

# UNIQUE PROPERTIES OF THE FIBONACCI AND LUCAS SEQUENCES

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September 7, 2007

## **Abstract**

The algebraic structure of the set of all Fibonacci-like sequences, which includes the Fibonacci and Lucas sequences, is developed, utilizing an isomorphism between this set and a subset of the 2-by-2 integer matrices. Using this isomorphism, determinants of sequences, and Fibonacci-like matrices, can be defined. The following results are then obtained: (1) the Fibonacci sequence is the only such sequence with determinant equal to 1, (2) the set of all Fibonacci-like sequences forms an integral domain, (3) even powers of Lucas matrices are multiples of a Fibonacci matrix, (4) only powers of multiples of Fibonacci matrices or Lucas matrices are multiples of Fibonacci matrices.