

Alexandru Hening and Michael Kelly were my REU students this summer. In June they started to work on the following problem. Suppose we have a round garden of radius R where the trees are located at the points of integer lattice (we assume that all trees are round and have the same thickness). It is clear that if the trees are thin enough, the center of the garden is not shaded (which means exactly that there exists a line through the center of the garden that does not touch any tree).

The problem is to find the minimal thickness of the trees for which the center of the garden will be shaded.

This problem was posed by Pólya in 1918 (G. Pólya, Zahlentheoretisches und wahrscheinlichkeits-theoretisches über die Sichtweite im Walde, Arch.Math. und Phys.,27, Series 2 (1918) 135-142.) The same problem was mentioned again in the famous Encyclopedia of Elementary Mathematics in the article by Boltyanskii and Yaglom published in the fifties of the last century. In this article, Boltyanskii and Yaglom gave only the following estimate for the radius r of the trees.

$$\frac{1}{\sqrt{(R^2 + 1)}} \leq r \leq \frac{1}{R}. \quad (1)$$

The natural problem was to find the exact value of the radius.

A.Hening and M.Kelly obtained in some sense a complete solution for any garden which is compact and convex and satisfies one more natural condition (see the article). In the case of round garden the problem has been solved by T.T. Allen (see T.T. Allen, Pólya's Orchard Problem, Amer. Math. Monthly(93), 1986, pp. 98-104.). The proof given by A.Hening and M.Kelly differs from the original proof by Allen. They also tried to generalize the problem to the three-dimensional case. However, in this case, for some reasons explained in their paper, the "two-dimensional" proof does not work, and the problem becomes much more complicated. Nevertheless, they obtained a partial solution in this case in the sense that they proved a three-dimensional analog of the inequality mentioned above. In their paper, A.Hening and M.Kelly also study the same problem for some specific domains.

The paper is written on a good mathematical level and is interesting to read.

Boris Bekker, Central Michigan University, e-mail: boris.bekker@cmich.edu