Panel 1

ES204 Mechanical Systems

Rotating Axes Acceleration
Lecture 28

Dr. Fisher

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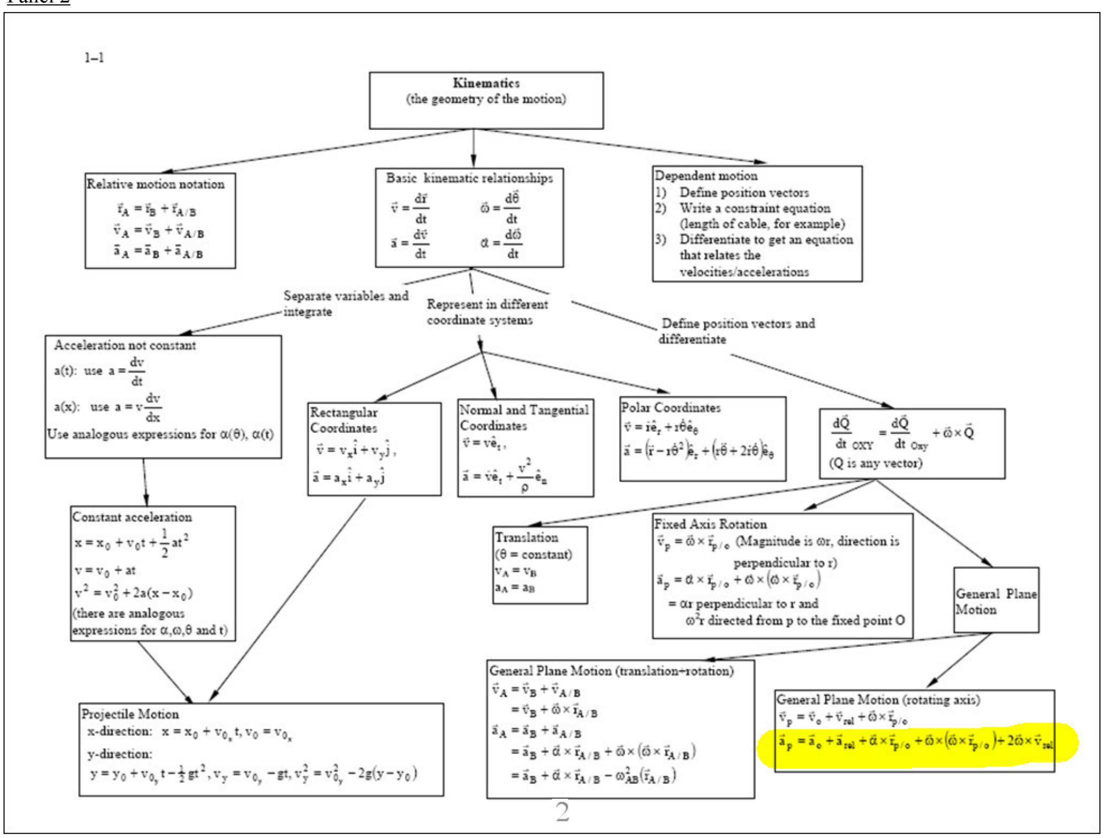
Panel 2



Panel 3

Rotating Axis Velocity equation

$$\vec{V}_P = \vec{V}_O + \vec{V}_{rel} + \vec{\omega} \times \vec{r}_{P/O}$$

Each term is something with respect to (wrt) something else

$$\vec{V}_{P/OXY} = \vec{V}_{oxy/OXY} + \vec{V}_{rel\,P/oxy} + \vec{\omega}_{oxy/OXY} \times \vec{r}_{P/oxy}$$

P+ P / Stationary Origin

Moving Origin / Stationary Origin

P+ P / Moving origin

Moving origin / Stationary origin

P+P / Moving origin

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Panel 4

Acceleration

$$\bar{a}_A = \frac{d}{dt}(\bar{V}_A) = \frac{d}{dt}\left(\bar{V}_B + \bar{V}_{rel} + \bar{\omega} \times \bar{r}_{A/B}\right)$$

$$= \bar{a}_B + \left(\bar{a}_{rel} + \bar{\omega} \times \bar{V}_{rel}\right) + \bar{\alpha} \times \bar{r}_{A/B} + \left(\bar{\omega} \times \underbrace{\bar{V}_{A/B}}_{\vec{V}_{rel} + \bar{\omega} \times \bar{r}_{A/B}}\right)$$

$\bar{a}_B$ = acceleration of the origin of the rotating frame

coincident acceleration $\begin{cases} \bar{\alpha} \times \bar{r} = \text{tangential acceleration due to rotating fra} \\ \bar{\omega} \times (\bar{\omega} \times \bar{r}) = \text{normal acceleration " " " "} \end{cases}$

$2\bar{\omega} \times \bar{V}_{rel}$ = Coriolis acceleration

$\bar{a}_{rel}$ = acceleration relative to rotating frame

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Panel 5

Rotating Axis Acceleration equation

$$\vec{a}_P = \vec{a}_o + \vec{a}_{rel} + \vec{\alpha} \times \vec{r}_{P/O} - \omega^2 \vec{r}_{P/O} + 2\vec{\omega} \times \vec{V}_{rel}$$

Each term is something with respect to (wrt) something else

Not any harder. Just more.

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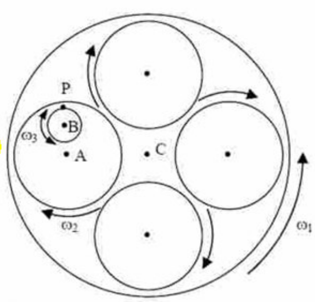
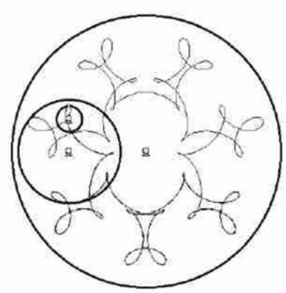
Panel 6

At Disney World there is a ride called "The Mad Hatter's Tea Party". The ride consists of a large spinning disk. Attached to this disk are additional spinning disks and attached to these disks are a number of tea cups which also spin (naturally). A photograph of the ride is shown to the right and a schematic diagram of a top view is shown below. I don't remember how many disks there were (or in what direction or how fast they rotated), so I am just guessing. I've only included one cup to keep the figure from becoming cluttered. The rider had the option of controlling the direction and speed of rotation of the cup. Personally, I hated this ride because it made me very dizzy, but unfortunately, my children loved it. Naturally, being a good father I took them on it, but while riding I couldn't help but think about the dynamics (as I'm sure you would). Let's assume all the angular velocities are constant and the large disk rotates at 0.2 rad/s (CCW), the medium disk rotates at 1.4 rad/s (CW) with respect to the large disk, and the cup rotates at 6 rad/s (CCW) with respect to the medium disk. Let's define the distance from A to C to be 12 feet, the distance from A to B to be 5 feet and the distance from B to P to be 2 feet. A snapshot of a Working Model simulation of these conditions is shown below so you can see the actual path of point P.

a) At the instant shown, determine the velocity and acceleration of point P.





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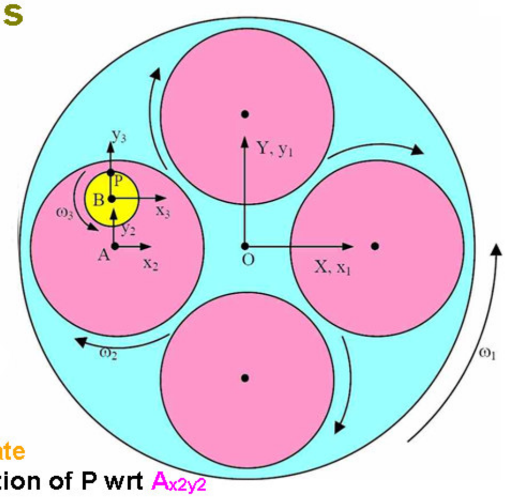
Panel 7

Using 4 coordinate systems

OXY = fixed to ground
Ox_1y_1 = fixed to large disk
Ax_2y_2 = fixed to the medium disk with its origin at A
Bx_3y_3 = fixed to the cup with its origin at B



Step #1 Think of A as stationary and relate
Acceleration of P wrt B_{x3y3} to Acceleration of P wrt A_{x2y2}

Step #2 Think of O as stationary and relate
Acceleration of P wrt A_{x2y2} to Acceleration of P wrt O_{x1y1}

Step #3 The ground is stationary so relate
Acceleration of P wrt O_{x1y1} to Acceleration of P wrt OXY

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Panel 8

Step #1 Think of A as stationary and relate
Acceleration of P wrt B_{x3y3} to Acceleration of P wrt A_{x2y2}

$$\vec{a}_P = \vec{a}_O + \vec{a}_{rel} + \vec{\alpha} \times \vec{r}_{P/O} - \omega^2 \vec{r}_{P/O} + 2\vec{\omega} \times \vec{V}_{rel}$$

$\vec{a}_P$ =

$\vec{a}_{rel_z}$

$\vec{a}_O$ =

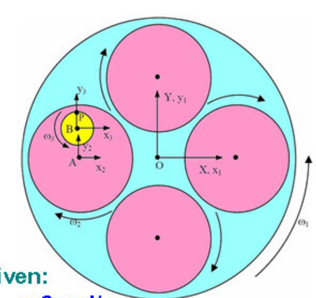
$\vec{a}_{rel}$ =

$\vec{\alpha}$ =

$\vec{r}_{P/O}$ =

ω =

$\vec{V}_{rel}$ =



Given:
ω_3 = 6 rad/s
P to B is 2 ft

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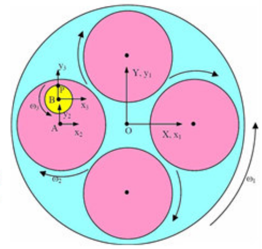
Panel 9

Step #2 Think of O as stationary and relate
Acceleration of P wrt A_{x2y2} to Acceleration of P wrt O_{x1y1}



$$\vec{a}_P = \vec{a}_o + \vec{a}_{rel} + \vec{\alpha} \times \vec{r}_{P/O} - \omega^2 \vec{r}_{P/O} + 2\vec{\omega} \times \vec{V}_{rel}$$

Knowns:
ω_2 = 1.4 rad/s CW   P to B is 2 ft   B to A is 5 ft

$\vec{a}_{rel_2} = -72\hat{j}$
Vrel = $-12\hat{i}$

Plug in values into equation. Don't solve

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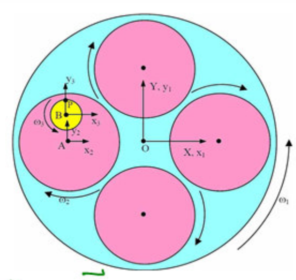
Panel 10

Step #3 The ground is stationary so relate
Acceleration of P wrt O_{x1y1} to Acceleration of P wrt OXY



Given:
ω_1 = 0.2 rad/s
P to B is 2 ft
B to A is 5 ft
A to O is 12

$\vec{a}_{rel_1} = -52.12\hat{j}$
$\vec{V}_{rel_1} = -2.2\hat{i}$

$$\vec{a}_P = \vec{a}_o + \vec{a}_{rel} + \vec{\alpha} \times \vec{r}_{P/O} - \omega^2 \vec{r}_{P/O} + 2\vec{\omega} \times \vec{V}_{rel}$$

Motion of P (defined in Ox_1y_1) w.r.t. OXY (that is, with respect to our fixed coordinate system).

$$\bar{a}_o = 0 \qquad \bar{\alpha} = 0$$
$$\bar{a}_{rel} = \bar{a}_{rel_1} = -52.12\hat{j} \qquad \bar{v}_{rel} = \bar{v}_{rel_1} = -2.2\hat{i}$$
$$\bar{\omega} = \omega_1\hat{k} = 0.2\hat{k} \qquad \bar{r} = -12\hat{i} + 7\hat{j}$$

Substituting into our acceleration equation we get

$$\begin{aligned}\bar{a}_p &= 0 + (-52.12\hat{j}) + 0 - (0.2)^2(-12\hat{i} + 7\hat{j}) + 2(0.2\hat{k}) \times (-2.2\hat{i}) \\ &= -52.12\hat{j} + 0.48\hat{i} - 0.28\hat{j} - 0.88\hat{j} \\ &= 0.48\hat{i} - 53.28\hat{j} \ \text{ft/s}\end{aligned}$$

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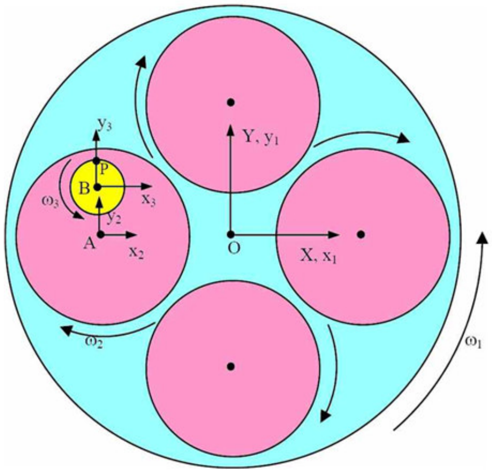
Panel 11

Note:
This problem is very simple because all of the $\hat{i}$ directions are the same and all of the $\hat{j}$ directions are the same. Rotating Axes problems become more challenging when the $\hat{i}$ directions are at funny angels.



$$\hat{i}_3 = \hat{i}_2 = \hat{i}_1 = \hat{I}$$

$$\hat{j}_3 = \hat{j}_2 = \hat{j}_1 = \hat{J}$$

The stationary coordinate system is often a capital letter

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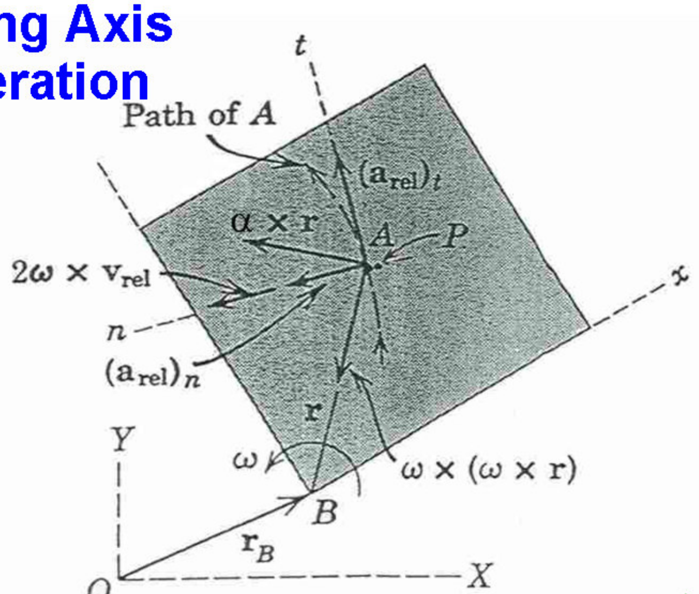
Panel 12

Rotating Axis Acceleration



$$\vec{a}_P = \vec{a}_o + \vec{a}_{rel} + \vec{\alpha} \times \vec{r}_{P/O} - \omega^2 \vec{r}_{P/O} + 2\vec{\omega} \times \vec{V}_{rel}$$

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Panel 13

15.126 In the automated welding setup shown, the position of the two welding tips G and H is controlled by the hydraulic cylinder D and rod BC. The cylinder is bolted to the vertical plate which at the instant shown rotates counterclockwise about A with a constant angular velocity of 1.6 rad/s. Knowing that at the same instant the length EF of the welding assembly is increasing at the constant rate of 300 mm/s, determine (a) the velocity of tip H, (b) the acceleration of tip H.

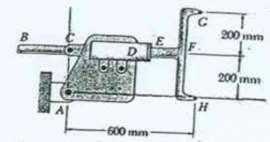

Fig. P15.126

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Panel 14

15.126 In the automated welding setup shown, the position of the two welding tips G and H is controlled by the hydraulic cylinder D and rod BC. The cylinder is bolted to the vertical plate which at the instant shown rotates counterclockwise about A with a constant angular velocity of 1.6 rad/s. Knowing that at the same instant the length EF of the welding assembly is increasing at the constant rate of 300 mm/s, determine (a) the velocity of tip H, (b) the acceleration of tip H.

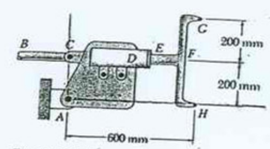

Fig. P15.126

Determine the velocity of H

What is the general form equation for velocity

Solve for $\vec{V}_H$

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Panel 15

15.126 In the automated welding setup shown, the position of the two welding tips G and H is controlled by the hydraulic cylinder D and rod BC. The cylinder is bolted to the vertical plate which at the instant shown rotates counterclockwise about A with a constant angular velocity of 1.6 rad/s. Knowing that at the same instant the length EF of the welding assembly is increasing at the constant rate of 300 mm/s, determine (*a*) the velocity of tip H, (*b*) the acceleration of tip H.

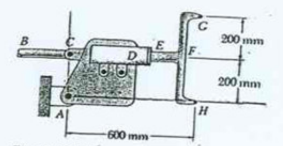

Fig. P15.126

Determine the acceleration of H

What is the general form equation for acceleration?

Solve for $\vec{a}_H$

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