Referree's Report
An analytic result in a star-shaped domain

This is a nice result, but the exposition should be better and cleaner. At the moment, it is not appropriate for reading by any one other than an expert. There is little or no explanation of why this is interesting. I doubt that many undergraduates could read this and understand this paper. Please carefully read the following excerpt from the guide for authors and consider whether your introduction fulfills the requirements.

> *Your paper should be accessible and interesting to a wide range of readers. All papers should have introductory paragraphs that describe the work, give background on the particular problem, and provide motivation for the more detailed aspects of the paper. Furthermore, your paper should include appropriate references.*

More elaborate suggestions for the author follow below.

However, after reading the paper, I have two simple questions that should be addressed in revisions to the paper.

1. The construction apparently does not require star-shaped region (see last bullet in list below). Why concentrate on this class of domains?
2. The construction of the domain is proved for $\mathbf{R}^2$, but why can it not be extended to $\mathbf{R}^n$? The extension to $\mathbf{R}^3$ should almost be immediate by revolving the domain constructed in $\mathbf{R}^2$ about the x-axis in Figure 1, suitably adjusting the chosen constants. Likewise by revolving the two-dimensional domains in higher dimensions, one should be able to construct a domain in $\mathbf{R}^n$.

Suggestions for the Author

- The paper could use a better title (more descriptive) such as "Construction of domains that admit non-constant stable solutions to diffusion equations".
- The abstract needs to be enlarged. It is supposed to entice the reader to read the paper. It needs more details. Lead the reader to understand why this is important. Briefly state the known results from Matano's paper, concerning examples of domains admitting stable non-constant steady-state solutions. You can even reference Matano's paper in the abstract.
- The introduction needs to better introduce the material and the problem. In particular, the introduction needs to be expanded so that the reader understands the result in the paper and its significance. Do not assume the reader is an expert in semilinear partial differential equations. This is an undergraduate journal.
- The significance of the result should be mentioned. Why is the construction of stable non-constant steady-state solutions interesting? From Matano's paper, there are results on the existence/non-existence of non-constant steady-state solutions. References to these results should appear in the introduction. For instance, that there are no convex domains that admits stable non-constant steady-state solutions should be mentionned.

- There also should be some mention that conditions on $f(u)$ are needed for global existence, as there are nonlinearities which blow-up in finite time irrespective of the boundary conditions.

- Is the entire second section one paragraph??

- There should be more discussion on Theorem 1 and the methods from the Matano's paper. Either reference the theorem from Matano's paper without stating or discuss it. At the moment, you state the theorem and then continue on and start discussing nonconvex domains (in the same paragraph) while the reader is still digesting the statement of the theorem for convex domains, which leads to some confusion. There should be some break between the theorem and the problem to be considered next.

- In equation (4), please state that $0 = g(a) = g(0) = g(b)$. Don't make the reader draw out that conclusion. Make it easy on the reader. It might help, to give an example of a function $g(u)$ that satisfies the conditions, $g(u) = -mu(u-b)(u-a)$ where m is an appropriately chosen positive constant to guarantee $|g(u)| \leq |u|$. You do not want someone to question whether the result is vacuous (a result for which the hypotheses are never satisfied).

- End the paragraph on the conditions on $f(u)$, before starting the paragraph on the conditions on Ω. There also should be some mention that k is a positive constant to be specified later.

- In stating conditions P, please state Poincaré's second inequality, not everyone knows the difference between Poincaré's first and second identity. I (the referee) am an analyst working in nonlinear PDEs and I had to look up the difference, as it is typical to refer to a Poincaré inequality not the first Poincaré inequality or the second Poincaré inequality.

- Please check the equation above theorem 2. I believe you should have D_2 in it somewhere. Also, it might be worthwhile to state that $\lambda_1(D_i) = 0$ corresponds to the constant eigenfunction, so everyone knows the convention you are using. [I have seen some mathematicians refer to the zero eigenvalue for Neumann conditions as λ_0 and the first non-zero eigenvalue as λ_1, as the zero-eigenvalue does not really count for the application that they are considering.]

- In the construction of the domain starting on bottom of page 3, the set does not have to be star shaped until you require it to be star-shaped. Take a dumb-bell shape, two balls connected by a thin rod and smooth out the singularities so the domain has a smooth boundary. Such a set satisfies your conditions at this point, but is not star-shaped. Apparently, unless I am missing something, your construction works on such a domain.