

COMMENTS ON WEI REN'S PAPER

In the following, "p.m, l. \pm n" means page m and line n from the top or from the bottom.

1. p.1, l.+5. Perhaps the introductory paragraphs might be better as:

Let N be an integer. For non-negative integers n , let $P_N(n)$ be the coefficients of the series defined by

$$\prod_{n=1}^{\infty} \frac{1}{(1 - q^n)^N} = \sum_{n=0}^{\infty} P_N(n)q^n$$

and let $P_N(r) = 0$ for a rational number r not equal to any non-negative integer.

Recently Farkas and Kra gave five three-term identities for the coefficients $P_N(n)$:

Theorem 1 (Farkas and Kra [1]). For all

2. p.2, l.+8. In the statement of Theorem 2, "positive integers" should be "non-negative integers".

3. p.2, l.+9 to +12. In Theorem 2, the line numbers (1.7), . . . , (1.10) and the corresponding equations should be on the same line.

4. p.2, l.-9. Perhaps it would be better to change *Remarks* to Note that, and then have (i) (ii); also use "our" instead of "Our" and a comma at the end rather than a period. I would also like to suggest that you add (iii) which would be "from Theorem 2, it follows immediately" and then you would state the corollary. Note that if you use this construction, you will need an "and" at the end of (ii)

5. p.2, l.-5. Instead the sentence "It immediately follows that" , how about "From Theorem 2 immediately follows" ?

6. p.3, l.+4 Remove the comma and "denoted Γ " should be read "denoted by Γ " ?

7. No change

8. p.3, l.+13. "N be a positive integer." should be read "N a positive integer."

9. p.3, l.-11 to l.-8. m and n should be exchanged. For example, not $\chi(n) = \chi(m)$ but $\chi(m) = \chi(n)$.

10. p.3, l.-5. $\chi(d)f$ should be read $\chi(d)f(z)$.

11. p.3, l.-2. "A modular form $f(z)$ " would be preferable to "A modular form".

12.No change

13. p.4, l.+8. ...is the family of Hecke...

14. p.4, l.+14. "A modular form" should be "A modular form $f(z)$ ".

15. No change

16. p.4, l.-7. The exponent 24 should be deleted, i.e.,

$$\eta(z) = q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1 - q^n)^{24}, \quad q = e^{2\pi iz}$$

should be read

$$\eta(z) = q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1 - q^n), \quad q = e^{2\pi iz}$$

17. p.4, l.-6. "define integers ... by" should be read
"define the integers $a(n), b(n), c(n)$ and $d(n)$ by $a(0) = b(0) = c(0) = d(0) = 0$ and by"

18. p.4, l.-1. We would prefer "Lemma 1" to "a Theorem", as mentioned in the next Comment No.19. And Lemma 1 should be Lemma 2 and so on.

19. p.5, l.+1. Theorem 3 should be read Lemma 1. Because, considering "Lemma 1" in the original paper, we immediately see that Theorems 2 and 3 are equivalent to each other.

20. p.5, l.+8 to +11. The sides should be exchanged.

21. p.5, l.-12 The exponent $2n$ should be replaced by $2n - 1$, i.e.,

$$\dots = \sum_{n=0}^{\infty} P_{-12}(n-1)q^{2n} = \dots$$

should be read

$$\dots = \sum_{n=0}^{\infty} P_{-12}(n-1)q^{2n-1} = \dots$$

22. p.5, l.-11. $\eta^4(4z)$ should be read $\eta^4(6z)$.

23. p.5, l.-10. "identities" is preferable to "statements". (Also in other places)

24. p.5, l.-9 to l.-8. Instead "We now define ... by", how about "For a positive integer N divisible by 4, we define the character χ_N by"

25. p.5, l.-5. Instead "mod N as the following:", how about " χ_N^{triv} by".

26. p.5, l.-2. Instead "the trivial character", how about χ_N^{triv} .

27. p.6, l.+1 to l.+4. The statements in Lemma 2 are upside down according to Lemma 1.

28. p.6, l.+7. Instead "that ... hold.", how about "the identities (2.2), (2.3) and (2.4) hold with $N = 16$ and $\chi = \chi_{16}$."

29. p.6, l.+8. The reference [4] should be replaced by Rademacher's book "Topics in Analytic Number Theory, Springer-Verlag, 1973",

because the authors of the paper [4] just quoted the statements from the Rademacher's book.

30. p.6, l.+14. Instead "Suppose $\gamma \in \Gamma_0(16)$, and $c > 0$," how about "Suppose $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_0(16)$ and $c > 0$."

31. No change

32. p.6, l.-10. The parentheses are not correct.

33. p.6, l.-6. "Note that we still need to consider the ..." is preferable to "Note. We ...".

34. p.6, l.-4. "... $\Gamma_0(16)$ and $c < 0$, then we have by (2.1) and (3.7)" is preferable.

35. p.7, l.+2.

$$f(z) \Big| \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -f(z).$$

should be read

$$f(z) \Big| \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}_3 = -f(z).$$

36. p.7, l.+6. Use whereas rather than where as. Also "the identity (2.1) gives" is preferable to "(2.1) gives".

37. p.7, l.+9. Instead " $A, D \in \mathbf{Z}^+$," it is preferable to use "positive integers A, D and an integer B "

38. p.8, l.+10. $f|[\gamma_0]_3$ should be read $f(z)|[\gamma_0]_3$.

39. p.8, l.+12. "on $\Gamma_0(16)$ " should be read "for $\Gamma_0(16)$ ".

40. p.8, l.-3 to l.-2. The equal mark "=" should be put on the left side of the second line when the equation is separated.

41. p.8, l.-2. In the last term the condition $x^2 + 1 \equiv 0$ should be read $x^2 + x + 1 \equiv 0$

42. p.8, l.-1 to p.9, l.+1. "(resp. of the conductor ... " should be read "(resp. f , the conductor of the character χ ")

43. p.9. l.+1 to l.+3. Though the notation $\lambda(r_p, s_p, p)$ is used in the paper [5], simpler notation $\lambda(p)$ is preferable to $\lambda(r_p, s_p, p)$. (?)

44 p.9, l.+4. "if $2s_p \geq r_p$," should be read "if $2s_p > r_p$,".

45. p.9, l.+5 to l.+7. The definition of μ_k should be read

$$\mu_k := \begin{cases} 0 & \text{if } k \equiv 1 \pmod{3}, \\ -\frac{1}{3} & \text{if } k \equiv 2 \pmod{3}, \\ \frac{1}{3} & \text{if } k \equiv 0 \pmod{3}. \end{cases}$$

46. p.9, l.+13 to l.+14. The sentences " Since ... We then get" should be changed as follows:

Since $\dim M_{-1}(\Gamma_0(16), \chi_{16}) = 0, r_2 = 4, s_2 = 2, \varepsilon_3 = 0$ and $\mu_3 = \frac{1}{3}$, we get by the formula above

$$\dim S_3(\Gamma_0(16), \chi_{16}) = \dots = 1.$$

47. p.9, l.+16. $S_2(\Gamma_0(36), \chi_{36})$ and $S_6(\Gamma_0(4), \chi_4)$ should be read $S_2(\Gamma_0(36), \chi_{36}^{\text{triv}})$ and $S_6(\Gamma_0(4), \chi_4^{\text{triv}})$, respectively.

48. p.9, l.-13. To be more accurate, "they" should be replaced by " $\eta^4(6z), \eta^6(4z)$ and $\eta^{12}(2z)$ ".

49. p.9, l.-11. "follows Theorem 3" and "conclude by" should be read "follows from Theorem 3" and "conclude our argument by".

50. p.9, l.-10. "on $\Gamma_0(16)$ " should be read "for $\Gamma_0(16)$ ".

51. p.9, l.-9. "... eigenform, by (2.6), we know that ..." would be better to be changed to "... eigenform. We know by (2.6) that ...".

52. p.9, l.-7.
The last term

$$\dots = \lambda_l \sum_{n=1}^{\infty} \lambda_l b(n)q.$$

should be read

$$\dots = \sum_{n=1}^{\infty} \lambda_l b(n)q.$$

53. p.9, l.-4 to l.-3. The sentence "Notice that ... which is (3.2)" could be changed with "Noticing that $\chi_{16}(l) = \left(\frac{-4}{l}\right)$ for all primes l we have

$$b(l)b(n) = \dots,$$

for all primes l and non-negative integers n . Thus we obtain the desired identity (3.2)."

54. p.10. The referred papers should be arranged in alphabetical order by authors.

55. p.10, l.+6. As shown in the Comment No.29, the paper [5] should be replaced by the Rademacher's book.

56. p.10, l.-4. The pages "70-73" should be read "69-78".