitted to simplify the title.

In what follows, I include just a short list of typos or comments numbered by a page number followed by a line number. All of these can be easily addressed.

1. (2,-11): with *** denoting product of **2**-cycles ...

2. (3,-5): each pair c_i, d_i **is** as ...
3. (4, 5): ρ_1 should be ρ'_1 and all the elements in the 2-cycles are missing primes above them as well
4. (4, 6): ρ_2 should be ρ'_2 and all the elements in the cycles are missing primes as well