

## ABSTRACT

TIMOTHY FERDINANDS AND LANDON KAVLIE

In 1953, Beckman and Quarles proved a well-known result in Euclidean Geometry that any transformation preserving a distance  $\rho$  must be a rigid motion. In 1991, June Lester published an analogous result for circle-preserving transformations in the complex plane. In our paper, we introduce the notion of dual numbers and the geometry of the dual plane. We focus on the set of vertical parabolas and non-vertical lines  $\mathcal{P}$  with a distance between pairs of parabolas defined to be the difference of slopes at their point(s) of intersection. We then prove that any bijective transformation from  $\mathcal{P}$  to itself which preserves our distance 1 induces a fractional linear or Laguerre transformation of the dual plane.

DEPARTMENT OF MATHEMATICS AND STATISTICS, 1740 KNOLLCREST CIRCLE SE, CALVIN COLLEGE, GRAND RAPIDS, MICHIGAN 49546-4403 USA  
*E-mail address:* `tdf3@calvin.edu`

DEPARTMENT OF MATHEMATICS AND STATISTICS, 1740 KNOLLCREST CIRCLE SE, CALVIN COLLEGE, GRAND RAPIDS, MICHIGAN 49546-4403 USA  
*E-mail address:* `ljk7@calvin.edu`

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*Date:* April 15, 2009.

Both authors supported by the National Science Foundation under Grant No. DMS-0702939, Faculty Advisor Michael Bolt.