Referee's Report on Linear N-Graphs

This paper takes a recently-defined concept – that of a *linear N-graph* – and establishes the following results:

- The join of two linear N-graphs is also a linear N-graph.
- Vertex multiplications preserve linearity.
- Linear N-trees are precisely those trees that have at most two vertices of degree greater than one.

The paper should be published.

To improve readability and the writing, I recommend the following changes.

(1) Italicize N (use math notation) in the title and the header of section 2.
(2) Abstract, line 2: delete "to".
(3) Abstract, line 3: "... taking joins and multiplying vertices..."
(4) Change all occurences of := and =: to =. The reader will understand when the authors intend to present a definition.
(5) p. 1, l 5: delete "of it"
(6) p. 1, l 15: replace "on" with "by"
(7) p. 1, l 16: use the standard $|V|$ and $|E|$ for the order and size of G, respectively.
(8) None of the graphs G_U (or other induced graphs), C_n, P_n, or $K_{n_1,n_2,...,n_r}$ appear to be used elsewhere in the paper; remove these items from the Preliminaries section.
(9) p. 1, l -3: remove the space between "however" and the comma; "... we consider here only one of them..."
(10) p. 2, l 3: remove the comma after "Figure 1"
(11) p. 2, l 19: provide the (specific) reference (see next item re specificity).
(12) Proposition 2.2: help the reader who seeks out Civan and Yalçin's paper by providing a specific reference (i.e. [1, Prop. 81])
(13) p. 2, l 20: It might be worth an extra phrase to help the reader see why the coloring in Figure 2b is not a linear coloring.
(14) p. 2, l 25: change "proved" to "prove"
(15) p. 2, l 26: change "it" to "G"
(16) p. 3, l 1: provide specific reference for first sentence.
(17) p. 3, first paragraph: reword second sentence (break into two sentences and work with last phrase) for readability
(18) Prop. 2.3: specific reference
(19) p. 3, l 11: delete "clearly"
(20) p. 3, l 12: "Examining the graphs depicted in Figure 3..."

(21) p. 3, definition of join: I believe it is common to use $G \vee H$ to denote the join of G and H. Also, the example $K_2 \vee K_3$ is not particularly illustrative; consider another.

(22) p. 3, l 26: "... it is enough to show that $k_1 + k_2$ colors are sufficient..." (and again in the proof of Prop. 2.7)

(23) p. 4, first paragraph: the only papers available via MathSciNet that deal with vertex multiplication use a common notation and definiton for what the authors of this paper denote as $G \circ m$: I suggest reviewing these papers' abstracts and adopting their notation and definition.

(24) p. 4, first paragraph: the sentence beginning "It is customary..." is confusing. Is it needed?

(25) p. 4, l 12: "Now, for any i, j with $1 \leq i, j \leq m_t$..."

(26) p. 5: "Acknowledgment" (singular); "authors' fourth-year project" (possessive)

The referee wishes to commend the authors: this level of fine criticism of the writing could not occur were their mathematics not correct and well-presented!