Panel 1

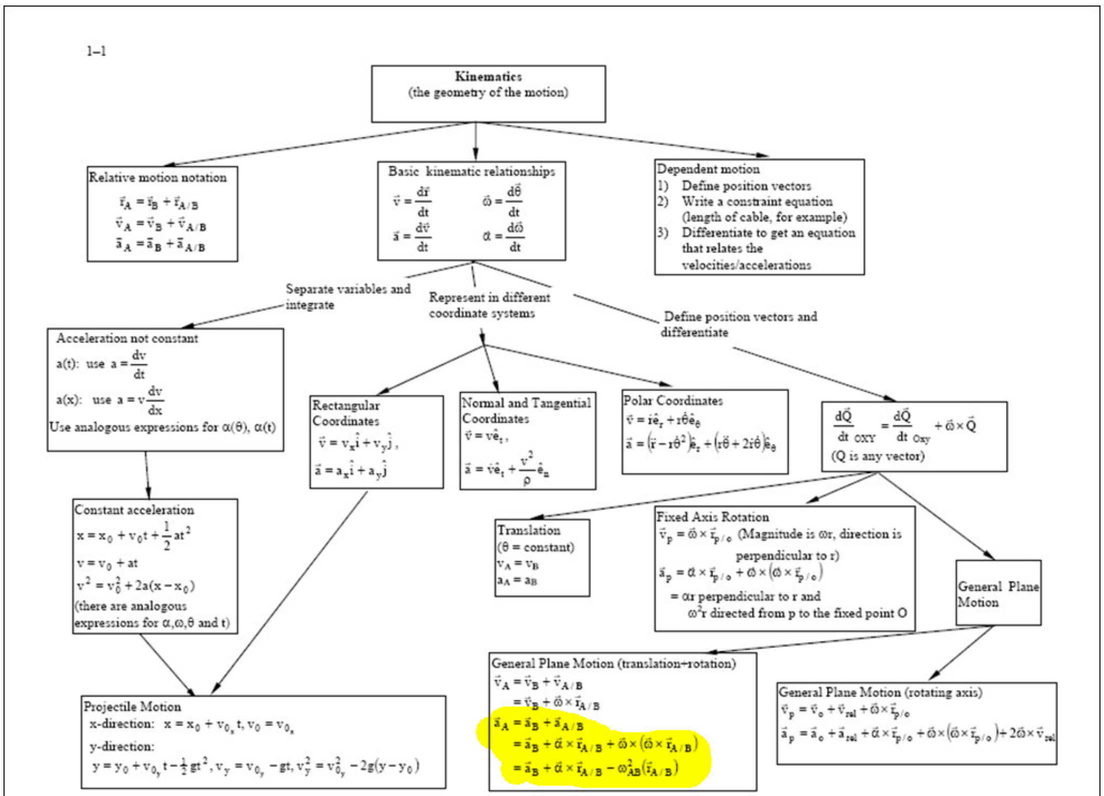
ES204 Mechanical Systems

General Plane Motion
Acceleration
Lecture 21

Dr. Fisher

1

Panel 2



2

Panel 3

Review velocity kinematics

- Instantaneous center of velocity (IC)

$v_A = \omega r_{A/IC}$ direction is perpendicular to r

- Vector algebra

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

  – Substitute in what you know
  – Take cross product
  – Equate components to get scalar equations

3

Panel 4

Equation to relate the velocity of two points on the same rigid body is:

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$
$$= \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

Equation to relate the acceleration of two points on the same rigid body

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

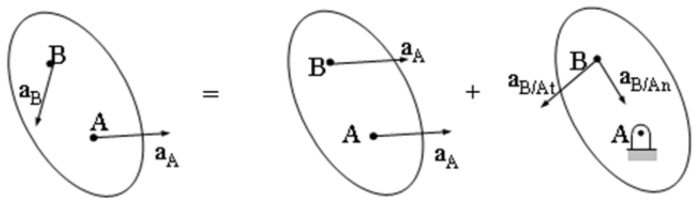
$$\vec{a}_B =$$

$$\vec{a}_B =$$

4

Panel 5

Accelerations



General Plane Motion = Translation + Fixed Axis Rotation

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})$$

Observations:

- $\vec{a}_{B/A}$ has a normal and a tangential component
- Tangential component = $\vec{\alpha} \times \vec{r}_{B/A}$
  - Magnitude = $\alpha r_{B/A}$
  - Direction = perpendicular to $r_{B/A}$
- Normal component = $\vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})$
  - Magnitude = $\omega^2 r_{B/A}$
  - Direction = towards point A
- You cannot use IC's for accelerations!!

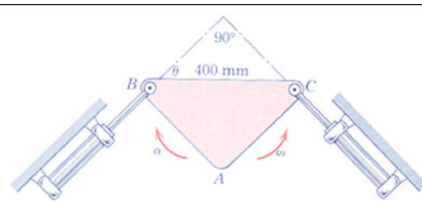
5

Panel 6

At the instant shown, θ=45 deg and the plate ABC has a counterclockwise angular velocity of 20 rad/s and a clockwise angular acceleration of 100 rad/s^2.
Determine:

a) the magnitude of the velocity of the piston attached to C,
b) the magnitude of the acceleration of the piston attached to C

(*taken from Dynamics, 3rd Edition by Merriam & Kraige*)



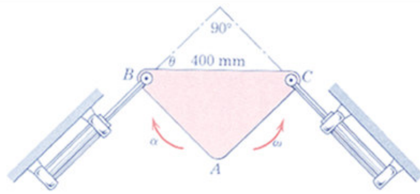
6

Panel 7

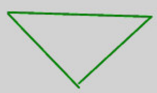
At the instant shown, θ=45 deg and the plate ABC has a counterclockwise angular velocity of 20 rad/s and a clockwise angular acceleration of 100 rad/s^2.
Determine:

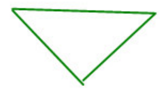
a) the magnitude of the velocity of the piston attached to C,
b) the magnitude of the acceleration of the piston attached to C

(*taken from Dynamics, 3rd Edition by Merriam & Kraige*)



Draw on velocity directions





Pick a coordinate system that puts $\vec{V}_b$ and $\vec{V}_c$ in the $\hat{i}$ and $\hat{j}$ directions, respectively

$\vec{V}_B =$

$\vec{V}_C =$

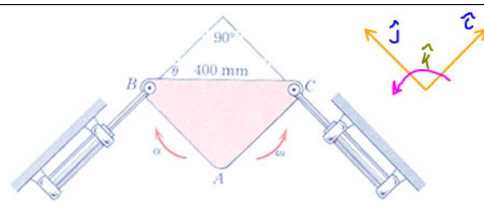
7

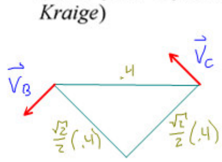
Panel 8

At the instant shown, θ=45 deg and the plate ABC has a counterclockwise angular velocity of 20 rad/s and a clockwise angular acceleration of 100 rad/s^2.
Determine:

a) the magnitude of the velocity of the piston attached to C,
b) the magnitude of the acceleration of the piston attached to C

(*taken from Dynamics, 3rd Edition by Merriam & Kraige*)





Solve using IC method

Solve using the vector approach

$\vec{V}_C = \vec{V}_B + \vec{\omega}_{BC} \times \vec{r}_{C/B}$

$\vec{V}_B =$

$\vec{V}_C =$

$\vec{\omega}_{BC} =$

$\vec{r}_{C/B} =$

$\hat{j}$

8

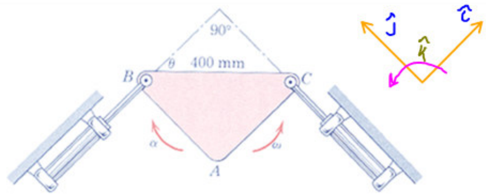
Panel 9

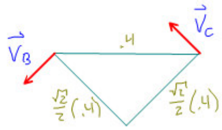
At the instant shown, θ=45 deg and the plate ABC has a counterclockwise angular velocity of 20 rad/s and a clockwise angular acceleration of 100 rad/s^2.

Determine:

a) the magnitude of the velocity of the piston attached to C,
b) the magnitude of the acceleration of the piston attached to C

(*taken from Dynamics, 3rd Edition by Merriam & Kraige*)





Solve using the vector approach

$\vec{V}_C = \vec{V}_B + \vec{\omega}_{BC} \times \vec{r}_{C/B}$

Solve using IC method

$V_C = \omega\, r_{C/IC}$

$V_C = 20(.2\sqrt{2})$

$V_C = 5.66$ m/s

9

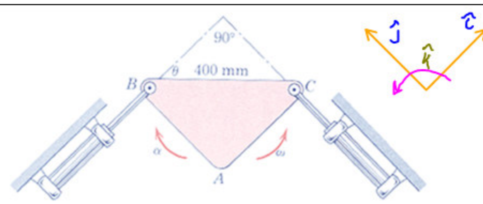
Panel 10

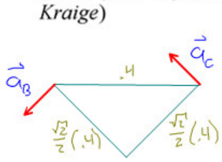
At the instant shown, θ=45 deg and the plate ABC has a counterclockwise angular velocity of 20 rad/s and a clockwise angular acceleration of 100 rad/s^2.

Determine:

a) the magnitude of the velocity of the piston attached to C,
b) the magnitude of the acceleration of the piston attached to C

(*taken from Dynamics, 3rd Edition by Merriam & Kraige*)





$\vec{\omega} = 20\,\hat{k}$

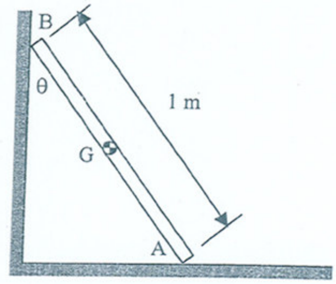
$\vec{r}_{C/B} = .2\sqrt{2}\,\hat{i} - .2\sqrt{2}\,\hat{j}$

Find the acceleration, $\vec{a}_c$

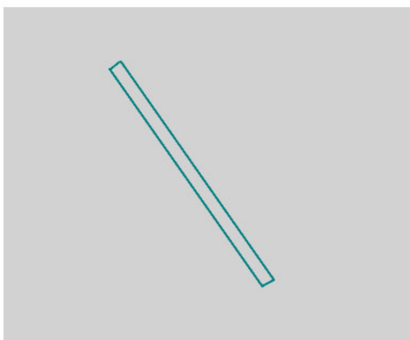
10

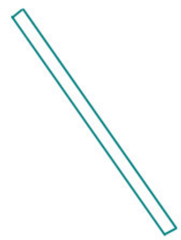
Panel 11

Point A has a constant velocity of 10 m/s to the right. Determine the velocity and acceleration of point B when θ = 30°. Hint: Don't you dare say the acceleration of point B is zero!



Draw on velocity and acceleration vectors

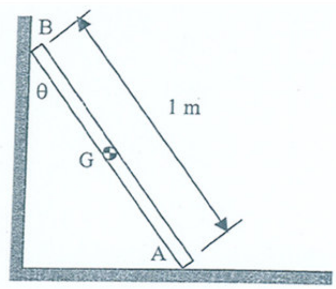


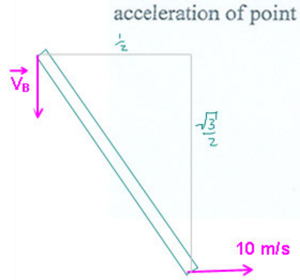


11

Panel 12

Point A has a constant velocity of 10 m/s to the right. Determine the velocity and acceleration of point B when θ = 30°. Hint: Don't you dare say the acceleration of point B is zero!





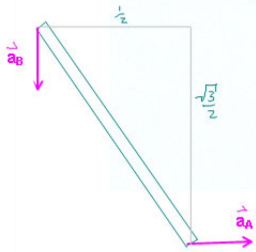
$$\vec{V}_B = \vec{V}_A + \vec{\omega} \times \vec{r}_{B/A}$$

Find the velocity of Vb using any method you like

12

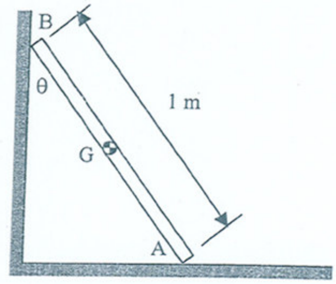
Panel 13

Point A has a constant velocity of 10 m/s to the right. Determine the velocity and acceleration of point B when θ = 30°. Hint: Don't you dare say the acceleration of point B is zero!



$\vec{\omega} = \frac{20}{\sqrt{3}}\hat{k}$

$\vec{r}_{B/G} = -\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$

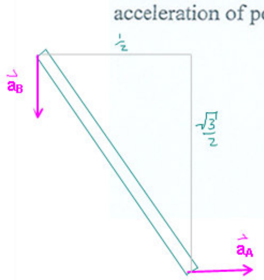


Find the acceleration of a_b

13

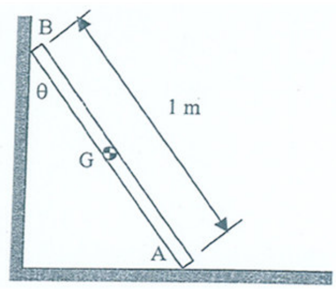
Panel 14

Point A has a constant velocity of 10 m/s to the right. Determine the velocity and acceleration of point B when θ = 30°. Hint: Don't you dare say the acceleration of point B is zero!



$\vec{\omega} = \frac{20}{\sqrt{3}}\hat{k}$

$\vec{r}_{B/G} = -\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$



Find the acceleration of a_b

$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$

$\hat{i}$

$\hat{j}$

14