You are invited to attend and refreshments will be available.

• R H I T •

M A T H E M A T I C S   C O L L O Q U I U M

Wednesday, October 4th, 2006

Room G-317 Crapo Hall

9th period

(3:25 - 4:15 p.m.)

David Finn

Rose Hulman Math Professor

will present

Concavity, Maxima and Differential Equations

Abstract: In single variable calculus to show that there is a maximum of a function $f(x)$, we want to find a point x where the derivative $f'(x)$ is equal to zero and the second derivative $f''(x)$ is negative. Geometrically, we want the graph $y = f(x)$ to have a point with a horizontal tangent line and is concave down at that point. This can be used to show when solving the boundary value problem $y'' = g(y)$, $y(0) = y(1) = 0$ that there will a unique maximum in the interval $0 < y < 1$ provided g is a continuous function and $g(y) < 0$.

How do these ideas extend to function of more than one variable? What do we mean to say the graph of a function is concave down? What conditions imply that there is a unique maximum? Is concavity the correct condition to request for a unique maximum for a unique maximum to a partial differential equation? One goal of this talk is to provide an example of how simple ideas in single variable calculus do not extend nicely to multi-variable calculus and different ideas are required. The other goal is to provide simple geometric conditions on the graph of a function $u(x, y)$ that satisfies $u = 0$ on some curve closed C to ensure that there is a local unique maximum inside the curve C and then apply this condition to solutions to some partial differential equations. This talk will emphasize the geometrical aspects of concavity and maxima, but there will analysis for those that desire analysis.