

Figure 1: The immersion of a single-punctured torus into the plane.



Figure 2: The reversal of the overlap of a single-punctured torus through a non-flat deformation.

I.



a.



b.



c.



d.



e.



f.

II.



a.



b.



c.



d.



e.



f.

Figure 3: Three flat deformations of a flattened single-punctured torus.

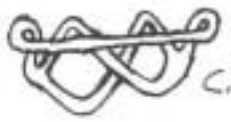
III.



a.



b.



c.



d.

Figure 4: Two distinct flattenings of a three-punctured sphere, and three distinct flattenings of a two-punctured torus.

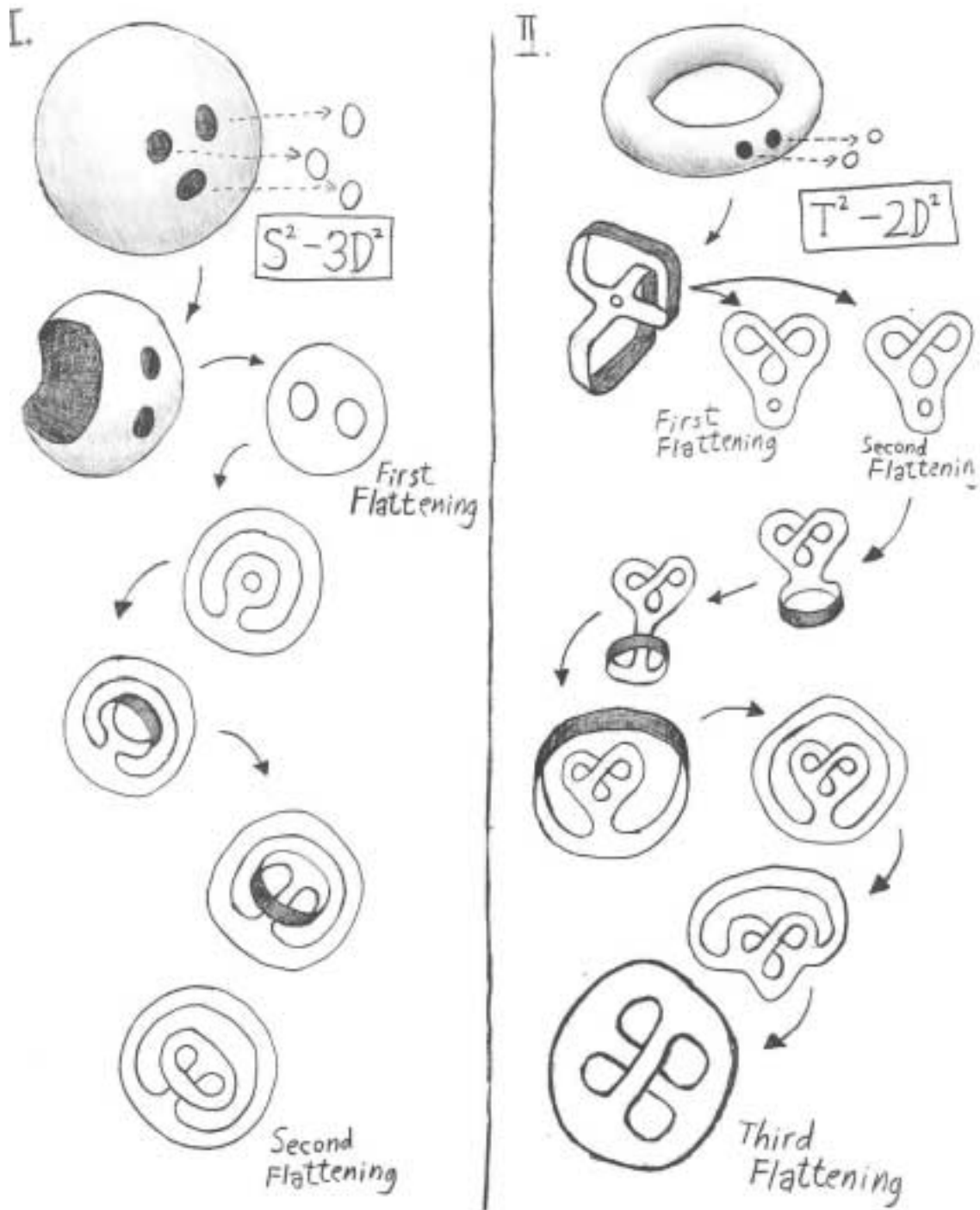
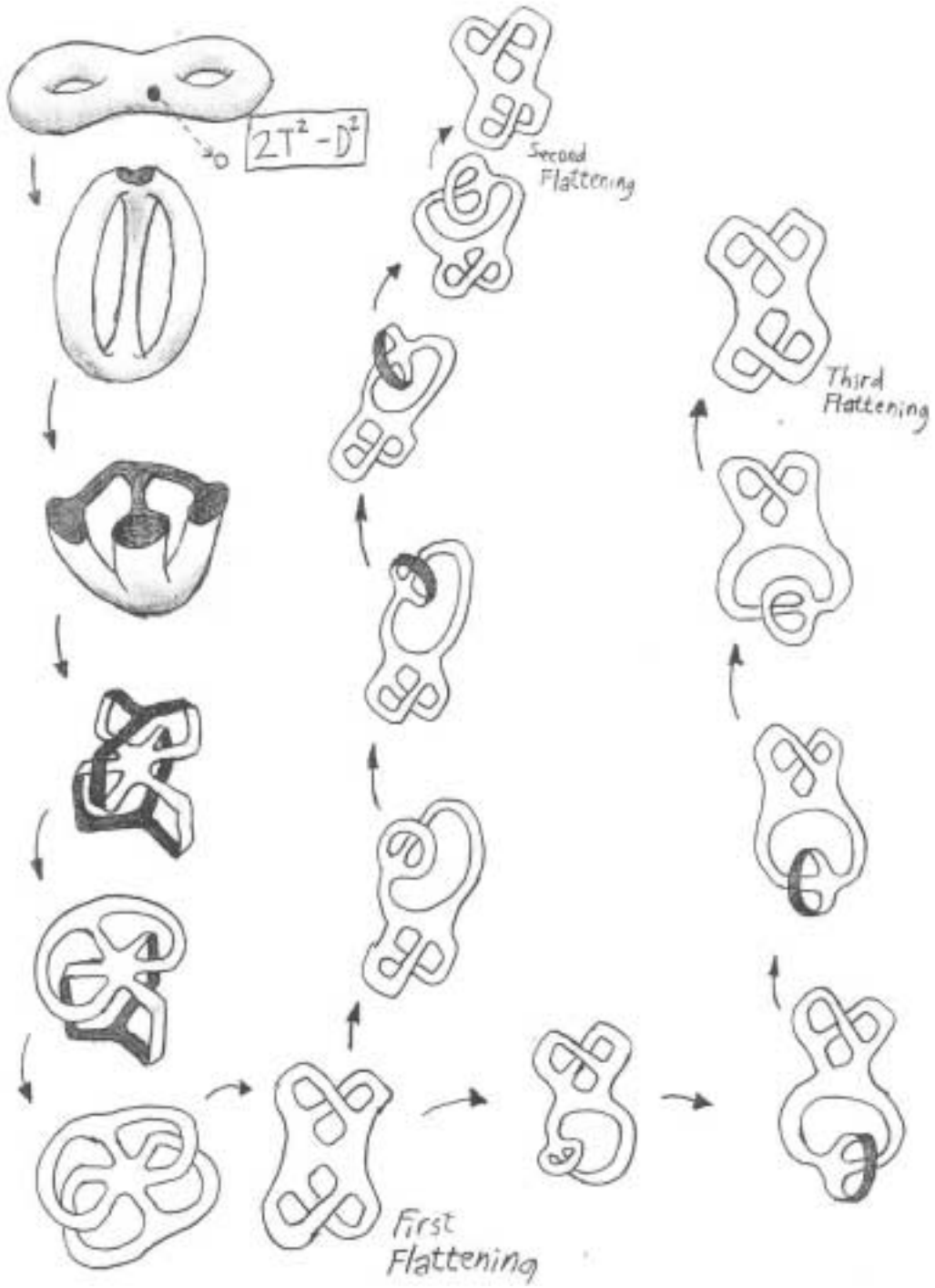


Figure 5: Three distinct flattenings of a single-punctured double-torus.



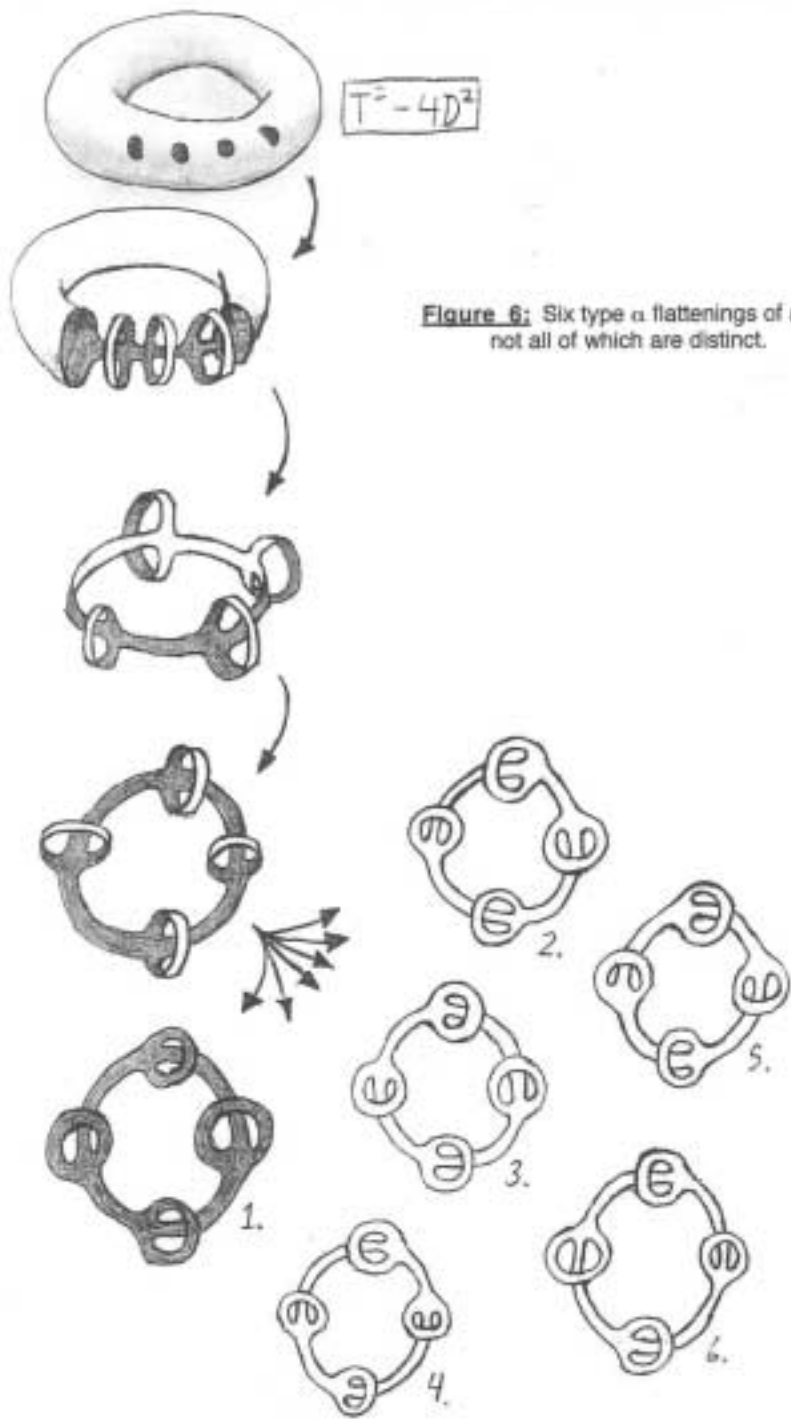


Figure 6: Six type α flattenings of a four-punctured torus, not all of which are distinct.

Figure 7: Flat deformation of a four-punctured torus, demonstrating that forms #1, #2, and #3 from figure 6 are distinct.

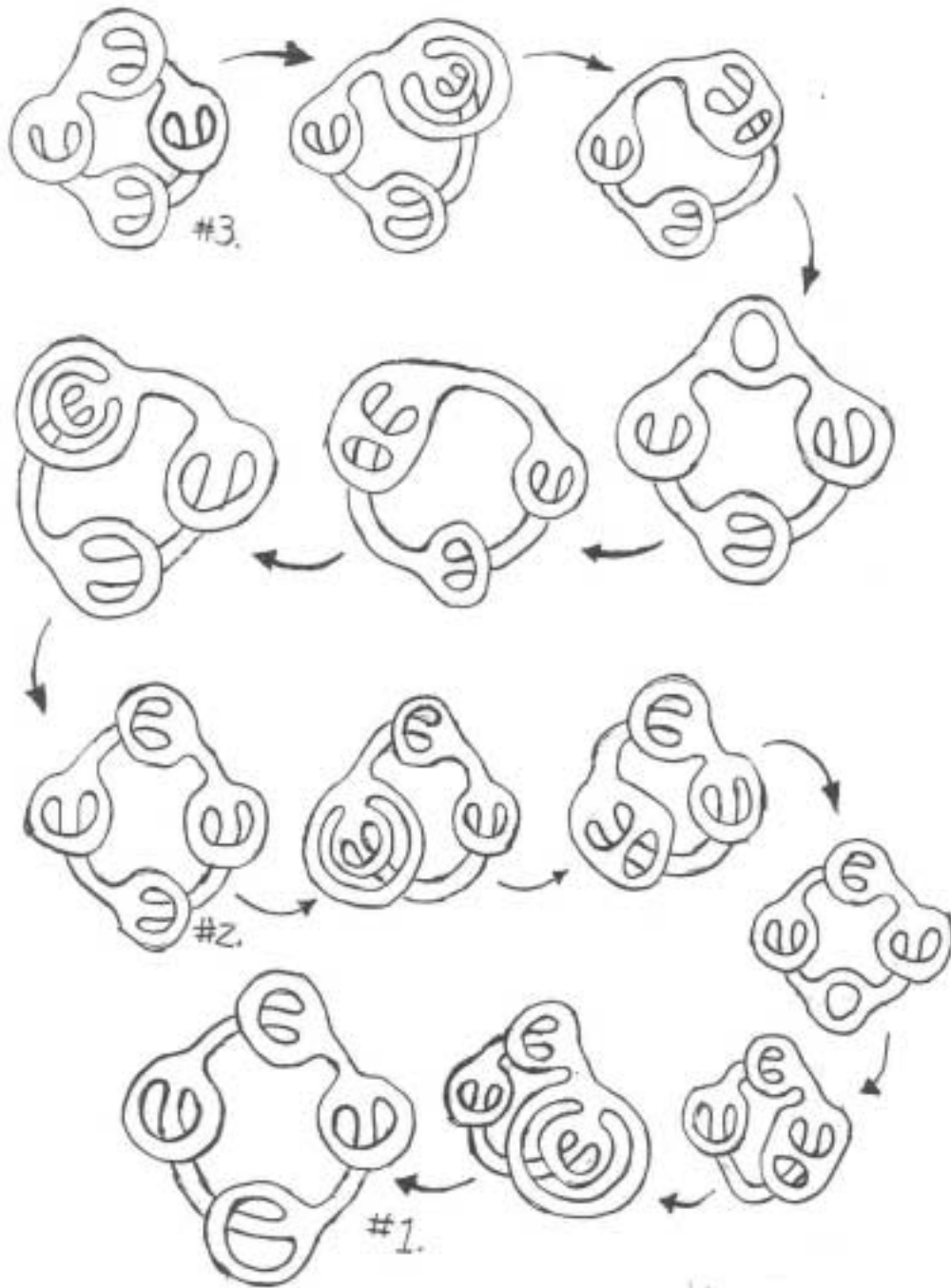
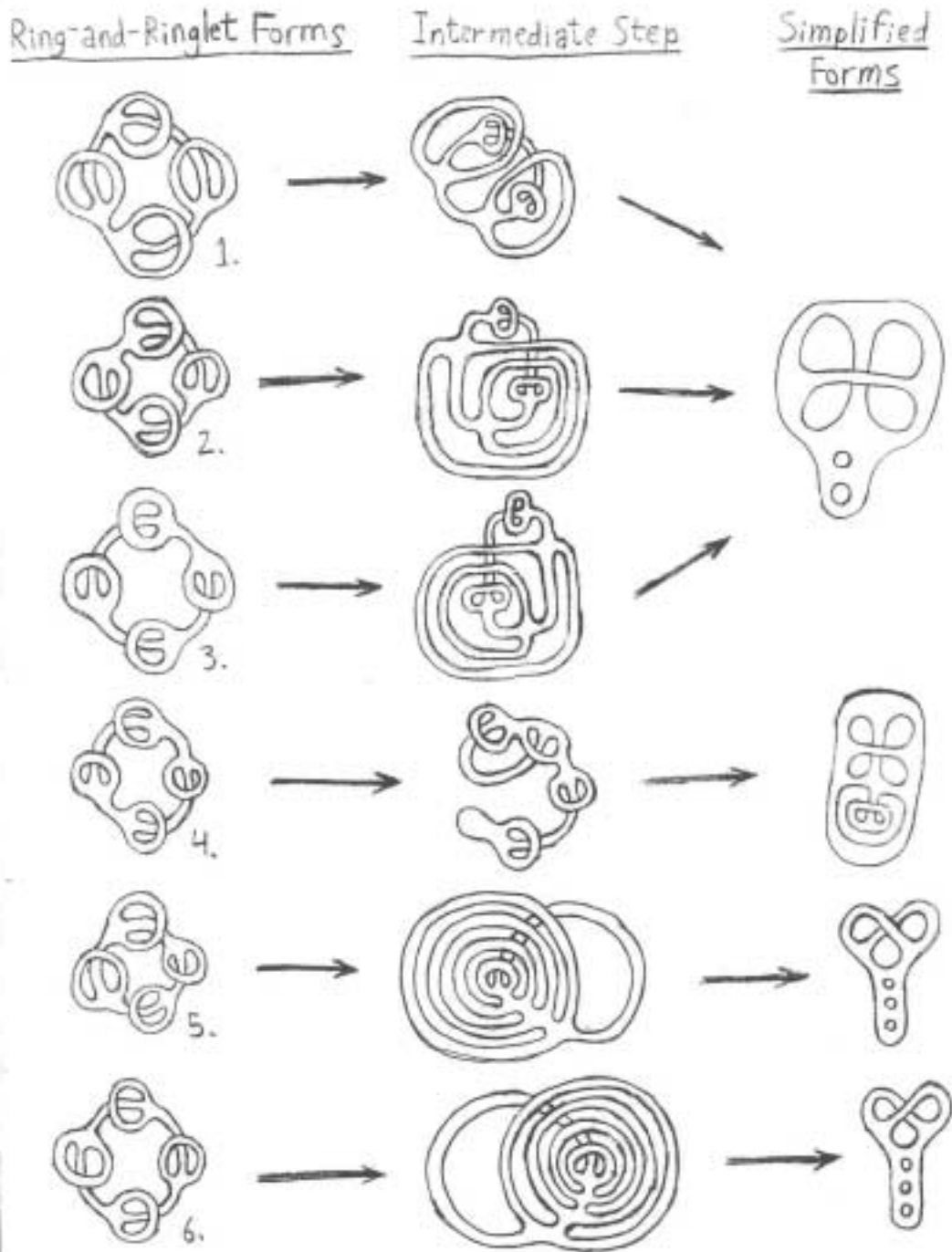


Figure 8: Simplification of the six ring-and-ringlets forms from figure 6. Detailed processes are not shown except for form #4 (see figure 9).



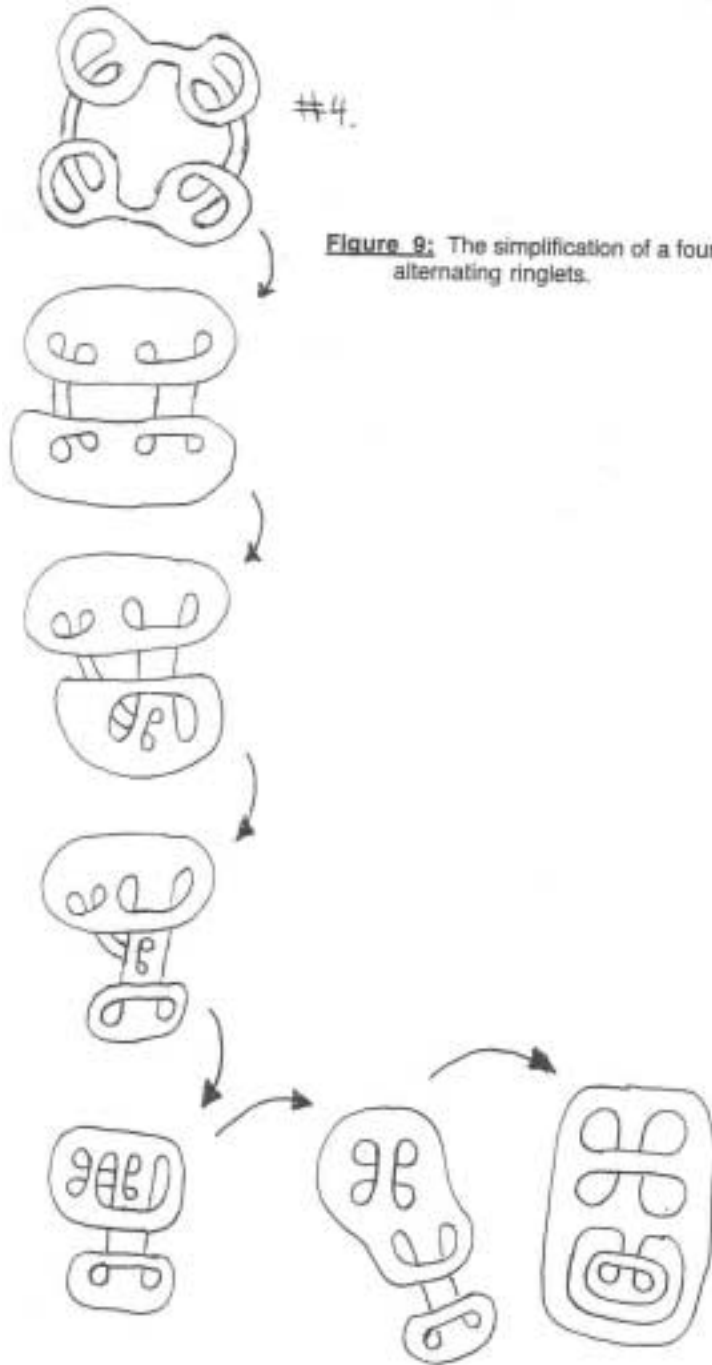


Figure 9: The simplification of a four-punctured torus type α flattening with alternating ringlets.

Figure 10: Complete list of simplified type α flattenings of tori with up to six punctures. Vertical columns contain similar forms with similar properties.

n:
Number of
Punctures
in TORUS

Type α Flattenings (Simplified Forms) —

1	
2	
3	
4	
5	
6	

Figure 11: Simplification of type α flattenings with alternating ringlets.

Ring-and-Ringlet Form

Simplified Form

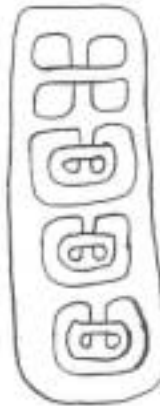
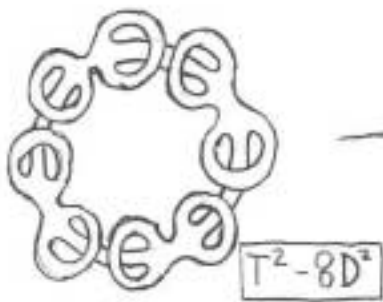
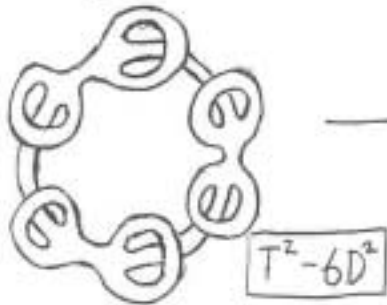
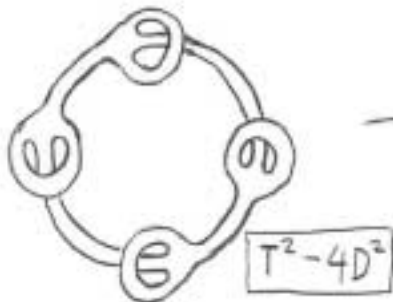
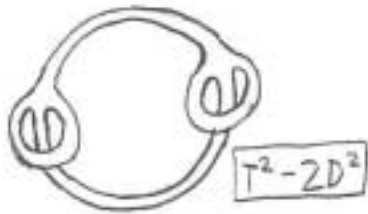
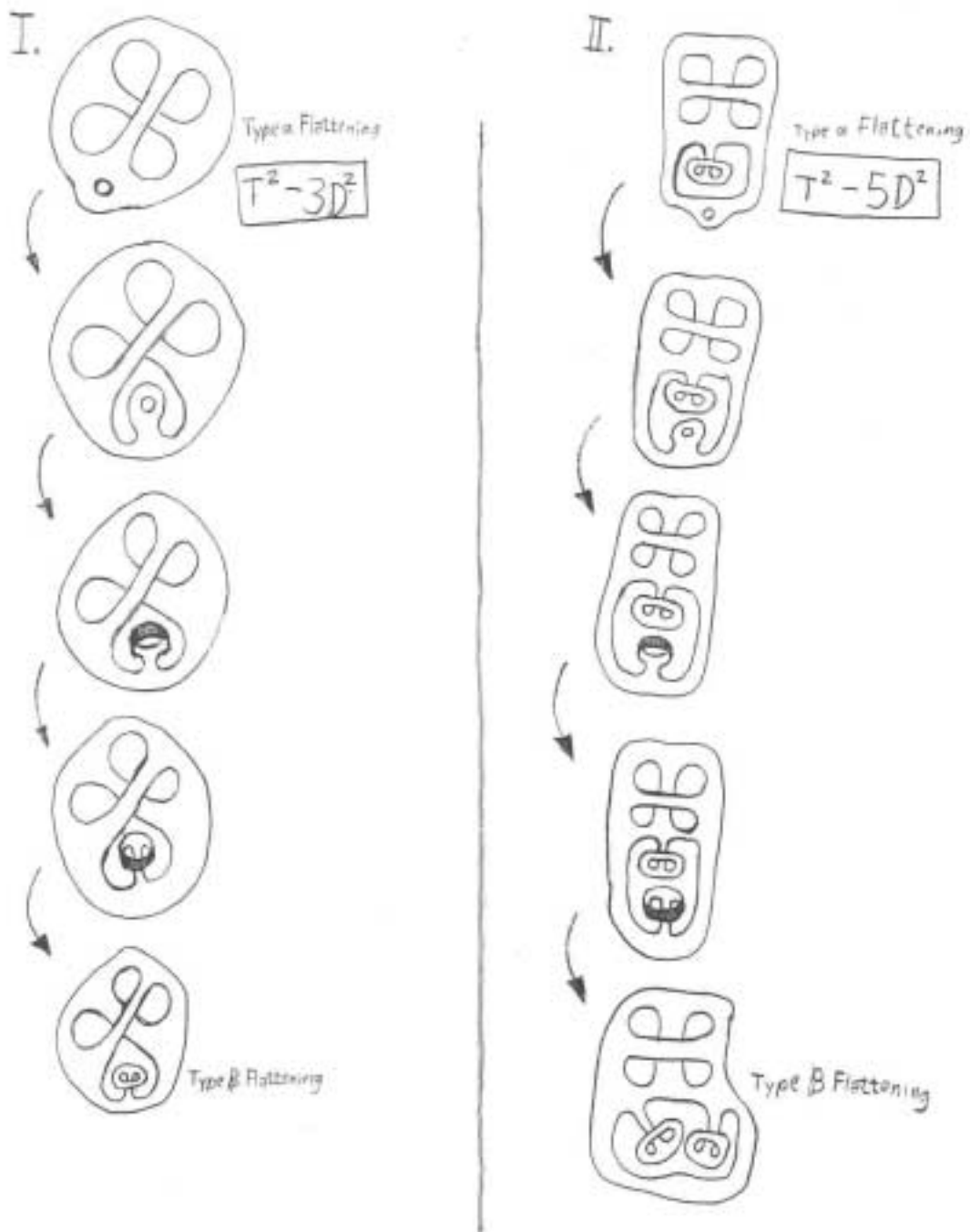


Figure 12: Two methods of deforming a type α flattening into a type β flattening through a non-flat deformation.



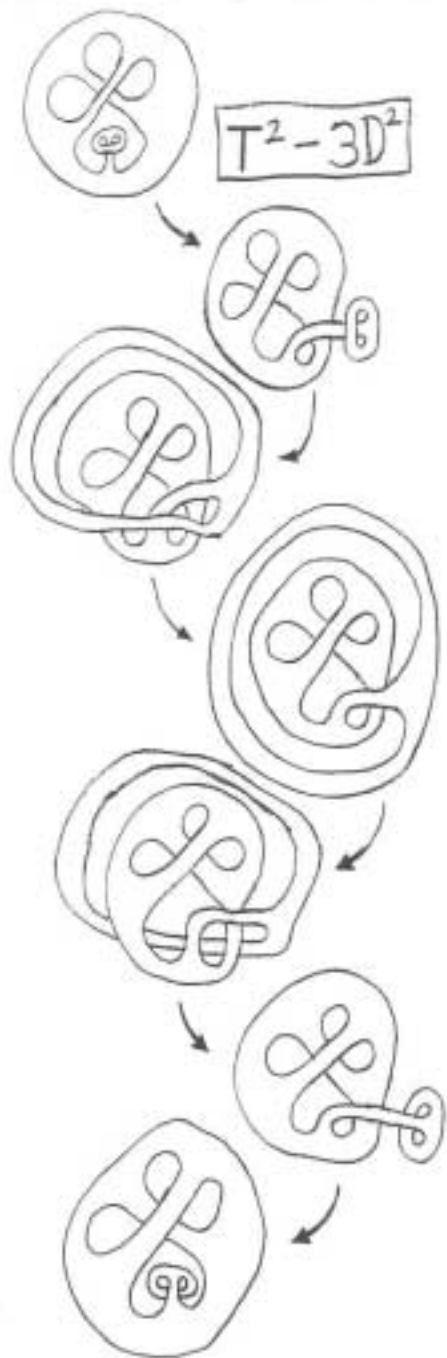

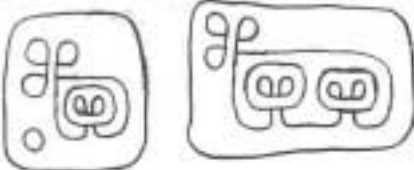
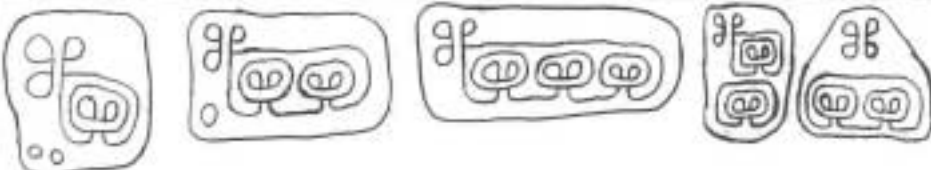


Figure 13: The reversal of the overlap of a mushroom through a flat deformation.



Figure 14: Proof that a type β flattening cannot be produced by making a mushroom on a type α flattening whose ring-and-ringlets form has all its ringlets pointing in the same direction.

Figure 15: Complete list of simplified type β flattenings of tori with up to six punctures. Unlike figure 9, this is not organized into columns.

n : Number of Punctures in Torus	Type β Flattenings (Simplified Forms)
1	None
2	None.
3	
4	
5	
6	