

Take Me Out to/of the Ball Game

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Abstract

When should a person leave a baseball game in order to maximize his/her enjoyment? How does this decision depend on the score of the game? Assuming a modified logistic rate of departure from the stadium, and a constant maximum exit rate from the parking lot, we find optimal strategies for leaving games of various scores.

1 Introduction

Baseball has been billed as America's national past time. The number of fans who annually pack stadiums across the country is sufficient evidence to validate this claim. However, as the games come to a close an interesting phenomenon occurs. Many spectators begin to exit before the final outs have been recorded, presumably to avoid traffic problems in the parking lot and neighboring streets. Anyone who has attended a baseball game has wondered at some point: When should I leave so I can witness as much of the game as possible, yet avoid a long wait in the parking lot? With this project we will attempt to mathematically determine the best time to leave a baseball game.

2 Problem

Our objective is to determine the time at which a baseball fan should leave a game in order to maximize his or her enjoyment. Two major factors will be

considered. The first is the enjoyment of the sights, sounds and smells of the game. The second, in contrast, is the frustration of waiting in the stadium and in the parking lot after the game has ended. Our analysis will center around the interaction between a modified logistic equation, representing the number of people who have left the stadium, and a linear equation, corresponding to the number of people who have left the parking lot.

3 Assumptions

We begin our analysis with the assumption that each inning lasts 25 minutes. We also assume that the game will not end early due to weather or other extraneous circumstances, nor will it proceed into extra innings due to a tie. Thus, a 9-inning game will last 225 minutes. Next, we suppose that everyone who attends the game arrives in a vehicle. Since people rarely attend a baseball game alone, we assume there are two spectators per car. It is also reasonable to assume those present will not leave before the top of the sixth inning and the stadium will be nearly empty 45-60 minutes after the game has concluded. We made these two assumptions after speaking with several Major League Baseball team representatives. They were unable to give exact values, but agreed on these approximations.

Another assumption is that due to the large size of the exits at a baseball stadium, spectators can get to their vehicles as quickly as they wish. Therefore, we can neglect the lag time between getting up to leave and arriving at one's car. We also assume that frustration from waiting in the parking lot, frustration from waiting in the stadium, and enjoyment from watching the game will not necessarily be of the same magnitude. In particular, we assume the parking lot frustration level to remain a constant value over time, the frustration due to waiting in the stadium to be half of this value, and the enjoyment of spectating, depending on the score, to be one to two times this value.

4 Stadium Curve

Since people wish to avoid long delays in the parking lot, it is reasonable to assume that as seated fans observe other spectators leaving or having left the

stadium, they will be more inclined to leave themselves. Thus, to model this event we use a simple logistic differential equation. This equation states that the rate at which people leave, $\frac{dP}{dt}$, is proportional to the number of people who have left the stadium, $P(t)$, and to the number of people still in the stadium, $(K - P(t))$, where K is the total number of fans.

Additional factors also have an effect on when the spectators leave the stadium. Later in the game, people will become more conscious of wanting to avoid traffic. Thus, as the game nears a close, the people leaving the stadium have a greater influence on those who are still seated. This naturally increasing tendency to leave as the game progresses is accounted for in the added factor t^α .

At any time, the game's appeal will be determined primarily by the difference between the scores of the two teams. People will be more inclined to stay when a game is competitive and less inclined if it ceases to be. The function $S(t)$ is the magnitude score difference between the two teams at time t after the fifth inning, where $S(t)$ assumes values from 1 to 10. The score functions are scaled using the function $R(S)$, which is described below. Combining all of these factors, the differential equation becomes:

$$\frac{dP}{dt} = \beta P(t) (K - P(t)) t^\alpha R(S). \quad (1)$$

In its general form, the solution to this differential equation is:

$$P(t) = \frac{K}{1 + K e^{-K\beta \int (R(S) t^\alpha) dt}}. \quad (2)$$

With the general solution in hand, the next step is to determine the constants. Since an integral is in the solution, the resulting integration constant must be determined. As stated previously, we assume that fans do not generally leave before the sixth inning, so we apply the initial condition $P(130) = 10$. This means that 130 minutes into the game, just after the sixth inning begins, 10 spectators will have left.

The next constants, α and β , were determined by the information gained from phone interviews with team representatives. In particular, the constants were chosen so that almost all of the spectators will have left the

stadium 45-60 minutes after the game ends. The values of $\beta = 2.623 \times 10^{-6}$ and $\alpha = \frac{1}{20}$ give a good representation of the spectators' departure. It seems reasonable to take $R(S) = \frac{S(t)+26}{36}$, so that $R(S)$ takes on values from $\frac{3}{4}$ to 1 depending on the score of the game.

The final constant to determine is K . At the time this paper was written, approximately one-third of the Major League Baseball games for the 2003 season had been played. The median attendance of the games was about 25000 and thus K was chosen accordingly.

With the determination of the constants complete, we now consider three games with different score functions.

- The first game, G_1 , has a score difference of one run at all times. Therefore it has a score function of $S(t) = 1$.
- The score of the second game, G_2 , increases linearly from 1 to 10 from the beginning of the sixth inning to the end of the game. Thus, the score function is $S(t) = \frac{9}{100}t - \frac{41}{4}$.
- In the final game, G_3 , one team finds itself down 10 runs at the beginning of the sixth inning but comes back to make the score competitive. This linearly decreasing score function is $S(t) = \frac{-9}{100}t + \frac{85}{4}$.

At times before the sixth inning the score functions yield unrealistic values. However, the initial condition of $P(130) = 10$ rectifies this problem.

The following are the stadium curves for games G_1 , G_2 , and G_3 .

As expected, the different score functions lead to differences in the number of people who stay until the end of the game. In G_1 , fewer than 5,000 people have left by the end of the ninth inning while in G_2 more than 9,000 have left for home. This is a reasonable result since more people would be willing to face the traffic to watch a close game than a blowout. Due to the large early lead in G_3 , there are more spectators that leave early in the game than in G_1 and G_2 . However, as the game becomes more competitive, people decide to stay and watch the game. The spectators at G_2 react in the opposite manner. While their game is competitive early on, few head for home. As the game progresses and becomes less interesting, more people decide to leave. The

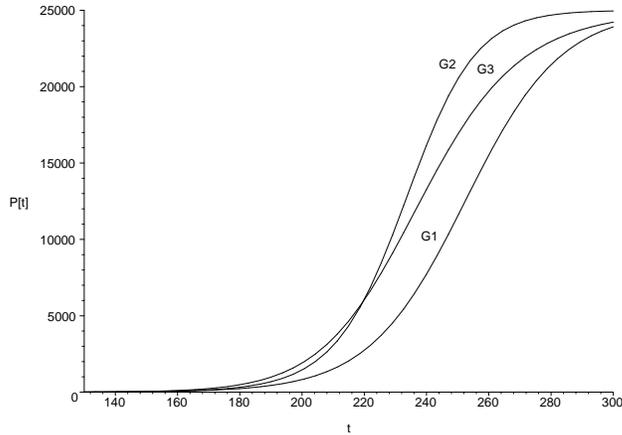


Figure 1: Number of Departed Spectators for G_1 , G_2 , and G_3

change of events in both games accounts for the intersection of the graphs of G_2 and G_3 .

5 Parking Lot Curve

Now that we know how spectators leave the stadium, we need to know how these people will exit the parking lot. As with any parking lot, the stadium parking lot has a maximum rate at which cars are able to exit. We denote this maximum rate, m . We then define $C(t)$, the parking lot curve, as the number of people who have left the parking lot at time t :

$$C(t) = m t + b. \tag{3}$$

Initially, the rate of departure from the stadium is less than m . During this time $P(t)$ and $C(t)$ coincide and the spectators are able to leave as quickly as they wish. When the rate of people leaving the stadium becomes greater than m , the two curves diverge and some spectators have to wait in the parking lot before they can leave. Through the data collected from various baseball stadium representatives, we found that typically 100 cars are able to leave the parking lot per minute. Combining this with our assumption

that there are two people per car, the maximum rate of departure from the parking lot, m , is 200 people per minute. Knowing the slope value and the point of divergence, b is easily calculated. The graph of $C(t)$ and $P(t)$ for G_2 is shown below.

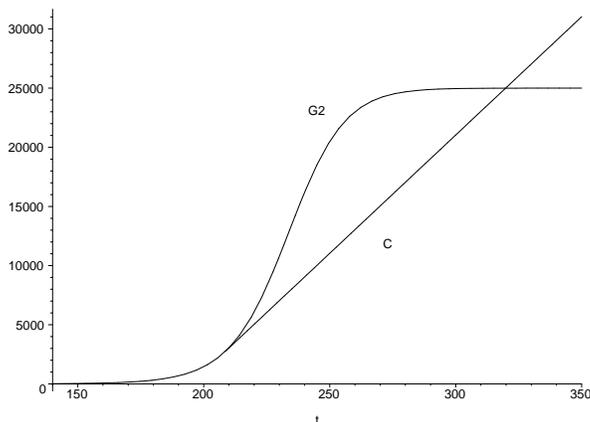


Figure 2: Stadium and Parking Lot Curves for a Linearly Increasing Score Difference

In this example the two curves diverge 208 minutes after the game started. In comparison, the time of divergence for G_1 occurs approximately 225 minutes after the game starts. This is a reasonable value since most of the fans will remain in the stadium to see how the game ends. In game G_3 , the divergence point is located at 209 minutes. Spectators left earlier than those attending G_1 , but later than those attending the less interesting game, G_2 .

6 Enjoyment

Now that we have determined how people exit the stadium and the parking lot, we can derive the overall enjoyment of the ballpark experience. Our model for a spectator's overall enjoyment is:

$$E(t) = \int_{125}^t Q(S(t))[1 - HVSD(t - 225)]dt - \int_{225}^t \frac{1}{2}[HVSD(t - 225)]dt - \int_t^{t_E} 1dt. \quad (4)$$

The first integral, $\int_{125}^t Q(S(t))[1 - HVSD(t - 225)]dt$, represents a spectator's enjoyment from the game. The function $Q(S) = \frac{-1}{9}(S(t) - 1) + 2$ scales the score function, $S(t)$. For each minute spent watching the game, the value is one to two times that of a minute spent waiting in the parking lot. This is multiplied by the Heaviside function $[1 - HVSD(t - 225)]$ to ensure a spectator's enjoyment does not increase after the game has ended. Since the value per minute of enjoyment is determined by $Q(S)$, the integral of $Q(S)[1 - HVSD(t - 225)]$ from the time our score function is applicable, $t = 125$, until the time the spectator leaves the stadium, time t , represents a spectator's total positive enjoyment from the game. Setting the lower integral bound at $t = 125$ ensures that the score function's unrealistic values before $t = 125$ will not impact the outcome of the enjoyment function.

For various reasons, some spectators choose to wait in the stadium rather than in the parking lot. The frustration of this period is represented in the second integral, $\int_{225}^t \frac{1}{2}[HVSD(t - 225)]dt$. The constant $\frac{1}{2}$ represents the frustration of waiting in the stadium for one minute. This constant was chosen less than one since watching post game activities is less frustrating than waiting in one's car. The Heaviside function ensures that a spectator's frustration begins after the game ends. The integral from the end of the game, $t = 225$, till the time the spectator leaves the game denotes the spectator's total frustration due to waiting in the stadium after the game has ended.

The final integral, $\int_t^{t_E} 1dt$, represents spectators' frustration from the time they leave the stadium until the time they exit the parking lot, $t = t_E$. The constant value of 1 represents the value of frustration per minute. Note that the value of this integral is equal to $\frac{P(t)-C(t)}{m}$. Subtracting the parking lot curve, $C(t)$, from the stadium curve, $P(t)$, yields the number of people who are waiting in the parking lot. Dividing by the parking lot exit rate, m , determines the amount of time a spectator waits, therefore determining the value of frustration. Alternately, notice from the graph that $\frac{P(t)-C(t)}{m} = \Delta t$, and Δt (the horizontal distance between the graphs) represents the time difference between leaving the stadium and leaving the parking lot.

By subtracting frustration from positive enjoyment, a spectator's total enjoyment is calculated. The graph below depicts the overall enjoyment of a spectator in each of the three example games depending on his/her time of departure. The highest point of each plot corresponds to the time when a spectator should leave the game in order to maximize his/her enjoyment.

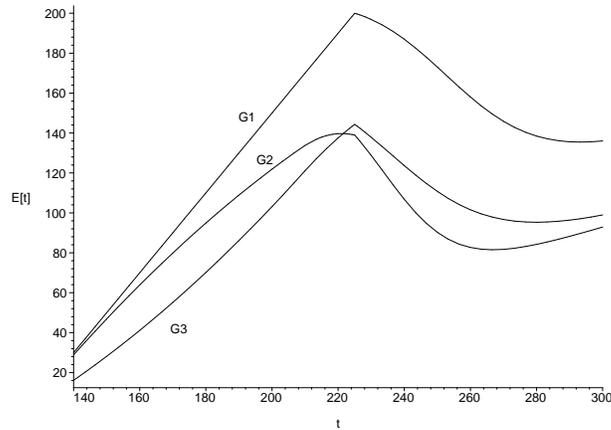


Figure 3: Maximum Enjoyment and Best Time of Departure

Our model predicts that a spectator should leave slightly before $t = 225$, just before the game ends. This result is somewhat surprising but is justifiable for all three games. In G_1 , fans are willing to face the possibility of crowds in order to see a close game in its entirety. The enjoyment gained from watching the game outweighs the frustration of a longer wait in the parking lot. Similarly, the score is very close at the end of G_3 . In addition, more people left earlier in the game, therefore there are fewer people to deal with in the parking lot once the game ends. The combination of a shorter wait in the parking lot and an increasingly competitive game yields a late time of departure. The smaller number of remaining spectators also accounts for the best time of departure in G_2 . Although G_2 may not be as appealing as G_1 or G_3 , the number of departed spectators is great enough that the frustration of waiting in the parking lot is small just before the game has ended.

The graph also shows at certain points an early departure from G_3 yields a smaller amount of enjoyment than that of G_2 . However, if the spectators attending G_3 stay through the end of the game, they are able to witness an incredible comeback and their total enjoyment increases beyond that of G_2 . This is a direct result of the different score functions in each game.

Interestingly enough, increasing or decreasing the value of frustration due to waiting in the stadium has little or no effect on the departure time of a spectator from the stadium. We also explored the possibilities that a spectator's frustration could have an exponential or logarithmic growth. These changes also had very little effect on the time when a spectator should leave to maximize his/her enjoyment. This results from the late divergence of the parking lot and the stadium curves which leads to a small back up in the parking lot right before the game ends. Consequently a spectator's total enjoyment does not decrease significantly as long as the spectator leaves within 10 minutes of the game's conclusion. In conclusion, the next time you attend a baseball game and begin pondering when you should leave, we suggest that you give your mind a rest, relax, and enjoy the entire game.