

# Referree's Report

## Isoperimetric Regions in Gauss Sectors

This is a very nice paper, and should be accepted with minor revisions. My suggestions for some revisions are given below.

A general comments first:

- It would be useful to either refer to the Gauss plane or  $G^2$  not changing between these two different ways of referencing the same mathematical object.

*Title and Abstract:* The abstract should be enlarged to describe some of the results and ... to attract readers to the paper.

*Section 1:* It would help if Figures in the conjecture in the introduction were given much earlier. Why does the conjecture in the introduction refer to figures 2 and 4, when figure 1 on the next page provides almost the same information? I would suggest considering a simplified conjecture for the introduction. *Conjecture:* Consider an  $\alpha$ -sector of the Gauss plane. A minimizer to the isoperimetric problem is either a circular arc, a ray orthogonal to the boundary, a ray emanating from the origin, or a half-edge of a rounded  $n$ -gon. Then describe that each is further conjectured to occur only for certain cases, depending on the area that is contained and the angle  $\alpha$ . This allows you to only need to reference Figure 1.

*Section 2:* Please explain in more detail how a  $2\alpha$ -cone is constructed from two  $\alpha$ -sectors. What do you mean by reflected? A figure would help, so the reader does not have to draw their own to figure it out. In proposition 2.5, you need to explain what you mean by a symmetric minimizer. You do not want the reader to have to figure out terms and draw pictures, else they may stop reading before they really start.

Do you really mean to prove examples 2.9 and 2.10?

Is Proposition 2.11 (Variation formulae) supposed to change font style in the middle of the statement of the proposition?

*Section 3:* The second sentence on page 6, needs to be rewritten as it is confusing in structure. I would suggest if Proposition 3.2 remains, starting with "A different proof is required for the case  $m = 1$ , because ...". I personally see no reason for Proposition 3.2 as it does not lead to greater understanding of the results of the paper. It is worth noting that Proposition 3.1 generalizes to higher dimensions and different proof is needed for the case of  $m = 1$ , but that is it.

It would help to reference in definition 3.3, the existence of rounded  $n$ -gons is obtained in Section 4.

After stating the main conjecture, it would useful to remind the reader what Theorem 3.13 states as motivation for the proposition and lemmas that are used to prove it. As soon as I read the first sentence of Page 9, I had to skip ahead to find Theorem 3.13 to understand what where you were heading with Propositions and Lemmas 3.5 to 3.12. I had not remembered that you had stated on page 5, that Theorem 3.13 shows that minimizers must be monotonic in the distance from the origin. It was four pages before and about 10 minutes after I had read the description of Theorem 3.13. Eliminating Proposition 3.2 would cut down on the time for the reader to forget.

Figure 8 should reference to Proposition 3.12. Figure 9 is not referenced, and states a conjecture that is not contained in the text. Is it a formal conjecture?

*Section 4:* This is a great section, showing the experimental validation of the conjecture and the construction of rounded  $n$ -gons.