Rose-Hulman Institute of Technology – 22'nd Undergraduate Mathematics Conference (2005)
Student Speaker Abstracts; Listed according to Presentation Times

Friday, 1:35 – 1:55 p.m.
Modeling the Photochemical Transformation of Dissolved Organic Matter from an Antarctic Lake
Jennifer Guerard, Rose-Hulman Institute of Technology

Abstract: Transformation of dissolved organic matter (DOM) through photochemical processes may be an important factor controlling carbon availability to microorganisms in aquatic systems where carbon input is limited, such as in Antarctic lakes. When DOM undergoes photochemical turnover, corresponding changes can be observed in the absorption spectra, which are a measure of the amount of light a sample absorbs as a function of wavelength, in this case in the ultraviolet/visible regions. This technique can provide useful tools to probe the mechanisms by which DOM is transformed by sunlight as well as analyze the general chemical content of the DOM. Absorption spectra were collected during the course of the photolysis to determine how the spectral slope (S) of the absorption curve changes as a function of irradiation time. Since absorption spectra tend to follow an exponential curve, we determined S by fitting exponential models using nonlinear regression. Various models and data collection schemes were developed and applied in order to control for instrumental drift and other regression model violations in order to obtain more accurate estimates of S.

Friday, 2:00 – 2:20 p.m.
The Road Coloring Problem
Matthew Phillips, University of Evansville

Abstract: The Road Coloring Problem was proposed in the 1970s and has remained unsolved ever since. The problem calls for the determination of whether every aperiodic, strongly connected k-out digraph is synchronizable, and is associated with studies of graph theory, combinatorics, group theory, and automata theory. This talk will focus on the background information associated with the problem and a careful explanation of what the posed problem entails. The presentation will also include a discussion on the specific case when the digraph is an Eulerian graph.

Friday, 2:25 – 2:45 p.m.
The Knight's closed tour on a chessboard
Rozalia Tadjer, Goshen College

Abstract: In this presentation, I am going to talk about the knight's closed tour on a m x n board. A closed tour is a Hamilton circuit - each square of the board is visited only once and the knight's last move takes it back to the first square. There are some specific properties of the board, which ensure a closed tour, specified in the following theorem: a board supports a closed (knight's) tour if and only if its area mn is an even integer > 24 and neither m, nor n is 1, 2, or 4. I will explain the proof in one direction and show an example of the second direction.

Friday, 3:00 – 3:20 p.m.
The Feasibility of Noah's Ark
Tammy J. Johns, Siena Heights University

Abstract: This presentation models the mathematical feasibility of Noah's Ark. The ark will be addressed as well as the flood from both an evolutionist and creationist viewpoint.

Friday, 3:25 – 3:45 p.m.
Sequences, Ratios, and Multiplying Rabbits

Elizabeth Blankenship, Benedictine University
Abstract: The Fibonacci Sequence is one of the most interesting sequences because it can be found in many natural processes. What began as a simple problem in Liber Abaci has become more important than Leonardo of Pisa could have ever known. Even more interesting is the relationship between the Fibonacci Sequence and Golden Ratio - the most irrational number. I will be looking at both the Fibonacci Sequence and the Golden Ratio and demonstrating how the Golden Ratio can be obtained from the limit of the ratios of consecutive Fibonacci numbers.

Friday, 3:50 – 4:10 p.m.
Orthogonality in the Conjugate Gradient Method
Justin Droba, Rose-Hulman Institute of Technology

Abstract: I will examine how choosing conjugate orthogonal search directions in an attempt to generate an iterative method to solve a symmetric positive definite system leads to some very nice properties, especially the finite termination of the method. I will discuss some of the reasons why the method is useful (particularly parallelization), and walk though a couple examples.

Saturday, 10:10 – 10:30 a.m.
What's a quasi π-group?
Jesse Pratt, Northern Kentucky University

Abstract: In 1957, Abhyankar defined quasi p-groups. Ben Harwood in undergraduate research at Northern Kentucky University examined the elementary properties of quasi p-groups. He also posed the question "What does it mean for a group to be a quasi p-group for all primes p dividing its order?" We will examine that question and hint at how many such groups exist.

Saturday, 10:35 – 10:55 a.m.
Coxeter Groups and Finiteness Properties
Seth Case, University of Illinois

Abstract: Graph groups are commonly studied in combinatorial group theory. This talk will focus on introducing the audience to Coxeter groups. These groups were first studied by Coxeter as reflection groups in the plane. We will also address the notion of subgroup separability, and if time allows produce a beautifully simple result (due to Cooper, Long, and Reid) on the separability of subgroups fixed by automorphisms.

Saturday, 11:10 – 11:30 a.m.
Knot Knot K Equals K
Rachelle Barr, Siena Heights University

Abstract: A fundamental problem in knot theory is proving two distinct knot projections represent distinct knots. Additionally, it is difficult to see that two projections of the same knot are in fact equivalent. To solve this problem an equivalence invariant must be discovered describing knot properties that are the same under different projections chosen of the same knot. Unfortunately, calculating several of these invariants can be quite a challenging task and more often than not the invariant does knot classify by equivalence. This talk will cover several invariants and how they treat equivalent different projections of the same knot.

Saturday, 11:10 – 11:30 a.m.
The Properties of the Graph of Zero Divisors of Z \ pqr, where pqr are distinct primes
Peter Fine, Rose-Hulman Institute of Technology

Saturday, 11:35 – 11:55 a.m.
Black Box Linear Algebra: The Search for an Efficient Rank Preconditioner
Austin Somers, Devin Chalmers, Wabash College

Abstract: Black box linear algebra treats matrices as "black boxes," relying on external actions (the matrix-vector product) rather than explicit manipulation of the matrix itself for analysis. The properties of black box matrices may be made apparent through the use of "preconditioners," devised matrices which, in action on black boxes, result in new matrices related to the original black box in particular ways. Preconditioning is a generally probabilistic method, a "Monte Carlo" algorithm, sacrificing surety for ease of computation. One useful type of preconditioner is a "rank preconditioner," which makes apparent the rank of the black box matrix it is applied to. The computationally expensive Toeplitz matrix is a rank preconditioner.

Another effective method of preconditioning is to aggregate small "switches" into a larger matrix, though this is fairly expensive as well. Our 8-week research with Dr. William Turner was directed toward devising and testing different candidate rank preconditioners, to seek out computationally "cheaper" matrices. We employed both theoretic tools of analysis and concrete experimentation on candidate matrices, with the help of the computer algebra system Mathematica. Several promising and fast matrices were devised and tested over the course of our internship, though none proved to reliably precondition black boxes in the desired fashion. We will describe the various matrices we devised, demonstrate their computational efficiency, and show how they fail to precondition with the same amount of success as Toeplitz or switching matrices.