Report on
"On Fourier series using functions other than sine and cosine" by Henry Scher.

The paper contains a complete characterization of sets of functions $\{f_1, \ldots, f_n\}$ such that $\{f_i(jx)\colon i = 1, \ldots, n,\ j \geq 1\}$ is an orthogonal basis of $L^2[0, 2\pi]$. This kind of basis (formed by dilations of a finite set of functions by factors $j = 1, 2, 3, \ldots$) is called a dilation basis. The author proves that any orthogonal dilation basis for $L^2[0, 2\pi]$ consists of a constant function and two orthogonal functions of the form $\alpha \cos x + \beta \sin x$. I cannot say for certain that the result is new, but I have not seen it anywhere. In the wavelet theory one considers dilations by action of a group, which does not include the system of dilation by all integers $j \geq 1$.

I found the proofs to be correct and clearly written, and the entire paper to be well-organized. I recommend it to be published. My comments and suggestions for the author are listed on the next page.

Comments for the author:

The title "Fourier series using functions other than sine and cosine" immediately brings to mind generalized Fourier series (such as Fourier-Legendre) which are non-trigonometric orthogonal bases for L^2. Since this is not what the paper is about, I suggest considering a different title, perhaps something like "A characterization of orthogonal dilation bases".

page 1, the line preceding Figure 1: the formula is awkwardly placed. It should be either in line (with $(-1)^{n+1}n^{-1}$ instead of the fraction) or displayed.

page 2, the last line (and several other places): I suggest using parentheses when the generic term of a series involves summation, e.g.

$$\sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

page 3, the last two lines of section 2: The statement "any basis of the entire set of periodic functions corresponds ..." is not true unless you mean "any **dilation** basis".

page 5, formula (11). I found it confusing that f_1 is expanded into a Fourier series twice: first in the unnumbered formula between (6) and (7) and then in (11). I would remove (11) and the sentence leading to it. Then (12) can be written as

$$A_{ik} = \frac{1}{\pi} \int_0^{2\pi} f_i(x) \cos kx \, dx = \dots$$

This way, the sentence "A similar argument can be made for each of the $f_i(x)$" becomes unnecessary.

page 5, formula (13) is missing dx.

Finally, enumeration of displayed formulas appears inconsistent. In most papers, either all displayed formulas are numbered, or only those that are cited in the text. The latter option seems more common in mathematics.