

Review of “Two identities of derangements” by Le Anh Vinh

Two main comments:

Theorem 1 is really just equation (1) with the indices changed, so it probably shouldn't be stated as a new theorem. Specifically, we can show Theorem 1 using equation (1) as follows:

$$\begin{aligned}\sum_{k=0}^n \binom{n-l}{k-l} d(n-k) &= \sum_{k=l}^n \binom{n-l}{k-l} d(n-k) \\ &= \sum_{k=0}^{n-l} \binom{n-l}{k} d(n-(k+l)) \\ &= \sum_{k=0}^{n-l} \binom{n-l}{k} d((n-l)-k) = (n-l)!\end{aligned}$$

where the last line is equivalent to equation (1). Since the theorem isn't used in the rest of the paper it can be omitted.

The result of Theorem 2 should just be stated as:

$$\sum_{k=0}^n k^i \binom{n}{k} d(n-k) = B_i n!$$

for any  $i$ . Equation (5) then follows by linearity. It would be nice (but not necessary) to have a combinatorial proof of this identity.

Some minor notes:

Page 1, after equation (2), should read “The number of ways a set of  $n$  elements...”

Page 2, in the statement of Theorem 2, and elsewhere, it would be useful to specify the limits on  $k$  in the summation.

Page 2, proof of Lemma 1: In 2(a) and 2(b), indicate which term in the statement of Lemma 1 these cases apply to ( 2(a) corresponds to the third

term and 2(b) corresponds to the second term).

Page 5, second to last line: Change “is the one sending” to “sends”.