

1. REFEREE'S REPORT TO THE EDITOR

The paper take the reader from an introduction to error-correcting codes to a generating function (encapsulating the most important features of the code) to a zeta function and a “Riemann Hypothesis”.

The introduction and motivations need **much** work. There is also significant editing needed for typos (big, small, in math and in text).

2. WHY THIS PAPER SHOULD BE PUBLISHED

Error-correcting codes are found in all types of electronic devices (cell phones, hard drives, CDs and DVDs, and the protocols that make the Internet work) as well as many military applications (some mentioned in the paper). The mathematical study of these codes brings together several branches of mathematics and this can be fun and exciting. This expository paper discusses error-correcting codes using the algebra of finite fields, generating functions, complex analysis and an analogy to the zeta functions of Riemann and Weil. There is no single text on the subject and Duursma's papers can be confusing, especially so to undergraduates.

Riemann's Hypothesis on the zeros of $\zeta(s) = \prod_p (1 - p^{-s})^{-1}$ is well-known. In 1949, Weil published an analogous zeta function for varieties defined over finite fields. In the case of curves, this takes the form $Z(T) = \frac{P(T)}{(1-T)(1-qT)}$ where $P(T) \in \mathbb{C}[T]$ is a polynomial and q is the size of the finite field. (If you put $T = q^{-s}$ and expand in “primes dividing q ”, you get a product similar to $\zeta(s)$.) Weil's “Riemann Hypothesis” (now proven true) is that the zeros of $P(T)$ all have modulus exactly equal to $1/2$.

The *weight enumerator* of an error-correcting code is the most important generating function of a code. Duursma has constructed a zeta function $Z(T) = \frac{P(T)}{(1-T)(1-qT)}$ of an error-correcting code where $P(T)$ is related to the weight enumerator of the code. For certain special codes, the “Riemann Hypothesis” ($P(\alpha) = 0$ implies $|\alpha| = 1/2$) is a natural issue to discuss.

This paper looks at these generating functions, their zeta functions, and in one case proves the “Riemann Hypothesis” property. Example codes, weight enumerators and zeta functions satisfying the “Riemann Hypothesis” property are all provided. Source code using the freely-available SAGE CAS is also included.

There are 15 references ranging from standard reference books in the field to a collection of Duursma's relevant articles. (Iwan Duursma is quite well-known in the field.)

DISCLAIMER: I do not know SAGE and have never used it. The examples in §4 are mathematically correct (I verified them with Maple), but I don't know if the SAGE code is correct. I did read the SAGE code, and there's one puzzlement I've noted.

Respectfully submitted,
–John Massman