Panel 1

ES204 Mechanical Systems

Rotating Axes Velocity
Lecture 25

Dr. Fisher

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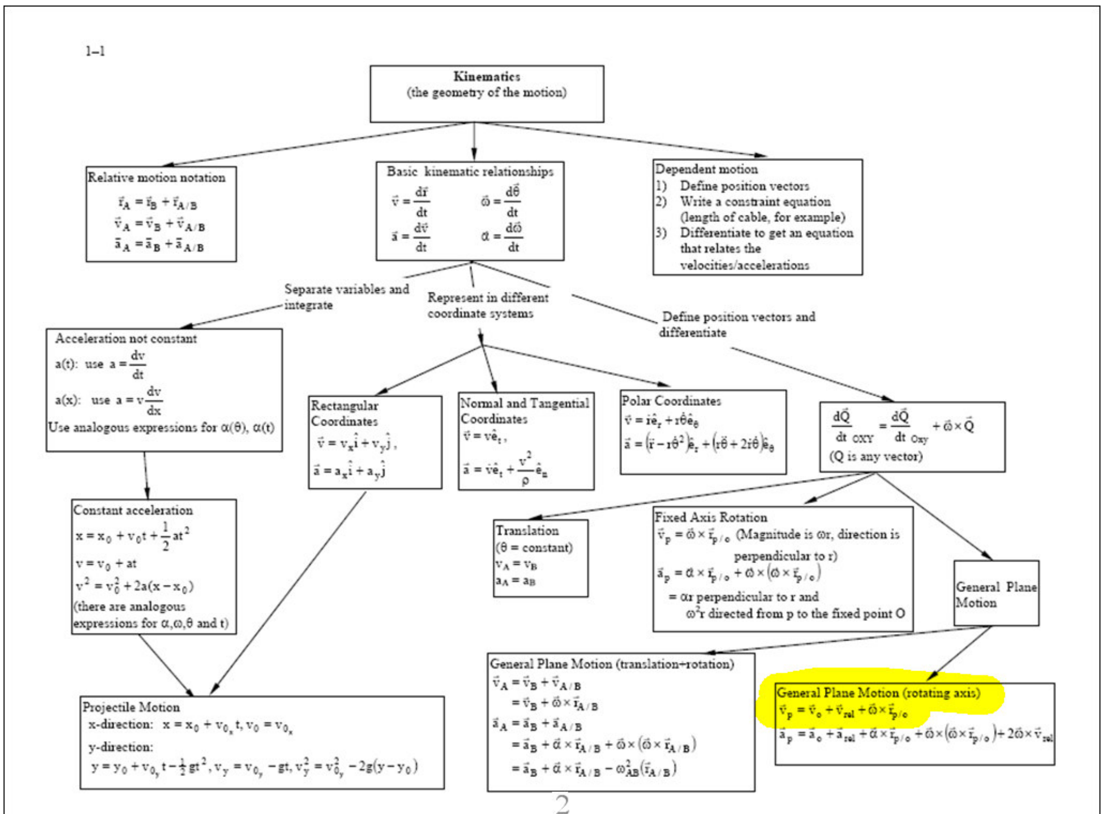
Panel 2



Panel 3

General Plane Motion Velocity equation

$$\vec{V}_B = \vec{V}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

Each term is something with respect to (wrt) something else

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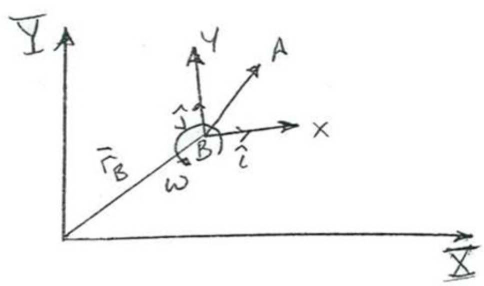
Panel 4

Using a rotating coordinate system (ie Rotating Axes)



x-y - rotating coordinate system

X-Y - fixed " "

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Panel 5

Rotating Axis Velocity equation

$$\vec{V}_P = \vec{V}_O + \vec{V}_{rel} + \vec{\omega} \times \vec{r}_{P/O}$$

Each term is something with respect to (wrt) something else

$$\vec{V}_{P/OXY} = \vec{V}_{oxy/OXY} + \vec{V}_{P_{rel}/oxy} + \vec{\omega}_{oxy/OXY} \times \vec{r}_{P/oxy}$$

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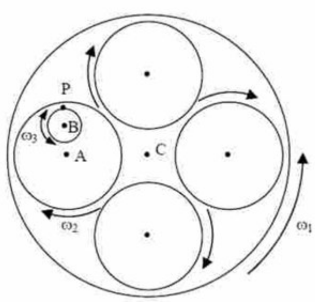
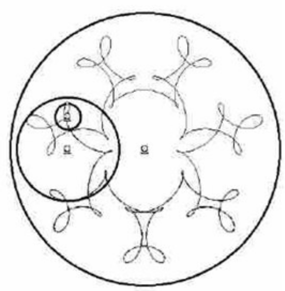
Panel 6

At Disney World there is a ride called "The Mad Hatter's Tea Party". The ride consists of a large spinning disk. Attached to this disk are additional spinning disks and attached to these disks are a number of tea cups which also spin (naturally). A photograph of the ride is shown to the right and a schematic diagram of a top view is shown below. I don't remember how many disks there were (or in what direction or how fast they rotated), so I am just guessing. I've only included one cup to keep the figure from becoming cluttered. The rider had the option of controlling the direction and speed of rotation of the cup. Personally, I hated this ride because it made me very dizzy, but unfortunately, my children loved it. Naturally, being a good father I took them on it, but while riding I couldn't help but think about the dynamics (as I'm sure you would). Let's assume all the angular velocities are constant and the large disk rotates at 0.2 rad/s (CCW), the medium disk rotates at 1.4 rad/s (CW) with respect to the large disk, and the cup rotates at 6 rad/s (CCW) with respect to the medium disk. Let's define the distance from A to C to be 12 feet, the distance from A to B to be 5 feet and the distance from B to P to be 2 feet. A snapshot of a Working Model simulation of these conditions is shown below so you can see the actual path of point P.





a) At the instant shown, determine the velocity and acceleration of point P.

b) The first time I went on this ride I let my kids control the speed and direction of the cup (hence my getting sick). The second time, however, I took the controls and adjusted the speed and direction of the cup so that it was in translation, that is, it was not rotating at all. I found the ride rather enjoyable but my kids said it was boring. What angular velocity did I choose for the cup?

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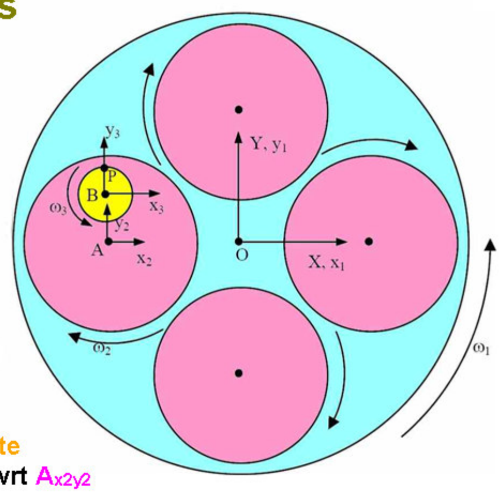
Panel 7

Using 4 coordinate systems

OXY = fixed to ground
Ox_1y_1 = fixed to large disk
Ax_2y_2 = fixed to the medium disk with its origin at A
Bx_3y_3 = fixed to the cup with its origin at B



$$\vec{v}_p = \vec{v}_o + \vec{v}_{rel} + \vec{\omega} \times \vec{r}_{p/o}$$

Step #1 Think of A as stationary and relate
Velocity of P wrt B_{x3y3} to Velocity of P wrt A_{x2y2}

Step #2 Think of O as stationary and relate
Velocity of P wrt A_{x2y2} to Velocity of P wrt O_{x1y1}

Step #3 The ground is stationary so relate
Velocity of P wrt O_{x1y1} to Velocity of P wrt OXY

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Panel 8

Step #1 Think of A as stationary and relate
Velocity of P wrt B_{x3y3} to Velocity of P wrt A_{x2y2}

Motion of P (defined in Bx_3y_3) w.r.t. Ax_2y_2 (that is, assuming Ax_2y_2 were fixed, what would the velocity of P be?) We will call this quantity $\vec{v}_{rel_2}$, although in some books you will see ${}^2\vec{v}^P$ where the superscript on the upper right refers to the point, and the one on the upper left, the coordinate system. For this part of the problem we have:

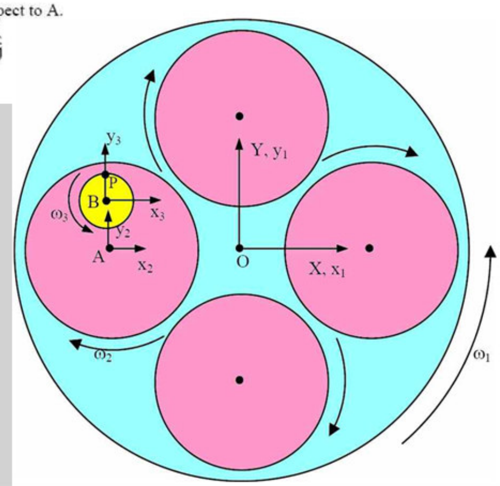
Given:
ω_3 = 6 rad/s
P to B is 2 ft

$$\vec{v}_o = 0$$
$$\vec{v}_{rel} = 0$$
$$\vec{\omega} = \omega_3\hat{k} = 6\hat{k}$$
$$\vec{r} = 2\hat{j}$$

Substituting into our velocity equation we get the relative velocity of P with respect to A.

$$\vec{v}_{rel_2} = 0 + 0 + (6\hat{k}) \times 2\hat{j}$$
$$= -12\hat{i}$$

Sounds lame but write the eq and solve problem



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Panel 9

Step #2 Think of O as stationary and relate

Velocity of P wrt A_{x2y2} to Velocity of P wrt O_{x1y1}

Motion of P (defined in Ax_2y_2) w.r.t. Ox_1y_1 (that is, assuming Ox_1y_1 were fixed, what would the velocity of P be?) The terms in our velocity equation are

Given:

- ω_2 = 1.4 rad/s
- P to B is 2 ft
- B to A is 5 ft

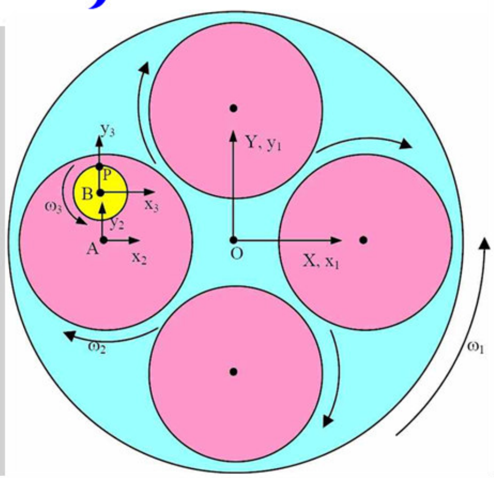
$$\vec{v}_o = 0$$

$$\vec{v}_{rel} = \vec{v}_{rel\,2} = -12\hat{i}$$

$$\vec{\omega} = \omega_2\hat{k} = -1.4\hat{k}$$

$$\vec{r} = 7\hat{j}$$

A little harder. All the vectors are given.
Write eq and subs in to find $Vrel_1$



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Panel 10

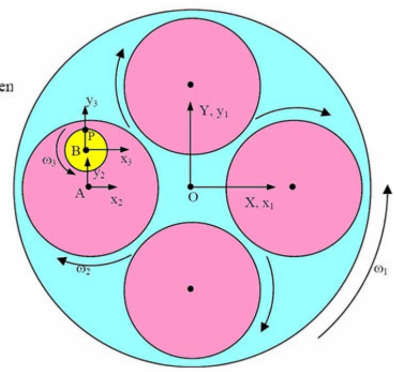
Step #3 The ground is stationary so relate

Velocity of P wrt O_{x1y1} to Velocity of P wrt OXY

Motion of P (defined in Ox_1y_1) w.r.t. OXY (that is, with respect to our fixed coordinate systen

Given:

- ω_1 = 0.2 rad/s
- P to B is 2 ft
- B to A is 5 ft
- A to O is 12



Write equation and put in values

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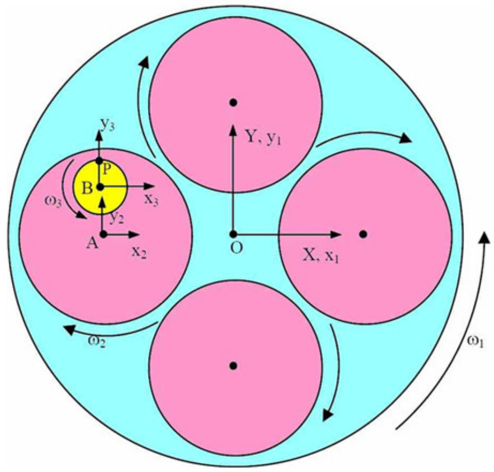
Panel 11

Note:
This problem is very simple because all of the $\hat{i}$ directions are the same and all of the $\hat{j}$ directions are the same. Rotating Axes problems become more challenging when the $\hat{i}$ directions are at funny angels.



$$\hat{i}_3 = \hat{i}_2 = \hat{i}_1 = \hat{I}$$

$$\hat{j}_3 = \hat{j}_2 = \hat{j}_1 = \hat{J}$$

The stationary coordinate system is often a capital letter

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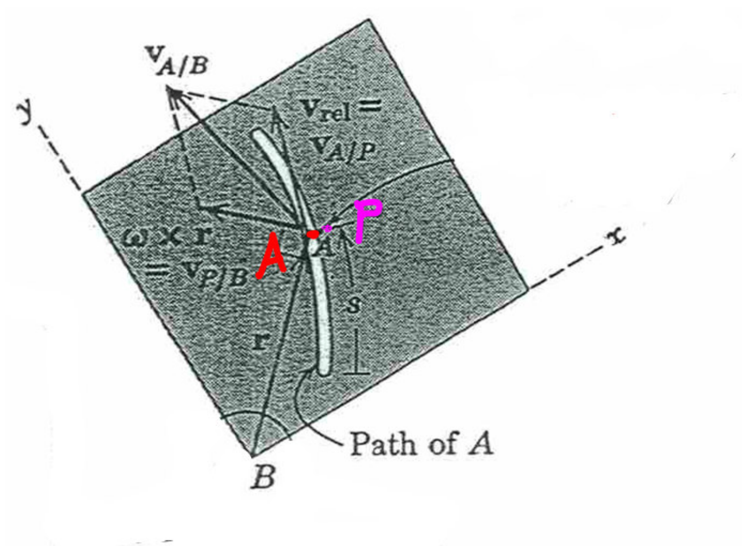
Panel 12

A graphical look at the rotating axis velocity equation



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Panel 13

Practice

Rotating Axis Velocity equation

$$\vec{V} = \vec{V} + \vec{V} + \vec{\omega} \times \vec{r}$$

Each term is something with respect to (wrt) something else

$$\vec{V}_{/} = \vec{V}_{/} + \vec{V}_{/} + \vec{\omega}_{/} \times \vec{r}_{/}$$

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