

This is an impressive article on an interesting subject. I recommend publication, subject to the revisions suggested below.

#### General suggestions:

1. I believe that readers will find the opening (toy car) example to be interesting and helpful. However, a better transition from this example to the general context would be useful. This example suggests a very concrete problem. In the chart on page two, a reader may be led to assume that the number of cars must be a multiple of 2 and also a multiple of 3. Fair enough. But then the author talks about convex combinations of certain divisions. If we are thinking about real toy cars, it is hard to know what to make of this. Some sort of explanation for this transition would be helpful. On a related note, the term “single homogeneous object” in the Introduction will probably be confusing to readers, since it is hard to view a collections of toy cars as a single homogeneous object.
2. The beginning example provides a nice example of the set  $D$ . I suggest the author explicitly make this point following the proof of the theorem.
3. The wording of Definition 5 could use a little work. Does it mean that if you have  $n+1$  vectors and take all convex combinations, then what you get is called a simplex if and only if it is  $n$ -dimensional? Is this the same as simply saying that you get a simplex whenever you take all convex combinations of a linearly independent set?
4. On page 5, in the proof of Claim 1.3, I don't understand how a division is a closed half space. But the proof of Claim 1.3 needs nothing fancy. It is easy to see the truth of this claim quite directly. I think an informal explanation (of at most a few sentences) would suffice.
5. I'm a little stumped by Section 3. Is this just, in effect, saying that the statement of the theorem makes sense? Perhaps there needs to be some clarification of terminology. When the author refers (in the statement of the theorem) to “... the simplex determined by  $D$ ”, is there some assumption that the set  $D$  is independent? Does any collection of vectors determine a simplex? And, in the first paragraph of Section 3, I'm not sure what exactly “the vertices for the set of envy-free divisions” means.

#### Small items and typos:

1. The chart on the top of page 3 can be better presented. It took a little work to parse the two lines of text on top.
2. The last sentence before “3. Fair Division” has a “the” that shouldn't be there.
3. In the last paragraph on page 3, should “...motivating example convex combination division...” be “...motivating example **of** convex combination division...”?
4. On page 4, I would suggest “By interchanging person indices **if necessary**...”

5. Additional care needs to be taken in the use of the word "portion". Sometimes it is used to mean only the part of the good being divided, and sometimes it includes the money given or received.
6. On page 5, in the first line of the proof, I'd suggest "... $(x,d)$  is a convex combination **of elements** of  $D$ ."
7. On page 5, in the proof of Claim 1.1, " $\frac{1}{n}b_j > \frac{1}{n}b_n$ " should be " $\frac{1}{n}b_j \geq \frac{1}{n}b_n$ ".
8. On page 5, in the proof of Claim 1.2, I think the author is almost right. The proof is **not** exactly the same as for Claim 1.1. I think one ">" becomes a "≥", and one "≥" becomes a ">", but the proof still works fine. I don't think it's worth going through the details again. I'd suggest simply saying something like "the proof is similar ...".
9. On page 6, at the end of the first paragraph in the proof of Claim 2.1, I believe it should read " $\alpha_m > 0$ " instead of " $\alpha_m = mx_m$ ".
10. On page 7, in the second-to-last line before Claim 2.4, there should be a "+" after the opening " $x_1$ ".
11. On page 8, in the second sentence of Section 3, I'd change "...is the convex combinations of the ..." to "...is the **set of** convex combinations of the ...".
12. On page 9, in the statement of Claim 3.1, some clarification is needed. Actually, each side of equation (10) is an ordered pair, and each component of each ordered pair is a vector. So, the reference to the " $j^{\text{th}}$  component of equations (10)" is not quite right.
13. In the conclusion, the author asserts that the Brams and Taylor procedure yields an efficient and envy-free division. I do not think "efficient" is correct here.