Rotating Axes - Velocity

GPM Form

$$\vec{V}_B = \vec{V}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

Rotating Axis Form

$$\vec{V}_P = \vec{V}_O + \vec{V}_{rel} + \vec{\omega} \times \vec{r}_{P/O}$$

$$\vec{V}_{P/OXY} = \vec{V}_{O_{xyz}/OXY} + \vec{V}_{P_{rel}/O_{xyz}} + \vec{\omega}_{O_{xyz}/OXY} \times \vec{r}_{P/O_{xyz}}$$

P + P / Stationary Origin

Moving Origin / Stationary Origin

P + P / Moving origin

Moving origin / Stationary origin

P + P / Moving origin

$$\vec{v}_P = \vec{v}_o + \vec{v}_{rel} + \vec{\omega} \times \vec{r}_{P/o}$$

where

$\vec{v}_o$ = velocity of the origin of the rotating frame, oxyz

$\vec{\omega}$ = angular velocity of the rotating frame measured in OXYZ

$\vec{r}_{P/o}$ = position vector from point o to point P

$\vec{v}_{rel}$ = relative velocity of point P with respect to point o

Rotating Axes - Acceleration

GPM Form

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$$

Rotating Axis Form

$$\vec{a}_P = \vec{a}_o + \vec{a}_{rel} + \vec{\alpha} \times \vec{r}_{P/o} - \omega^2 \vec{r}_{P/o} + 2\vec{\omega} \times \vec{v}_{rel}$$

$$\vec{a}_{P/OXY} = \vec{a}_{oxy/OXY} + \vec{a}_{rel\,P/oxy} + \vec{\alpha}_{oxy/OXY} \times \vec{r}_{P/oxy} - \omega^2_{oxy/OXY}\,\vec{r}_{P/oxy} + 2\vec{\omega}_{oxy/OXY} \times \vec{v}_{rel\,P/oxy}$$

Pt P / Stationary Origin; Moving Origin / Stationary Origin; Pt P / Moving Origin; Moving Origin / Stationary Origin, Pt P / Moving Origin; Moving Origin / Stationary Origin, Pt P / Moving Origin; Moving Origin / Stationary Origin; Pt P / Moving Origin

The acceleration of point P is

$$\vec{a}_P = \vec{a}_o + \vec{a}_{rel} + \vec{\alpha} \times \vec{r}_{P/o} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/o}) + 2\vec{\omega} \times \vec{v}_{rel}$$

where

$\vec{a}_o$ = acceleration of the origin of the rotating frame, oxyz

$\vec{a}_{rel}$ = relative acceleration of point P with respect to point o as measured by an observer attached to the rotating frame

$\vec{\alpha}$ = angular acceleration of the rotating frame measured in OXYZ

$2\vec{\omega} \times \vec{v}_{rel}$ = Coriolis acceleration

$\vec{\alpha} \times \vec{r}_{P/o} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/o})$ = coincident acceleration

For plane motion Eq. 4.16 reduces to

$$\vec{a}_P = \vec{a}_o + \vec{a}_{rel} + \vec{\alpha} \times \vec{r}_{P/o} - \omega^2 \vec{r}_{P/o} + 2\vec{\omega} \times \vec{v}_{rel}$$