

MOMENTS OF THE DISTRIBUTION OF OKAZAKI FRAGMENTS

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ABSTRACT. This paper is a continuation of [1] which provides formulae for the probability distributions of the number of Okazaki fragments at time t during the process of DNA replication. Given the expressions for the moments of the probability distribution of the number of Okazaki fragments at time t in the recursive form, we evaluated formulae for the 3rd and 4th moments, using *Mathematica*, and obtained results in the explicit form. Having done this, we calculated its skewness and kurtosis.

Okazaki fragments are small fragments of DNA remaining after the process of DNA replication. Denote by $N_t(\omega)$ the number of Okazaki fragments at the time $t \geq 0$. Let $g_i(t)$ denote the probability that at time t there are exactly i Okazaki fragments, $g_i(t) = P(N_t = i)$. In [2], [3], [4] it is shown that g_i , $i = 0, 1, \dots$ satisfy the following system of equations,

$$\begin{aligned} g_0(t) &= e^{-\lambda t} + \int_0^{at} g_0(t-y)\lambda e^{-\lambda y} dy \\ g_i(t) &= h_i(t) + \int_0^{at} g_i(t-y)\lambda e^{-\lambda y} dy, \end{aligned} \quad (1)$$

where $0 < a < 1$ and

$$h_i(t) = \begin{cases} e^{-\lambda t} & i = 0 \\ \int_0^{at} g_{i-1}(t-y)\lambda e^{-\lambda y} dy & i = 1, 2, 3, \dots \end{cases} \quad (2)$$

Denote $\mathbf{g} = \{g_i\}_{i=1}^{\infty}$. In [2] it has been proved that \mathbf{g} defines a probability distribution on positive integers and the recursive formula for its moments was derived. Let $n_k(t) = E(N_t^k)$. We consider the following formula for $n'_k(t)$ taken from [1],

$$\begin{aligned} n'_k(t) &= \lambda^2 \sum_{j=0}^{k-1} \binom{k}{j} \int_{at}^t n_j(t-s)e^{-\lambda s} ds + \\ &\lambda e^{-\lambda t} + \lambda \sum_{j=0}^{k-1} \binom{k}{j} \left[\int_{at}^t n'_j(t-s)e^{-\lambda s} ds - a n_j(bt)e^{-\lambda at} \right], \end{aligned} \quad (3)$$

in order to evaluate the explicit expression for the k -th moment, i.e. $n_k(t)$. We specifically aimed at calculating $n_3(t)$ and $n_4(t)$ ($n_1(t)$ and $n_2(t)$ were already calculated in [1]). For this purpose we created a short program in *Mathematica* (Figure 1), which also calculates their

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In[1]:=
nderiv0[t_] := 0
ptemp0[t_] := 0
n0[t_] := 1
nderiv1[t_] := L * (1 - a) * e-L*a*t
ptemp1[t_] :=  $\frac{1-a}{a} * (e^{-L*a*t})$ 
n1[t_] :=  $\frac{1-a}{a} * (1 - e^{-L*a*t})$ 
nderivk[t_] := nderivk[t] =
  L2 *  $\sum_{j=0}^{k-1} \left( \text{Binomial}[k, j] * \int_{a*t}^t n_j[t-s] * L * e^{-L*s} ds \right) +$ 
  L * e-L*t + L *  $\sum_{j=0}^{k-1} \left( \text{Binomial}[k, j] * \left( \int_{a*t}^t \text{nderiv}_j[t-s] * e^{-L*s} ds -$ 
  a * nj[(1-a)*t] * e-L*a*t \right) \right)
ptempm[t_] := ptempm[t] =  $\int \text{nderiv}_m[t] dt$ 
nm[t_] := nm[t] = ptempm[t] - ptempm[0]

r = 4
Do[t1 = TimeUsed[]; nderivm[t];
  ptempm[t_] =  $\int \text{nderiv}_m[t] dt$ ; nm[t_] = ptempm[t] - ptempm[0];
  t2 = TimeUsed[]; diff = t2 - t1; Print[m];
  Print[nm[t]]; Print[nm[0]]; Print[diff],
  {m, 2, r, 1}]

Mean[t_] := n1[t]
Mean[t]
Varian[t_] := n2[t] - n1[t] * n1[t]
Varian[t]
Skewness[t_] :=  $\frac{n_3[t] - 3 * n_1[t] * n_2[t] + 3 * n_1[t]^3 - n_1[t]^3}{\sqrt{\text{Varian}[t]^3}}$ 
Skewness[t]
Kurtosis[t_] :=  $\frac{n_4[t] - 4 * n_3[t] * n_1[t] + 6 * n_2[t] * n_1[t]^2 - 4 * n_1[t]^4 + n_1[t]^4}{\text{Varian}[t]^2} - 3$ 
Kurtosis[t]

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FIGURE 1. *Mathematica* program

limits $t \rightarrow \infty$ and the skewness and kurtosis of \mathbf{g} . The computational complexity of this formula is $T(n) = \sum_{j=0}^{n-1} T(j) + 1$ whose solution is $T(n) = 3^n$. Therefore the complexity is $\Theta(3^n)$ assuming that all *Mathematica* operations are done in $O(1)$ time. Exponential complexity is a result of the form of expression (3). Times required to compute consecutive moments on a Celeron 1.80GHz *Windows XP Home Edition Mathematica 5* are shown in Figure 2.

Using the program from Figure 2 we evaluated the formulae for $n_2(t)$, $n_3(t)$, and $n_4(t)$. We present these formulae at the end of the paper in the appendix due to their long length. Their graphs with parameters $a = 0.4$ and $\lambda = 1$ are shown in Figure 3. Calculating the limits $t \rightarrow \infty$ of $n_1(t)$, $n_2(t)$, $n_3(t)$, and $n_4(t)$ with the same parameters λ and a gave the

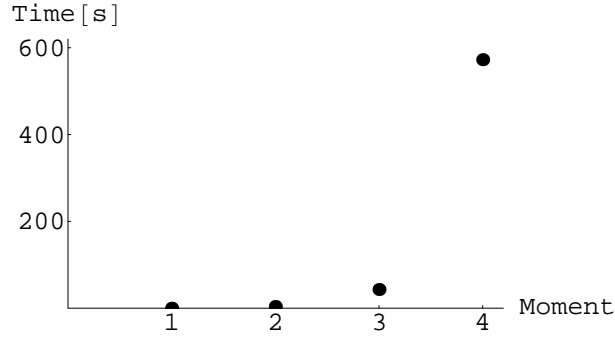


FIGURE 2. Time of computation of moments.

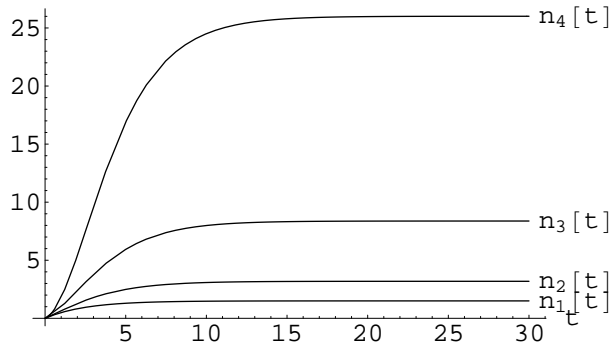


FIGURE 3. The 1st, 2nd, 3rd, 4th moments.

following results.

$$\begin{aligned} \lim_{t \rightarrow \infty} n_1(t) &= 1.5000 & \lim_{t \rightarrow \infty} n_2(t) &= 3.1875 \\ \lim_{t \rightarrow \infty} n_3(t) &= 8.3771 & \lim_{t \rightarrow \infty} n_4(t) &= 26.008 \end{aligned} \quad (4)$$

It was found in [2] (see also [1]) that $Var(\mathbf{g}) = \frac{1-a}{1-(1-a)^2}$. We further investigated other features of the distribution \mathbf{g} , particularly the skewness, $S(t)$, and kurtosis, $K(t)$.

$$S(t) = \frac{\mathbb{E}((X - \mathbb{E}X)^3)}{\sqrt{\mathbb{E}((X - \mathbb{E}X)^2)^3}} \quad K(t) = \frac{\mathbb{E}((X - \mathbb{E}X)^4)}{\mathbb{E}((X - \mathbb{E}X)^2)^2} - 3 \quad (5)$$

Again the formulae are very long so we present only their graphs with the graphs of the mean value and variance (as before, $\lambda = 1$ and $a = 0.4$) in Figure 4. The limits of mean, variance, skewness and kurtosis are the following,

$$\begin{aligned} \mathbb{E} &= \lim_{t \rightarrow \infty} n_1(t) = 1.5 & Var &= \lim_{t \rightarrow \infty} n_2(t) - n_1(t)^2 = 0.9375 \\ S &= \lim_{t \rightarrow \infty} S(t) = 0.86299 & K &= \lim_{t \rightarrow \infty} K(t) = 1.08352. \end{aligned} \quad (6)$$

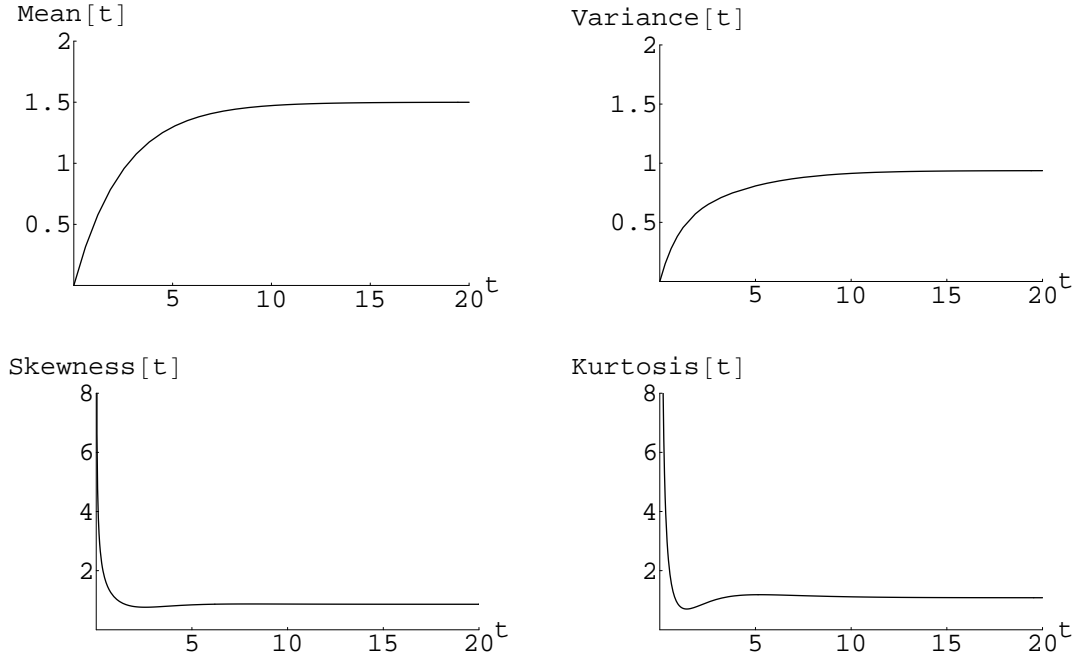


FIGURE 4. The mean, variance, skewness and kurtosis values.

We compared the distribution \mathbf{g} with the the normal distribution cut to the appropriate interval. In [1] the following formula was proved,

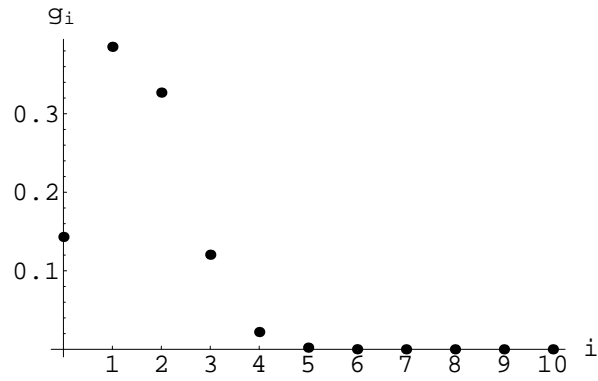
$$g_i = \prod_{j=1}^{\infty} (1 - b^j) \cdot \sum_{m=i}^{\infty} \frac{b^m}{1 - b^m} \Psi_{i,m}(b), \quad (7)$$

where for all $r \geq 1$ we put $\Psi_{1,r}(b) \equiv 1$ and for $s \geq i + 1$,

$$\Psi_{i+1,s}(b) = \sum_{r=i}^{s-1} \frac{b^r}{1 - b^r} \Psi_{i,r}(b). \quad (8)$$

Using this representation for g_i the authors of [1] computationally obtained the approximate values for g_i , $0 \leq i \leq 10$, where $g_i = \lim_{t \rightarrow \infty} g_i(t)$. We present these results in Figure 5. From the obtained results, it can be concluded that the distribution \mathbf{g} , with $a = 0.4$ and $\lambda = 1$ has some features of the normal distribution $N(1.5, 1)$ cut to the interval $[0, \infty)$. $\mathbb{E}(\mathbf{g}) = 1.5$ and $Var(\mathbf{g}) = 0.9375 \approx 1$. The degree of peakness of this distribution is not far from to the one for the normal distribution, since the kurtosis is about 1 (6), for the normal distribution it should be 0. However, the fact that the skewness is 0.86299 (6) indicates that the discussed distribution is positively skewed, i.e. if the distribution was "extended" to the interval $(-\infty, \infty)$, the right tail would be more pronounced than the left tail.

Further research to find an effective method of calculating the moments is necessary due to the exponential complexity of formula 3. Also it should be investigated what effect different values of a have on the moments.

FIGURE 5. g_i

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Appendix
Formulae for Moments

$$n_1(t) = -\frac{1-a}{a}e^{-\lambda t} + \frac{1-a}{a} \quad (9)$$

$$n_2(t) = \left(-\lambda \left(-1 + \frac{1}{\lambda} - \frac{(-2+a)(a-\lambda)}{a^2\lambda} - \frac{2(-2a+a^2+\lambda)}{a(-2a+a^2)\lambda} \right) + \lambda \left(-e^{-\lambda t} + \frac{e^{-\lambda t}}{\lambda} - \frac{(-2+a)e^{-a\lambda t}(a-\lambda)}{a^2\lambda} - \frac{2e^{(-2+a)a\lambda t}(-2a+a^2+\lambda)}{a(-2a+a^2)\lambda} \right) \right) \quad (10)$$

$$\begin{aligned} n_3(t) = & 1 - \frac{3}{-2a+a^2} - \frac{6}{-3a+6a^2-4a^3+a^4} - e^{-\lambda t} + \frac{6e^{-a(3-3a+a^2)\lambda t}}{-3a+6a^2-4a^3+a^4} + \frac{3e^{-\lambda t+(-1+a)^2\lambda t}}{-2a+a^2} + \\ & \frac{3(a-\lambda)}{(-1+a)a^2} + \frac{3(-1+a)^2(a-\lambda)}{a^2(1+a^2+2a^3)} - \frac{3(-1+a)(a-\lambda)}{a^2(-1+3a-4a^2+4a^3)} + \\ & \frac{6(-1+a)(a-\lambda)}{a^2(-3+12a-23a^2+30a^3-20a^4+8a^5)} - \frac{3e^{(-2+a)a\lambda t}(a-\lambda)}{(-1+a)a^2} - \frac{3(-1+a)^2e^{-(1+a)\lambda t}(a-\lambda)}{a^2(1+a^2+2a^3)} - \\ & \frac{6(-1+a)e^{-a(3-3a+2a^2)\lambda t}(a-\lambda)}{a^2(-3+12a-23a^2+30a^3-20a^4+8a^5)} + \frac{3(-1+a)e^{-(1-a+2a^2)\lambda t}(a-\lambda)}{a^2(-1+3a-4a^2+4a^3)} + \frac{3\lambda}{(-2+a)a^2} - \\ & \frac{3e^{-(-1+a)\lambda t+(1-a+a^2)\lambda t}\lambda}{(-2+a)a^2} - \left(-\frac{3}{\lambda} + \frac{3(-1+a)}{a^2\lambda} \right) \lambda^2 + e^{-(-1+a)\lambda t} \left(\frac{3(-1+a)e^{\lambda t}}{a^2\lambda} - \frac{3e^{a\lambda t}}{\lambda} \right) \lambda^2 + \\ & \frac{3\lambda(-a+\lambda)}{(-1+a)a^3} - \frac{3e^{(-2+a)a\lambda t}\lambda(-a+\lambda)}{(-1+a)a^3} + \frac{6(-2a+a^2+\lambda)}{a^2(-6+9a-5a^2+a^3)} - \\ & \frac{6e^{-a\lambda t+(-2+a)(-1+a)a\lambda t}(-2a+a^2+\lambda)}{a^2(-6+9a-5a^2+a^3)} - \frac{6\lambda(-2a+a^2+\lambda)}{(-1+a)^2a^3(-6+9a-5a^2+a^3)} + \\ & \frac{6e^{-a(3-3a+a^2)\lambda t}\lambda(-2a+a^2+\lambda)}{(-1+a)^2a^3(-6+9a-5a^2+a^3)} + \frac{3(a-2\lambda+a\lambda)}{(-2+a)a^2} - \frac{3e^{-a\lambda t+(-1+a)a\lambda t}(a-2\lambda+a\lambda)}{(-2+a)a^2} - \\ & \frac{3(-2+a)(a^2+2\lambda-2a\lambda+a^2\lambda)}{(-1+a)a} - \frac{\lambda(2a^2-a^3+6\lambda-12a\lambda+9a^2\lambda-3a^3\lambda)}{(-2+a)a^3} + \\ & \frac{e^{-a\lambda t}\lambda(2a^2-a^3+6\lambda-12a\lambda+9a^2\lambda-3a^3\lambda)}{(-2+a)a^3} + \\ & \frac{\lambda(11-10a+8a^2-3a^3-15\lambda+18a\lambda-12a^2\lambda+3a^3\lambda)}{(-1+a)^2} - \\ & \lambda \left(-\frac{3a(-1+\lambda)}{\lambda} + \frac{-6a+7a^2-2a^3-6\lambda+12a\lambda-9a^2\lambda+3a^3\lambda}{(-2+a)a^2\lambda} \right) + \\ & e^{-a\lambda t} \lambda \left(-\frac{3ae^{(-1+a)\lambda t}(-1+\lambda)}{\lambda} + \frac{-6a+7a^2-2a^3-6\lambda+12a\lambda-9a^2\lambda+3a^3\lambda}{(-2+a)a^2\lambda} \right) + \end{aligned}$$

$$\begin{aligned}
& e^{-\lambda t} \lambda^2 \left(-\frac{11 - 10a + 8a^2 - 3a^3 - 15\lambda + 18a\lambda - 12a^2\lambda + 3a^3\lambda}{(-1+a)^2\lambda} - \frac{3(-1+a)(-\lambda + \lambda^2)t}{\lambda} \right) + \\
& e^{-\lambda t} \lambda \left(\frac{3(-2+a)(a^2 + 2\lambda - 2a\lambda + a^2\lambda)}{(-1+a)a\lambda} + \frac{3(-1+a)(a\lambda - 2\lambda^2 + a\lambda^2)t}{a\lambda} \right)
\end{aligned} \tag{11}$$

$$\begin{aligned}
n_4 = & 1 - \frac{8}{(-2+a)(-1+a)^3} - \frac{72a}{(-2+a)(-1+a)^3} + \frac{94a^2}{(-2+a)(-1+a)^3} - \frac{48a^3}{(-2+a)(-1+a)^3} + \\
& \frac{10a^4}{(-2+a)(-1+a)^3} - e^{-\lambda t} + \frac{6(a-\lambda)}{(-1+a)a^2} + \frac{2(a-\lambda)}{(-1+6a)(-a+3a^2)^2} - \frac{6(-1+a)(a-\lambda)}{a(-1+3a)^2(-1-a+6a^2)} - \\
& \frac{6(-1+a)(a-\lambda)}{a^2(-1+3a-4a^2+4a^3)} + \frac{12(-1+a)(a-\lambda)}{a-4a^2+10a^3-19a^4+22a^5-20a^6+8a^7} - \\
& \frac{-a+9a^2-17a^3-21a^4+162a^5-540a^6+648a^7}{24(a-\lambda)} - \\
& \frac{a^2(12-66a+179a^2-324a^3+399a^4-348a^5+208a^6-80a^7+16a^8)}{12(a-\lambda)} + \\
& \frac{a^2(2-13a+47a^2-129a^3+247a^4-426a^5+540a^6-432a^7+324a^8)}{24(a-\lambda)} - \\
& \frac{a-3a^2+a^3+11a^4-66a^5+108a^6-216a^7+324a^8}{6e^{(-2+a)\lambda t}(a-\lambda)} + \frac{6(-1+a)e^{2(-1+a)\lambda t}(a-\lambda)}{a(-1+3a)^2(-1-a+6a^2)} - \\
& \frac{6e^{(-2+a)\lambda t}(a-\lambda)}{(-1+a)a^2} - \frac{2e^{6(-1+a)\lambda t}(a-\lambda)}{(-1+6a)(-a+3a^2)^2} + \\
& \frac{24e^{(-1+a)(1+3a)\lambda t}(a-\lambda)}{a-3a^2+a^3+11a^4-66a^5+108a^6-216a^7+324a^8} + \\
& \frac{6(-1+a)e^{-(1-a+2a^2)\lambda t}(a-\lambda)}{a^2(-1+3a-4a^2+4a^3)} - \\
& \frac{12e^{2(-1+a)a(2-a+3a^2)\lambda t}(a-\lambda)}{a^2(2-13a+47a^2-129a^3+247a^4-426a^5+540a^6-432a^7+324a^8)} + \\
& \frac{24e^{(-1+a)(1-a+6a^2)\lambda t}(a-\lambda)}{-a+9a^2-17a^3-21a^4+162a^5-540a^6+648a^7} + \\
& \frac{24(-1+a)e^{-a\lambda t+(-1+a)a(3-3a+2a^2)\lambda t}(a-\lambda)}{a^2(12-66a+179a^2-324a^3+399a^4-348a^5+208a^6-80a^7+16a^8)} - \\
& \frac{12(-1+a)e^{-a\lambda t+(-1+a)(1-a+2a^2)\lambda t}(a-\lambda)}{a-4a^2+10a^3-19a^4+22a^5-20a^6+8a^7} + \frac{76\lambda}{(-2+a)(-1+a)^3} + \frac{4\lambda}{(-2+a)a^2} - \\
& \frac{46a\lambda}{(-2+a)(-1+a)^3} - \frac{40a^2\lambda}{(-2+a)(-1+a)^3} + \frac{46a^3\lambda}{(-2+a)(-1+a)^3} - \frac{12a^4\lambda}{(-2+a)(-1+a)^3} -
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{24}{(-1+a)^2 a} - \frac{24}{(-1+a)^2 a^2 (3-3a+a^2)} \right) \lambda - \frac{4e^{-(1+a)\lambda t + (1-a+a^2)\lambda t} \lambda}{(-2+a)a^2} + \\
& e^{-(1+3a+a^3)\lambda t} \left(\frac{24e^{a(3+a^2)\lambda t}}{(-1+a)^2 a} - \frac{24e^{(1+3a^2)\lambda t}}{(-1+a)^2 a^2 (3-3a+a^2)} \right) \lambda + e^{(-1+a^2)\lambda t} \left(-12ae^{-a\lambda t - (-1+a)a\lambda t} + \right. \\
& \left. e^{-a\lambda t} \left(-\frac{12(-1+a)e^{-(1+a)\lambda t}}{(-2+a)a^2} + \frac{12(-1+a)^2(a-\lambda)}{(-a-a^2-3a^4-a^5+2a^6)\lambda} \right) \right) \lambda - \\
& \left(-\frac{12(a-\lambda)}{(-a+3a^2)^2(1+a+18a^3)\lambda} + \frac{12(a-\lambda)}{(-a+3a^2)^2(1-a+10a^2-6a^3+36a^4)\lambda} \right) \lambda - \\
& \left(-\frac{12(-1+a)}{(-2+a)a^2} - 12a + \frac{12(-1+a)^2(a-\lambda)}{(-a-a^2-3a^4-a^5+2a^6)\lambda} \right) \lambda + \\
& e^{-(1+2a+5a^2)\lambda t} \left(-\frac{12e^{8a^2\lambda t}(a-\lambda)}{(-a+3a^2)^2(1+a+18a^3)\lambda} + \right. \\
& \left. \frac{12e^{2a(1+3a^2)\lambda t}(a-\lambda)}{(-a+3a^2)^2(1-a+10a^2-6a^3+36a^4)\lambda} \right) \lambda - \\
& \frac{156\lambda^2}{(-2+a)(-1+a)^3} - \frac{24\lambda^2}{(-2+a)(-1+a)^3 a^2} + \frac{96\lambda^2}{(-2+a)(-1+a)^3 a} + \frac{132a\lambda^2}{(-2+a)(-1+a)^3} - \\
& \frac{60a^2\lambda^2}{(-2+a)(-1+a)^3} + \frac{12a^3\lambda^2}{(-2+a)(-1+a)^3} - \left(-\frac{4}{\lambda} + \frac{4(-1+a)}{a^2\lambda} \right) \lambda^2 + \\
& e^{-(1+a)\lambda t} \left(\frac{4(-1+a)e^{\lambda t}}{a^2\lambda} - \frac{4e^{a\lambda t}}{\lambda} \right) \lambda^2 - \left(-\frac{12}{(-2+a)a^3} + \frac{12(-1+a)^2(a-\lambda)}{a^3(-1-a-3a^3-a^4+2a^5)\lambda} \right) \lambda^2 + \\
& e^{(a+a^2)\lambda t} \left(-\frac{12e^{-3a\lambda t}}{(-2+a)a^3} + \frac{12(-1+a)^2 e^{-(1+2a)\lambda t}(a-\lambda)}{a^3(-1-a-3a^3-a^4+2a^5)\lambda} \right) \lambda^2 - \\
& \frac{12(-1+a)\lambda(-a+\lambda)}{a^3(-1+2a)^2(-1+2a-6a^2+7a^3-8a^4+4a^5)} + \\
& \frac{24(-1+a)\lambda(-a+\lambda)}{a^3(-1+2a)^2(-12+54a-149a^2+271a^3-360a^4+351a^5-253a^6+130a^7-44a^8+8a^9)} + \\
& \frac{12(-1+a)e^{(-1+a-3a^2+2a^3)\lambda t}\lambda(-a+\lambda)}{a^3(-1+2a)^2(-1+2a-6a^2+7a^3-8a^4+4a^5)} - \\
& \frac{24(-1+a)e^{(-4a+6a^2-5a^3+2a^4)\lambda t}\lambda(-a+\lambda)}{a^3(-1+2a)^2(-12+54a-149a^2+271a^3-360a^4+351a^5-253a^6+130a^7-44a^8+8a^9)} + \\
& \frac{12(a+\lambda)}{(-1+a)^2(3a-3a^2+a^3)} + \frac{12e^{-a(3-3a+a^2)\lambda t}(a+\lambda)}{(-1+a)^2(3a-3a^2+a^3)} + \\
& \frac{24(-2a+a^2+\lambda)}{(-1+a)^3(-2a+a^2)^2(2-2a+a^2)} - \frac{24e^{-\lambda t + (-1+a)\lambda t}(-2a+a^2+\lambda)}{(-1+a)^3(-2a+a^2)^2(2-2a+a^2)} - \frac{12(1+2\lambda)}{1+a+a^2+a^3} +
\end{aligned}$$

$$\begin{aligned}
& \frac{12e^{-2\lambda(-1+a^2)\lambda}(1+2\lambda)}{1+a+a^2+a^3} - \frac{24(-4a^2+8a^3-5a^4+a^5+3a\lambda-3a^2\lambda+a^3\lambda-\lambda^2)}{a^3(2-3a+a^2)^2(6-12a+11a^2-5a^3+a^4)} + \\
& \frac{24e^{-a\lambda+(-1+a)a(3-3a+a^2)\lambda}(-4a^2+8a^3-5a^4+a^5+3a\lambda-3a^2\lambda+a^3\lambda-\lambda^2)}{a^3(2-3a+a^2)^2(6-12a+11a^2-5a^3+a^4)} + \\
& \frac{12(2a^2-3a^3+a^4-2a\lambda+a^2\lambda+\lambda^2)}{(-2+a)^2a^3(1-4a+2a^2)(-1+2a-5a^2+2a^3)} - \\
& \frac{12(2a^2-3a^3+a^4-2a\lambda+a^2\lambda+\lambda^2)}{(-2+a)^2a^3(1-a+a^2)(-1+5a-10a^2+20a^3-16a^4+4a^5)} + \\
& \frac{12e^{2(-2+a)a(1-a+a^2)\lambda}(2a^2-3a^3+a^4-2a\lambda+a^2\lambda+\lambda^2)}{(-2+a)^2a^3(1-a+a^2)(-1+5a-10a^2+20a^3-16a^4+4a^5)} - \\
& \frac{12e^{(-1+2a-5a^2+2a^3)\lambda}(2a^2-3a^3+a^4-2a\lambda+a^2\lambda+\lambda^2)}{(-2+a)^2a^3(1-4a+2a^2)(-1+2a-5a^2+2a^3)} + \\
& \frac{24\lambda(4a^2-8a^3+5a^4-a^5-3a\lambda+3a^2\lambda-a^3\lambda+\lambda^2)}{(-2+a)^2(-1+a)^5a^4(6-12a+11a^2-5a^3+a^4)} - \\
& \frac{24e^{(-4a+6a^2-4a^3+a^4)\lambda}\lambda(4a^2-8a^3+5a^4-a^5-3a\lambda+3a^2\lambda-a^3\lambda+\lambda^2)}{(-2+a)^2(-1+a)^5a^4(6-12a+11a^2-5a^3+a^4)} + \\
& \frac{12(a^3-2a\lambda+2\lambda^2-a\lambda^2)}{a^3(3-3a+2a^2)(-1+3a-4a^2+4a^3)} - \\
& \frac{12e^{-a(3-3a+2a^2)\lambda}(a^3-2a\lambda+2\lambda^2-a\lambda^2)}{a^3(3-3a+2a^2)(-1+3a-4a^2+4a^3)} - \frac{3(-a+2\lambda-2a\lambda+a\lambda^2)}{a^2} + \\
& \frac{3e^{-2a\lambda}(-a+2\lambda-2a\lambda+a\lambda^2)}{a^2} + \frac{12(-2a^3+a^4+4a\lambda-3a^2\lambda+a^3\lambda-2\lambda^2+a\lambda^2)}{a^3(6-15a+14a^2-6a^3+a^4)} - \\
& \frac{12e^{-a\lambda-(2a-3a^2+a^3)\lambda}(-2a^3+a^4+4a\lambda-3a^2\lambda+a^3\lambda-2\lambda^2+a\lambda^2)}{a^3(6-15a+14a^2-6a^3+a^4)} - \\
& \frac{12\lambda(-2a^3+a^4+4a\lambda-3a^2\lambda+a^3\lambda-2\lambda^2+a\lambda^2)}{(-1+a)^3a^4(-6+9a-5a^2+a^3)} + \\
& \frac{12e^{-a(3-3a+a^2)\lambda}\lambda(-2a^3+a^4+4a\lambda-3a^2\lambda+a^3\lambda-2\lambda^2+a\lambda^2)}{(-1+a)^3a^4(-6+9a-5a^2+a^3)} - \\
& \frac{12(-2a^2+5a^3-4a^4+a^5+2a\lambda-3a^2\lambda+a^3\lambda-\lambda^2+a\lambda^2)}{(-2+a)^2a^3(-1-2a+a^2)(-1+a-4a^2+2a^3)} + \\
& \frac{12e^{(-1-2a+a^2)\lambda}(-2a^2+5a^3-4a^4+a^5+2a\lambda-3a^2\lambda+a^3\lambda-\lambda^2+a\lambda^2)}{(-2+a)^2a^3(-1-2a+a^2)(-1+a-4a^2+2a^3)} + \\
& \frac{3(a-a^2+2\lambda+4a\lambda-2a^2\lambda+a\lambda^2+a^2\lambda^2)}{a+a^2} - \frac{3e^{-2\lambda}(a-a^2+2\lambda+4a\lambda-2a^2\lambda+a\lambda^2+a^2\lambda^2)}{a+a^2} - \\
& \frac{4(-4a^2+4a^3-a^4+5a^2\lambda-7a^3\lambda+3a^4\lambda+6\lambda^2-12a\lambda^2+9a^2\lambda^2-3a^3\lambda^2)}{(-2+a)^2(-1+a)a^3} +
\end{aligned}$$

$$\begin{aligned}
& \frac{4e^{-\lambda t + (-1+a)^2 \lambda t} (-4a^2 + 4a^3 - a^4 + 5a^2 \lambda - 7a^3 \lambda + 3a^4 \lambda + 6\lambda^2 - 12a\lambda^2 + 9a^2 \lambda^2 - 3a^3 \lambda^2)}{(-2+a)^2(-1+a)a^3} \\
& \frac{2\lambda(24a^2 - 26a^3 + 7a^4 - 16a^2 \lambda + 17a^3 \lambda - 6a^4 \lambda - 12\lambda^2 + 24a\lambda^2 - 18a^2 \lambda^2 + 6a^3 \lambda^2)}{(-2+a)^2(-1+a)a^4} + \\
& \frac{1}{(-2+a)^2(-1+a)a^4} (2e^{(-2+a)\lambda t} \lambda(24a^2 - 26a^3 + 7a^4 - 16a^2 \lambda + 17a^3 \lambda - 6a^4 \lambda - 12\lambda^2 + \\
& 24a\lambda^2 - 18a^2 \lambda^2 + 6a^3 \lambda^2)) - (\lambda(18a^3 - 237a^4 + 792a^5 - 741a^6 - 887a^7 + 2902a^8 - \\
& 3275a^9 + 2016a^{10} - 676a^{11} + 96a^{12} + 288a\lambda - 1296a^2 \lambda + 3246a^3 \lambda - 5094a^4 \lambda + \\
& 4122a^5 \lambda + 748a^6 \lambda - 6066a^7 \lambda + 7518a^8 \lambda - 5134a^9 \lambda + 2108a^{10} \lambda - 488a^{11} \lambda + 48a^{12} \lambda + \\
& 72\lambda^2 - 468a\lambda^2 + 1488a^2 \lambda^2 - 2964a^3 \lambda^2 + 3828a^4 \lambda^2 - 2796a^5 \lambda^2 - 60a^6 \lambda^2 + 2916a^7 \lambda^2 - \\
& 3900a^8 \lambda^2 + 2928a^9 \lambda^2 - 1380a^{10} \lambda^2 + 384a^{11} \lambda^2 - 48a^{12} \lambda^2)) / ((-2+a)a^4(9-18a+ \\
& 27a^2 - 9a^3 - 16a^4 + 27a^5 - 16a^6 + 4a^7)) + (e^{-a\lambda t} \lambda(18a^3 - 237a^4 + 792a^5 - 741a^6 - \\
& 887a^7 + 2902a^8 - 3275a^9 + 2016a^{10} - 676a^{11} + 96a^{12} + 288a\lambda - 1296a^2 \lambda + 3246a^3 \lambda - \\
& 5094a^4 \lambda + 4122a^5 \lambda + 748a^6 \lambda - 6066a^7 \lambda + 7518a^8 \lambda - 5134a^9 \lambda + 2108a^{10} \lambda - 488a^{11} \lambda + \\
& 48a^{12} \lambda + 72\lambda^2 - 468a\lambda^2 + 1488a^2 \lambda^2 - 2964a^3 \lambda^2 + 3828a^4 \lambda^2 - 2796a^5 \lambda^2 - 60a^6 \lambda^2 + \\
& 2916a^7 \lambda^2 - 3900a^8 \lambda^2 + 2928a^9 \lambda^2 - 1380a^{10} \lambda^2 + 384a^{11} \lambda^2 - 48a^{12} \lambda^2)) / ((-2+a)a^4(9- \\
& 18a + 27a^2 - 9a^3 - 16a^4 + 27a^5 - 16a^6 + 4a^7)) - \frac{1}{(-2+a)a^2 \lambda} (2(-1+a)(-10a^2 \lambda + \\
& 17a^3 \lambda - 6a^4 \lambda + 24a\lambda^2 - 26a^2 \lambda^2 - 5a^3 \lambda^2 + 6a^4 \lambda^2 + \\
& 12\lambda^3 - 24a\lambda^3 + 18a^2 \lambda^3 - 6a^3 \lambda^3)) - \lambda \left(\frac{6(2+a\lambda)}{a(1+a)} - \frac{12(2+a\lambda)}{a(1+a+a^2+a^3)} \right) + \\
& e^{-(2+a^2)\lambda t} \lambda \left(-\frac{12e^{\lambda t}(2+a\lambda)}{a(1+a+a^2+a^3)} + \frac{6e^{a^2 \lambda t}(2+a\lambda)}{a(1+a)} \right) - \lambda \left(\frac{6(a^2 - 3a\lambda + a^2 \lambda + 2\lambda^2 - a\lambda^2)}{a^3(-1+3a-4a^2+4a^3)\lambda} - \right. \\
& \left. \frac{6(-a^3 + a^4 + 2a\lambda - 2a^2 \lambda - 2\lambda^2 + 3a\lambda^2 - a^2 \lambda^2)}{a^3(1+a)(1-a+2a^2)\lambda} \right) + \\
& e^{-(1+a)\lambda t} \lambda \left(\frac{6e^{-2(-1+a)a\lambda t}(a^2 - 3a\lambda + a^2 \lambda + 2\lambda^2 - a\lambda^2)}{a^3(-1+3a-4a^2+4a^3)\lambda} - \right. \\
& \left. \frac{6(-a^3 + a^4 + 2a\lambda - 2a^2 \lambda - 2\lambda^2 + 3a\lambda^2 - a^2 \lambda^2)}{a^3(1+a)(1-a+2a^2)\lambda} \right) - \lambda \left(-\frac{12a^2(-1+\lambda)}{\lambda} - \right. \\
& \left. \frac{2(-20a^2 + 20a^3 - 5a^4 + 16a^2 \lambda - 17a^3 \lambda + 6a^4 \lambda + 12\lambda^2 - 24a\lambda^2 + 18a^2 \lambda^2 - 6a^3 \lambda^2)}{(-2+a)^2 a^3 \lambda} \right) + \\
& e^{-a\lambda t + (-1+a)a\lambda t} \lambda \left(-\frac{12a^2 e^{-(-1+a)^2 \lambda t}(-1+\lambda)}{\lambda} - \right. \\
& \left. \frac{2(-20a^2 + 20a^3 - 5a^4 + 16a^2 \lambda - 17a^3 \lambda + 6a^4 \lambda + 12\lambda^2 - 24a\lambda^2 + 18a^2 \lambda^2 - 6a^3 \lambda^2)}{(-2+a)^2 a^3 \lambda} \right) + \\
& \lambda \left(\frac{839}{(-1+a)^5(-1+2a)^2(1-a+a^2)} + \frac{24}{(-1+a)^5 a^2(-1+2a)^2(1-a+a^2)} - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{216}{(-1+a)^5 a (-1+2a)^2 (1-a+a^2)} - \frac{1936a}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} + \\
& \frac{3298a^2}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} - \frac{5033a^3}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} + \\
& \frac{7263a^4}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} - \frac{9042a^5}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} + \\
& \frac{8692a^6}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} - \frac{5993a^7}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} + \\
& \frac{2780a^8}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} - \frac{772a^9}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} + \\
& \frac{96a^{10}}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} - \frac{2860\lambda}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} + \\
& \frac{24\lambda}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} + \frac{216\lambda}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} - \\
& \frac{936\lambda}{(-1+a)^5 a^3 (-1+2a)^2 (1-a+a^2)} + \frac{7142a\lambda}{(-1+a)^5 a^2 (-1+2a)^2 (1-a+a^2)} - \\
& \frac{14486a^2\lambda}{(-1+a)^5 a (-1+2a)^2 (1-a+a^2)} - \frac{22978a^3\lambda}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} + \\
& \frac{27912a^4\lambda}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} - \frac{25494a^5\lambda}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} + \\
& \frac{17210a^6\lambda}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} - \frac{8314a^7\lambda}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} + \\
& \frac{2716a^8\lambda}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} - \frac{536a^9\lambda}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} + \\
& \frac{48a^{10}\lambda}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} - \frac{168\lambda^2}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} + \\
& \frac{1272a\lambda^2}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} - \frac{4260a^2\lambda^2}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} + \\
& \frac{8664a^3\lambda^2}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} - \frac{12144a^4\lambda^2}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} + \\
& \frac{12264a^5\lambda^2}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} - \frac{9096a^6\lambda^2}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} + \\
& \frac{4896a^7\lambda^2}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} - \frac{1836a^8\lambda^2}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} + \\
& \frac{432a^9\lambda^2}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} - \frac{48a^{10}\lambda^2}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{(-1+a)\lambda} (2(-5a\lambda + 40a^2\lambda - 65a^3\lambda + 36a^4\lambda - 6a^5\lambda + 36\lambda^2 - 151a\lambda^2 + 212a^2\lambda^2 - \\
& 163a^3\lambda^2 + 66a^4\lambda^2 - 12a^5\lambda^2 + 36a\lambda^3 - 60a^2\lambda^3 + 60a^3\lambda^3 - 30a^4\lambda^3 + 6a^5\lambda^3)) - \\
& \lambda \left((-72a^2 + 234a^3 - 231a^4 - 324a^5 + 653a^6 + 515a^7 - 2386a^8 + 2943a^9 - 1904a^{10} + \right. \\
& 660a^{11} - 96a^{12} - 288a\lambda + 1296a^2\lambda - 3246a^3\lambda + 5094a^4\lambda - 4122a^5\lambda - 748a^6\lambda + \\
& 6066a^7\lambda - 7518a^8\lambda + 5134a^9\lambda - 2108a^{10}\lambda + 488a^{11}\lambda - 48a^{12}\lambda - 72\lambda^2 + 468a\lambda^2 - \\
& 1488a^2\lambda^2 + 2964a^3\lambda^2 - 3828a^4\lambda^2 + 2796a^5\lambda^2 + 60a^6\lambda^2 - 2916a^7\lambda^2 + 3900a^8\lambda^2 - \\
& 2928a^9\lambda^2 + 1380a^{10}\lambda^2 - 384a^{11}\lambda^2 + 48a^{12}\lambda^2) / (a^3(-18 + 45a - 72a^2 + 45a^3 + 23a^4 - \\
& 70a^5 + 59a^6 - 24a^7 + 4a^8)\lambda) - \frac{1}{\lambda} \left(-\frac{2a}{(-1+a)^2} - \frac{20a^2}{(-1+a)^2} + \frac{34a^3}{(-1+a)^2} - \frac{12a^4}{(-1+a)^2} - \right. \\
& \frac{48\lambda}{(-1+a)^2} + \frac{170a\lambda}{(-1+a)^2} - \frac{172a^2\lambda}{(-1+a)^2} + \frac{98a^3\lambda}{(-1+a)^2} - \frac{24a^4\lambda}{(-1+a)^2} - \frac{60a\lambda^2}{(-1+a)^2} + \frac{72a^2\lambda^2}{(-1+a)^2} - \\
& \left. \frac{48a^3\lambda^2}{(-1+a)^2} + \frac{12a^4\lambda^2}{(-1+a)^2} - \frac{12(1-2a+a^2)(-a\lambda+2\lambda^2-2a\lambda^2+a\lambda^3)}{\lambda} \right) + \\
& e^{-\lambda} \lambda \left(-\frac{1}{\lambda} \left(-\frac{8}{(-2+a)(-1+a)^3} - \frac{72a}{(-2+a)(-1+a)^3} + \frac{94a^2}{(-2+a)(-1+a)^3} - \right. \right. \\
& \frac{48a^3}{(-2+a)(-1+a)^3} + \frac{10a^4}{(-2+a)(-1+a)^3} + \frac{76\lambda}{(-2+a)(-1+a)^3} - \frac{46a\lambda}{(-2+a)(-1+a)^3} - \\
& \frac{40a^2\lambda}{(-2+a)(-1+a)^3} + \frac{46a^3\lambda}{(-2+a)(-1+a)^3} - \frac{12a^4\lambda}{(-2+a)(-1+a)^3} - \frac{156\lambda^2}{(-2+a)(-1+a)^3} - \\
& \frac{24\lambda^2}{(-2+a)(-1+a)^3 a^2} + \frac{96\lambda^2}{(-2+a)(-1+a)^3 a} + \frac{132a\lambda^2}{(-2+a)(-1+a)^3} - \frac{60a^2\lambda^2}{(-2+a)(-1+a)^3} + \\
& \left. \frac{12a^3\lambda^2}{(-2+a)(-1+a)^3} - \frac{1}{(-2+a)a^2\lambda} (2(-1+a)(-10a^2\lambda + 17a^3\lambda - 6a^4\lambda + 24a\lambda^2 - \right. \\
& 26a^2\lambda^2 - 5a^3\lambda^2 + 6a^4\lambda^2 + 12\lambda^3 - 24a\lambda^3 + 18a^2\lambda^3 - 6a^3\lambda^3)) \left. \right) + \\
& \frac{1}{(-2+a)a^2\lambda} (2(-1+a)(-10a^2\lambda + 17a^3\lambda - 6a^4\lambda + 24a\lambda^2 - 26a^2\lambda^2 - 5a^3\lambda^2 + 6a^4\lambda^2 + \\
& 12\lambda^3 - 24a\lambda^3 + 18a^2\lambda^3 - 6a^3\lambda^3)t) + e^{-\lambda} \lambda^2 \left(-\frac{1}{\lambda} \left(\frac{839}{(-1+a)^5(-1+2a)^2(1-a+a^2)} + \right. \right. \\
& \frac{24}{(-1+a)^5 a^2 (-1+2a)^2 (1-a+a^2)} - \frac{216}{(-1+a)^5 a (-1+2a)^2 (1-a+a^2)} - \\
& \frac{1936a}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} + \frac{3298a^2}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} - \\
& \left. \frac{5033a^3}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} + \frac{7263a^4}{(-1+a)^5 (-1+2a)^2 (1-a+a^2)} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{9042a^5}{(-1+a)^5(-1+2a)^2(1-a+a^2)} + \frac{8692a^6}{(-1+a)^5(-1+2a)^2(1-a+a^2)} - \\
& \frac{5993a^7}{(-1+a)^5(-1+2a)^2(1-a+a^2)} + \frac{2780a^8}{(-1+a)^5(-1+2a)^2(1-a+a^2)} - \\
& \frac{772a^9}{(-1+a)^5(-1+2a)^2(1-a+a^2)} + \frac{96a^{10}}{(-1+a)^5(-1+2a)^2(1-a+a^2)} + \\
& \frac{2860\lambda}{(-1+a)^5(-1+2a)^2(1-a+a^2)} - \frac{24\lambda}{(-1+a)^5a^3(-1+2a)^2(1-a+a^2)} + \\
& \frac{216\lambda}{(-1+a)^5(-1+2a)^2(1-a+a^2)} - \frac{936\lambda}{(-1+a)^5a(-1+2a)^2(1-a+a^2)} - \\
& \frac{7142a\lambda}{(-1+a)^5(-1+2a)^2(1-a+a^2)} + \frac{14486a^2\lambda}{(-1+a)^5(-1+2a)^2(1-a+a^2)} - \\
& \frac{22978a^3\lambda}{(-1+a)^5(-1+2a)^2(1-a+a^2)} + \frac{27912a^4\lambda}{(-1+a)^5(-1+2a)^2(1-a+a^2)} - \\
& \frac{25494a^5\lambda}{(-1+a)^5(-1+2a)^2(1-a+a^2)} + \frac{17210a^6\lambda}{(-1+a)^5(-1+2a)^2(1-a+a^2)} - \\
& \frac{8314a^7\lambda}{(-1+a)^5(-1+2a)^2(1-a+a^2)} + \frac{2716a^8\lambda}{(-1+a)^5(-1+2a)^2(1-a+a^2)} - \\
& \frac{536a^9\lambda}{(-1+a)^5(-1+2a)^2(1-a+a^2)} + \frac{48a^{10}\lambda}{(-1+a)^5(-1+2a)^2(1-a+a^2)} - \\
& \frac{168\lambda^2}{(-1+a)^5(-1+2a)^2(1-a+a^2)} + \frac{1272a\lambda^2}{(-1+a)^5(-1+2a)^2(1-a+a^2)} - \\
& \frac{4260a^2\lambda^2}{(-1+a)^5(-1+2a)^2(1-a+a^2)} + \frac{a^3\lambda^2}{(-1+a)^5(-1+2a)^2(1-a+a^2)} - \\
& \frac{12144a^4\lambda^2}{(-1+a)^5(-1+2a)^2(1-a+a^2)} + \frac{12264a^5\lambda^2}{(-1+a)^5(-1+2a)^2(1-a+a^2)} - \\
& \frac{9096a^6\lambda^2}{(-1+a)^5(-1+2a)^2(1-a+a^2)} + \frac{4896a^7\lambda^2}{(-1+a)^5(-1+2a)^2(1-a+a^2)} - \\
& \frac{1836a^8\lambda^2}{(-1+a)^5(-1+2a)^2(1-a+a^2)} + \frac{432a^9\lambda^2}{(-1+a)^5(-1+2a)^2(1-a+a^2)} - \\
& \frac{48a^{10}\lambda^2}{(-1+a)^5(-1+2a)^2(1-a+a^2)} - \frac{1}{(-1+a)a\lambda} (2(-5a\lambda + 40a^2\lambda - 65a^3\lambda + \\
& 36a^4\lambda - 6a^5\lambda + 36\lambda^2 - 151a\lambda^2 + 212a^2\lambda^2 - 163a^3\lambda^2 + 66a^4\lambda^2 - \\
& 12a^5\lambda^2 + 36a\lambda^3 - 60a^2\lambda^3 + 60a^3\lambda^3 - 30a^4\lambda^3 + 6a^5\lambda^3)) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{(-1+a)a\lambda} (2(-5a\lambda + 40a^2\lambda - 65a^3\lambda + 36a^4\lambda - 6a^5\lambda + 36\lambda^2 - 151a\lambda^2 + \\
& 212a^2\lambda^2 - 163a^3\lambda^2 + 66a^4\lambda^2 - 12a^5\lambda^2 + 36a\lambda^3 - 60a^2\lambda^3 + 60a^3\lambda^3 - 30a^4\lambda^3 + 6a^5\lambda^3)t) \\
& + \frac{6(-1+a)^3(-a\lambda^2 + 2\lambda^3 - 2a\lambda^3 + a\lambda^4)t^2}{a\lambda} \Big) + e^{-a\lambda t} \Big((-72a^2 + 234a^3 - 231a^4 - 324a^5 + \\
& 653a^6 + 515a^7 - 2386a^8 + 2943a^9 - 1904a^{10} + 660a^{11} - 96a^{12} - 288a\lambda + \\
& 1296a^2\lambda - 3246a^3\lambda + 5094a^4\lambda - 4122a^5\lambda - 748a^6\lambda + 6066a^7\lambda - 7518a^8\lambda + 5134a^9\lambda - \\
& 2108a^{10}\lambda + 488a^{11}\lambda - 48a^{12}\lambda - 72\lambda^2 + 468a\lambda^2 - 1488a^2\lambda^2 + 2964a^3\lambda^2 - 3828a^4\lambda^2 + \\
& 2796a^5\lambda^2 + 60a^6\lambda^2 - 2916a^7\lambda^2 + 3900a^8\lambda^2 - 2928a^9\lambda^2 + 1380a^{10}\lambda^2 - 384a^{11}\lambda^2 + \\
& 48a^{12}\lambda^2) / (a^3(-18 + 45a - 72a^2 + 45a^3 + 23a^4 - 70a^5 + 59a^6 - 24a^7 + 4a^8)\lambda) + \\
& e^{(-1+a)\lambda t} \Big(-\frac{1}{\lambda} \Big(-\frac{2a}{(-1+a)^2} - \frac{20a^2}{(-1+a)^2} + \frac{34a^3}{(-1+a)^2} - \frac{12a^4}{(-1+a)^2} - \frac{48\lambda}{(-1+a)^2} + \\
& \frac{170a\lambda}{(-1+a)^2} - \frac{172a^2\lambda}{(-1+a)^2} + \frac{98a^3\lambda}{(-1+a)^2} - \frac{24a^4\lambda}{(-1+a)^2} - \frac{60a\lambda^2}{(-1+a)^2} + \frac{72a^2\lambda^2}{(-1+a)^2} - \frac{48a^3\lambda^2}{(-1+a)^2} + \\
& \frac{12a^4\lambda^2}{(-1+a)^2} - \frac{12(1-2a+a^2)(-a\lambda + 2\lambda^2 - 2a\lambda^2 + a\lambda^3)}{\lambda} \Big) + \\
& \frac{12(1-2a+a^2)(-a\lambda + 2\lambda^2 - 2a\lambda^2 + a\lambda^3)t}{\lambda} \Big) \Big)
\end{aligned}$$

(12)