RHIT Undergraduate Mathematics Journal

Referee's Report – June 2006

Article: Subgroup Lattices That Are Chains
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This article is both well written and interesting. The comments below are intended to help put polish on the paper, since otherwise it is very good.

I recommend that this paper be re-submitted for review after the author addresses the comments given below. The review process for the re-submission will be quite quick.

I will begin with two general comments that should be applied wherever appropriate throughout the paper. The remaining comments will be more directed.

1. I recommend the citation style be changed. Instead of listing something like "[G] on Page 81" please number your bibliography and use a citation style like [2, pg. 81]. This seems a bit cleaner and less cluttered.

2. Whenever you have a lemma or theorem followed immediately by its proof, please insert an extra vertical space between the statement of the lemma/theorem and the start of the proof. I think that this looks better visually.

More specific comments:

Page One

3. At the end of the first paragraph of the introduction, please place each numbered item for Q_8 on a separate line.

4. At the bottom of the first page, the sentence "A group G is a group whose subgroup lattice is a *chain* if for all ..." is a rather awkward way to define the desired concept. I recommend rewarding, perhaps with something like "We say a group G has a subgroup lattice that is a chain if ....".

5. On the last line of the first page, "for which" should be replaced by "whose" (I think).

Page Two

6. In the first paragraph after Example 1.2 you mention that you will present some known results using elementary proofs. How are these known results typically proven? Where in the literature might the interested reader go to find these results proven?

7. The infinite group example whose subgroup lattice is a chain led to a few questions: You should state that the operation is addition. You need to specify whether or not the n and the i are fixed or varying, else it is not clear what the distinction between G and G_i is. Finally, can you insert more vertical dots in the diagram you have in Example 1.3 between $\{G_2\}$ and $\{G_n\}$?

Page Three

8. In Definition 1.4, I always like to include *nontrivial* when saying 'nontrivial proper subgroups" just so there is no possible confusion.

9. The sentence immediately before Example 1.5 – does your statement then imply that the only possible such lattice is the third one from Example 1.5? It might be worth explicitly mentioning this.

Page Four

10. In the first full sentence, you say "have more complicated structure" – more complicated than what?

11. The first sentence of the second paragraph should end in x not in G.

12. Immediately before Lemma 2.1, you again refer to the existence of other proofs but do not tell the reader where to find them.

13. In the proof of Lemma 2.2, shouldn't everything be done with additive notation as opposed to multiplicative? Does the same problem arise in Lemma 2.3? Did the same problem arise in Example 1.3?

14. Who is y in the proof of Lemma 2.2? I think you mean to use an x there.

Page Five

All good!

Page Six

15. On the last line before section III, I believe you mean $i \leq h$ instead of $i \leq n$.

16. On the last line before section III, please provide some explanation for the final sentence that begins with :"Therefore..."

17. In the statement of Lemma 3.4 you use the notation of HK. Please provide a short definition.

18. In the first paragraph of the proof of Theorem 3.1 you state "Observe that the subgroup lattice of an infinite cyclic group is not a chain, and..". The reader might find useful a quick explanation. Also, what about infinite non-cyclic groups? This would appear to be an important question to address for the proof of the theorem.

19. The last line of the page has "$g \neq e$", but the $\neq$ sign is too close to the e. Please put a space in. This appears to happen everytime you use the $\neq$ symbol.

Page Seven

20. At the end of the first paragraph you state "since x and g commute...", you may wish to justify this by pointing out that $x = g^m$.

21. In the last paragraph of the proof of Theorem 3.1, what is meant by $< (x, y) >$? Why does it have prime order?

22. You state "For example, for any prime p, $Z_p \times Z_p$ would be a group whose subgroup lattice is formed by p+1 chains." Please justify this - the example is helpful, but an argument would be more pleasing.