Panel 1

# ES204 Mechanical Systems

## Rotating Axes Practice - Angled Axes
## Lecture 29

Dr. Fisher

1

Panel 2

1–1

**Kinematics**
(the geometry of the motion)

Relative motion notation

$\vec{r}_A = \vec{r}_B + \vec{r}_{A/B}$

$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$

$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$

Basic kinematic relationships

$\vec{v} = \frac{d\vec{r}}{dt}$ $\quad \vec{\omega} = \frac{d\vec{\theta}}{dt}$

$\vec{a} = \frac{d\vec{v}}{dt}$ $\quad \vec{\alpha} = \frac{d\vec{\omega}}{dt}$

Dependent motion
1) Define position vectors
2) Write a constraint equation (length of cable, for example)
3) Differentiate to get an equation that relates the velocities/accelerations

Separate variables and integrate

Represent in different coordinate systems

Define position vectors and differentiate

Acceleration not constant

a(t): use $a = \frac{dv}{dt}$

a(x): use $a = v\frac{dv}{dx}$

Use analogous expressions for α(θ), α(t)

Rectangular Coordinates

$\vec{v} = v_x\hat{i} + v_y\hat{j}$,

$\vec{a} = a_x\hat{i} + a_y\hat{j}$

Normal and Tangential Coordinates

$\vec{v} = v\hat{e}_t$,

$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$

Polar Coordinates

$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$

$\frac{d\vec{Q}}{dt}_{OXY} = \frac{d\vec{Q}}{dt}_{Oxy} + \vec{\omega}\times\vec{Q}$

(Q is any vector)

Constant acceleration

$x = x_0 + v_0 t + \frac{1}{2}at^2$

$v = v_0 + at$

$v^2 = v_0^2 + 2a(x - x_0)$

(there are analogous expressions for α,ω,θ and t)

Translation
(θ = constant)

$v_A = v_B$

$a_A = a_B$

Fixed Axis Rotation

$\vec{v}_p = \vec{\omega}\times\vec{r}_{p/o}$ (Magnitude is ωr, direction is perpendicular to r)

$\vec{a}_p = \vec{\alpha}\times\vec{r}_{p/o} + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}_{p/o}\right)$

= αr perpendicular to r and $\omega^2$r directed from p to the fixed point O

General Plane Motion

Projectile Motion

x-direction: $x = x_0 + v_{0_x}t$, $v_0 = v_{0_x}$

y-direction:

$y = y_0 + v_{0_y}t - \frac{1}{2}gt^2$, $v_y = v_{0_y} - gt$, $v_y^2 = v_{0_y}^2 - 2g(y - y_0)$

General Plane Motion (translation+rotation)

$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$

$= \vec{v}_B + \vec{\omega}\times\vec{r}_{A/B}$

$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$

$= \vec{a}_B + \vec{\alpha}\times\vec{r}_{A/B} + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}_{A/B}\right)$

$= \vec{a}_B + \vec{\alpha}\times\vec{r}_{A/B} - \omega_{AB}^2\left(\vec{r}_{A/B}\right)$

General Plane Motion (rotating axis)

$\vec{v}_p = \vec{v}_o + \vec{v}_{rel} + \vec{\omega}\times\vec{r}_{p/o}$

$\vec{a}_p = \vec{a}_o + \vec{a}_{rel} + \vec{\alpha}\times\vec{r}_{p/o} + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}_{p/o}\right) + 2\vec{\omega}\times\vec{v}_{rel}$

2

Panel 3

**Rotating Axis Velocity equation**

$$\vec{V}_P = \vec{V}_O + \vec{V}_{rel} + \vec{\omega} \times \vec{r}_{P/O}$$

**Each term is something with respect to (wrt) something else**

$$\vec{V}_{P/OXY} = \vec{V}_{O_{XY}/OXY} + \vec{V}_{P_{rel}/OXY} + \vec{\omega}_{OXY/OXY} \times \vec{r}_{P/OXY}$$

P+P / Stationary Origin  Moving Origin / Stationary Origin  P+P / Moving origin  Moving origin / Stationary origin  P+P / Moving Origin

3

Panel 4

**Rotating Axis Acceleration equation**

$$\vec{a}_P = \vec{a}_O + \vec{a}_{rel} + \vec{\alpha} \times \vec{r}_{P/O} - \omega^2 \vec{r}_{P/O} + 2\vec{\omega} \times \vec{V}_{rel}$$

**Each term is something with respect to (wrt) something else**

$$\vec{a}_{P/OXY} = \vec{a}_{O_{XY}/OXY} + \vec{a}_{rel_{P/OXY}} + \vec{\alpha}_{OXY/OXY} \times \vec{r}_{P/OXY} - \omega^2_{OXY/OXY} \vec{r}_{P/OXY} + 2\vec{\omega}_{OXY/OXY} \times \vec{V}_{rel_{P/OXY}}$$

P+P / Stationary Origin  Moving Origin / Stationary Origin  P+P / Moving Origin  Moving Origin / Stationary Origin  P+P / Moving origin  Moving Origin / Stationary Origin  P+P / Moving Origin

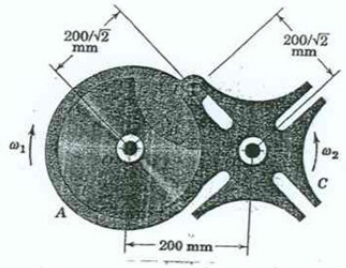
**Not any harder. Just more.**

4

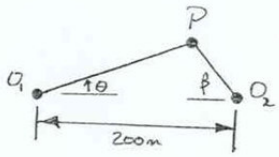
Panel 5

5/173 The Geneva wheel of Prob. 5/51 is shown again here. Determine the angular acceleration $\alpha_2$ of wheel $C$ for the instant when $\theta = 20°$. Wheel $A$ has a constant clockwise angular velocity of 2 rad/s.

*Ans.* $\alpha_2 = 16.53$ rad/s$^2$ CCW




Geometry




Picture is for $\theta = 45°$

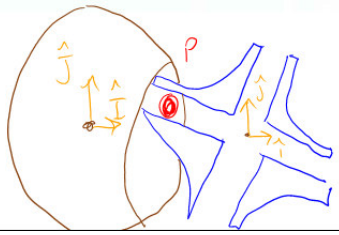
$\bar{r}_{P/O_1} = \frac{200}{\sqrt{2}}\cos 20\,\hat{i} + \frac{200}{\sqrt{2}}\sin 20\,\hat{j} = 132.9\,\hat{i} + 48.4\,\hat{j}$ mm

$\bar{r}_{P/O_2} = -\left(200 - \frac{200}{\sqrt{2}}\cos 20\right) + \frac{200}{\sqrt{2}}\sin 20\,\hat{j} = -67.1\,\hat{i} + 48.4\,\hat{j}$ mm

$\beta = \tan^{-1}\left(\frac{48.4}{67.1}\right) = 35.8°$



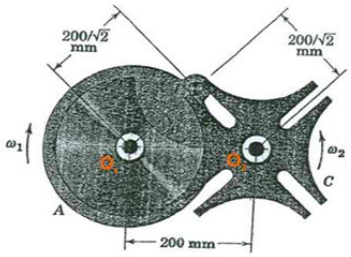





5

Panel 6

First we need to find the velocity and acceleration using GPM




$\bar{v}_P = \bar{v}_{O_1} + \bar{\omega}_A \times \bar{r}_{P/O_2}$

$\bar{v}_{O_1} =$

$\bar{\omega} =$

$\bar{r}_{P/O_1} = 132.9\,\hat{i} + 48.4\,\hat{j}$

$\bar{a}_P = \bar{a}_{O_1} + \bar{\alpha} \times \bar{r}_{P/O_1} - \omega^2 \bar{r}_{P/O_1}$

$\bar{a}_{O_2} =$

$\bar{\alpha} =$

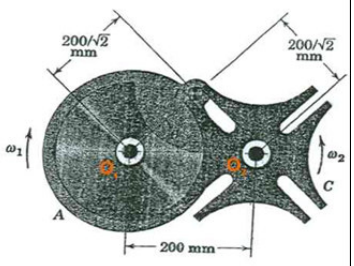
$\bar{r}_{P/O_1} = 132.9\,\hat{i} + 48.4\,\hat{j}$

$\omega =$

6

Panel 7

Use Rotating Axes to relate P to a moving origin at $O_2$




General Form equation for Velocity rotating axes

$\vec{V}_P = -266\hat{j} + 97\hat{i}$

$\vec{V}_{O_2} =$

$\vec{V}_{rel} =$

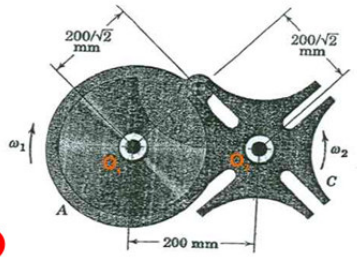
$\vec{\omega}_z =$

$\vec{r}_{P/O_2} = -67\hat{i} + 48\hat{j}$

7

Panel 8

Use Rotating Axes to relate P to a moving origin at $O_2$




General Form equation for Velocity rotating axes

$\vec{a}_{O_2} = -532\hat{i} - 193\hat{j}$

$\vec{a}_{rel} =$

$\alpha =$

$\vec{r}_{P/O_2} = -67.1\hat{i} + 48\hat{j}$

$\vec{\omega} = 1.92$

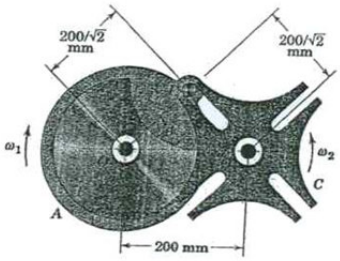
$\vec{V}_{rel} = 190\hat{i} - 137\hat{j}$

8

Panel 9

The Geneva wheel of Prob. 5/51 is shown again here. Determine the angular acceleration $\alpha_2$ of wheel $C$ for the instant when $\theta = 20°$. Wheel $A$ has a constant clockwise angular velocity of 2 rad/s.

*Ans.* $\alpha_2 = 16.53$ rad/s$^2$ CCW




**Find the velocity and acceleration of P using GPM from $O_1$**

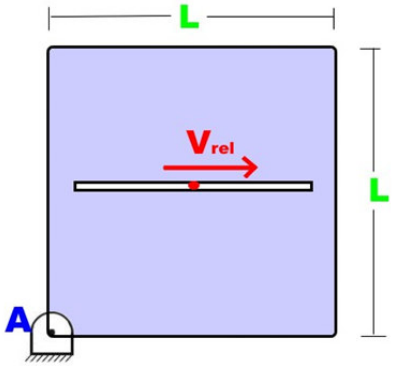
**Write the velocity Rotating Axes equations**
**Solve 2 eq/2 unk for $V_{rel}$ and $\omega_2$**

**Write the acceleration Rotating Axes equations**
**Solve 2 eq/2 unk for $a_{rel}$ and $\alpha$**

9

Panel 10

Assume that we have a square that has sides length L. In the middle of the block we have a particle that is moving at a constant rate of u relative to the block. The block is also rotating CCW at a constant value of omega.



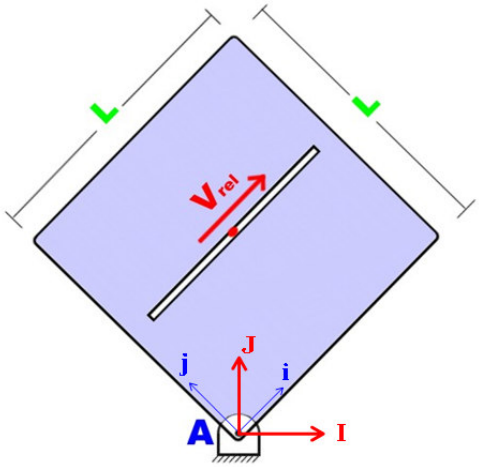

$$\vec{V}_P = \vec{V}_O + \vec{V}_{rel} + \vec{\omega} \times \vec{r}_{P/O}$$

10

Panel 11

If the block rotates 45 degrees we have a new value for V, but the first pass at the equations looks the same???

$$\vec{V}_P = \vec{V}_O + \vec{V}_{rel} + \vec{\omega} \times \vec{r}_{P/O}$$

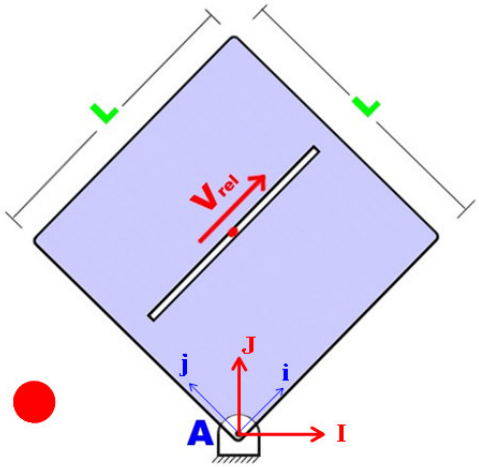



11

Panel 12

Relate i to I and J

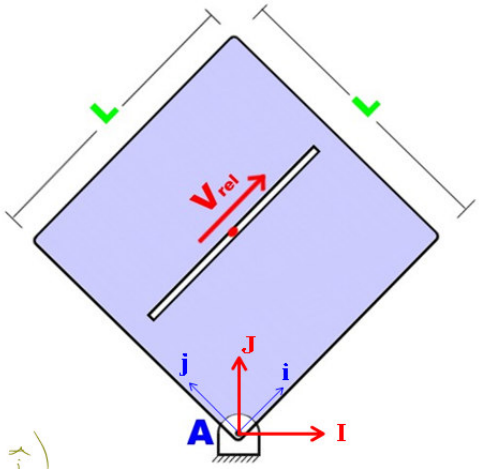
Relate j to I and J




12

Panel 13

If the block rotates 45 degrees we have a new value for V, but the first pass at the equations looks the same???




$$\vec{V}_P = \vec{V}_O + \vec{V}_{rel} + \vec{\omega} \times \vec{r}_{P/O}$$

$$\vec{V}_P = 0 + V_{rel}\,\hat{i} + \omega\,\hat{k} \times \left(\tfrac{L}{2}\,\hat{i} + \tfrac{L}{2}\,\hat{j}\right)$$

13

Panel 14

Solving for Vp you get...

$$\vec{V}_P = \frac{\sqrt{2}}{2}\left(V_{rel}\,\hat{I} + V_{rel}\,\hat{J} - \omega L\,\hat{I}\right)$$

$\hat{I}$ eq

$$V_{Px} = \frac{\sqrt{2}}{2}\left(V_{rel} - \omega L\right)$$

$\hat{J}$ eq

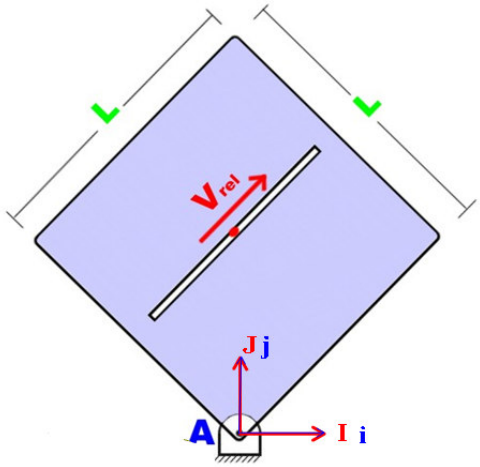
$$V_{Py} = \frac{\sqrt{2}}{2} V_{rel}$$

14

Panel 15

If you can it's usually easier to put the coordinates in the same direction. You should get the same answer and it's a little less messy.

$$\vec{V}_P = \vec{V}_O + \vec{V}_{rel} + \vec{\omega} \times \vec{r}_{P/O}$$



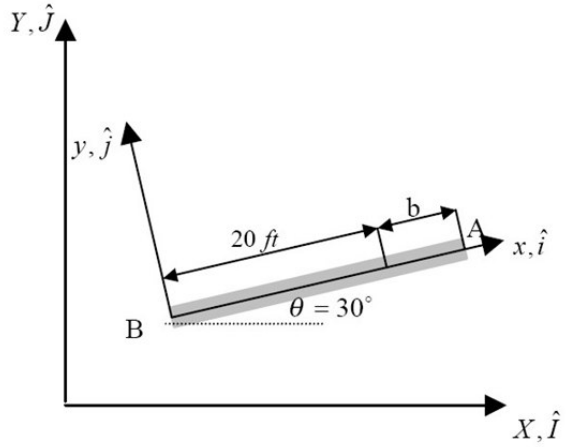

Before for Vp you got...

15

Panel 16

The fire truck is moving forward at a speed of 35 mi/hr (51.33 ft/sec) and is decelerating at the rate of 10 ft/sec$^2$. Simultaneously, the ladder is being raised and extended. At the instant considered, the angle is 30° and is increasing at the constant rate of 10 deg/sec. Also, at this instant the extension $b$ of the ladder is 5 ft, with an extension rate of 2 ft/sec and an extension acceleration of -1 ft/sec$^2$.

Find the velocity & acceleration of Pt A




At the instant shown :

$b = 5\,ft$

$\dot{b} = 2\,ft/s$

$\ddot{b} = -1\,ft/s^2$

$\omega = 10°/s = \frac{\pi}{18}\,rad/s$

$\alpha = 0$

16

Panel 17

The fire truck is moving forward at a speed of 35 mi/hr (51.33 ft/sec) and is decelerating at the rate of 10 ft/sec$^2$. Simultaneously, the ladder is being raised and extended. At the instant considered, the angle is 30° and is increasing at the constant rate of 10 deg/sec. Also, at this instant the extension $b$ of the ladder is 5 ft, with an extension rate of 2 ft/sec and an extension acceleration of -1 ft/sec$^2$.

General Form equation for Velocity rotating axes

Don't worry about the coordinate transformation yet just put in i, j, I, and J's

17

Panel 18

The fire truck is moving forward at a speed of 35 mi/hr (51.33 ft/sec) and is decelerating at the rate of 10 ft/sec$^2$. Simultaneously, the ladder is being raised and extended. At the instant considered, the angle is 30° and is increasing at the constant rate of 10 deg/sec. Also, at this instant the extension $b$ of the ladder is 5 ft, with an extension rate of 2 ft/sec and an extension acceleration of -1 ft/sec$^2$.

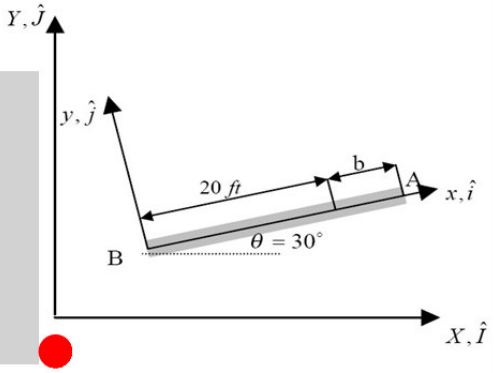
General Form equation for Velocity rotating axes

Don't worry about the coordinate transformation yet just put in i, j, I, and J's

18

Panel 19

The fire truck is moving forward at a speed of 35 mi/hr (51.33 ft/sec) and is decelerating at the rate of 10 ft/sec$^2$. Simultaneously, the ladder is being raised and extended. At the instant considered, the angle is 30° and is increasing at the constant rate of 10 deg/sec. Also, at this instant the extension $b$ of the ladder is 5 ft, with an extension rate of 2 ft/sec and an extension acceleration of -1 ft/sec$^2$.




Relate i to I and J

Relate j to I and J

19

Panel 20

The fire truck is moving forward at a speed of 35 mi/hr (51.33 ft/sec) and is decelerating at the rate of 10 ft/sec$^2$. Simultaneously, the ladder is being raised and extended. At the instant considered, the angle is 30° and is increasing at the constant rate of 10 deg/sec. Also, at this instant the extension $b$ of the ladder is 5 ft, with an extension rate of 2 ft/sec and an extension acceleration of -1 ft/sec$^2$.

General Form equation for Velocity rotating axes

Don't worry about the coordinate transformation yet just put in i, j, I, and J's

20

Panel 21

The fire truck is moving forward at a speed of 35 mi/hr (51.33 ft/sec) and is decelerating at the rate of 10 ft/sec$^2$. Simultaneously, the ladder is being raised and extended. At the instant considered, the angle is 30° and is increasing at the constant rate of 10 deg/sec. Also, at this instant the extension *b* of the ladder is 5 ft, with an extension rate of 2 ft/sec and an extension acceleration of -1 ft/sec$^2$.

**General Form equation for acceleration of rotating axes**

**Use the i and j information from earlier to solve for acceleration in I and J**

21

Panel 22

The fire truck is moving forward at a speed of 35 mi/hr (51.33 ft/sec) and is decelerating at the rate of 10 ft/sec$^2$. Simultaneously, the ladder is being raised and extended. At the instant considered, the angle is 30° and is increasing at the constant rate of 10 deg/sec. Also, at this instant the extension *b* of the ladder is 5 ft, with an extension rate of 2 ft/sec and an extension acceleration of -1 ft/sec$^2$.

$$\vec{a}_A = \vec{a}_B + \vec{a}_{rel} + \vec{\alpha} \times \vec{r}_{A/B} - \omega^2 \vec{r}_{A/B} + 2\vec{\omega} \times \vec{V}_{rel}$$

22