Panel 1

ES204 Mechanical Systems

GPM - Impact
Lecture 19

Dr. Fisher

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Panel 2

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Kinematics
(the geometry of the motion)

Relative motion notation

$\vec{r}_A = \vec{r}_B + \vec{r}_{A/B}$

$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$

$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$

Basic kinematic relationships

$\vec{v} = \frac{d\vec{r}}{dt}$ $\quad \vec{\omega} = \frac{d\vec{\theta}}{dt}$

$\vec{a} = \frac{d\vec{v}}{dt}$ $\quad \vec{\alpha} = \frac{d\vec{\omega}}{dt}$

Dependent motion
1) Define position vectors
2) Write a constraint equation (length of cable, for example)
3) Differentiate to get an equation that relates the velocities/accelerations

Separate variables and integrate

Represent in different coordinate systems

Define position vectors and differentiate

Acceleration not constant

a(t): use $a = \frac{dv}{dt}$

a(x): use $a = v\frac{dv}{dx}$

Use analogous expressions for α(θ), α(t)

Rectangular Coordinates

$\vec{v} = v_x\hat{i} + v_y\hat{j}$,

$\vec{a} = a_x\hat{i} + a_y\hat{j}$

Normal and Tangential Coordinates

$\vec{v} = v\hat{e}_t$,

$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$

Polar Coordinates

$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$

$\frac{d\vec{Q}}{dt}_{OXY} = \frac{d\vec{Q}}{dt}_{Oxy} + \vec{\omega} \times \vec{Q}$
(Q is any vector)

Constant acceleration

$x = x_0 + v_0 t + \frac{1}{2}at^2$

$v = v_0 + at$

$v^2 = v_0^2 + 2a(x - x_0)$

(there are analogous expressions for α,ω,θ and t)

Translation
(θ = constant)

$v_A = v_B$

$a_A = a_B$

Fixed Axis Rotation

$\vec{v}_p = \vec{\omega} \times \vec{r}_{p/o}$ (Magnitude is ωr, direction is perpendicular to r)

$\vec{a}_p = \vec{\alpha} \times \vec{r}_{p/o} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{p/o}\right)$

= αr perpendicular to r and ω^2r directed from p to the fixed point O

General Plane Motion

Projectile Motion

x-direction: $x = x_0 + v_{0_x} t$, $v_0 = v_{0_x}$

y-direction:

$y = y_0 + v_{0_y} t - \frac{1}{2}gt^2$, $v_y = v_{0_y} - gt$, $v_y^2 = v_{0_y}^2 - 2g(y - y_0)$

General Plane Motion (translation+rotation)

$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$

$= \vec{v}_B + \vec{\omega} \times \vec{r}_{A/B}$

$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$

$= \vec{a}_B + \vec{\alpha} \times \vec{r}_{A/B} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{A/B}\right)$

$= \vec{a}_B + \vec{\alpha} \times \vec{r}_{A/B} - \omega_{AB}^2\left(\vec{r}_{A/B}\right)$

General Plane Motion (rotating axis)

$\vec{v}_p = \vec{v}_o + \vec{v}_{rel} + \vec{\omega} \times \vec{r}_{p/o}$

$\vec{a}_p = \vec{a}_o + \vec{a}_{rel} + \vec{\alpha} \times \vec{r}_{p/o} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{p/o}\right) + 2\vec{\omega} \times \vec{v}_{rel}$

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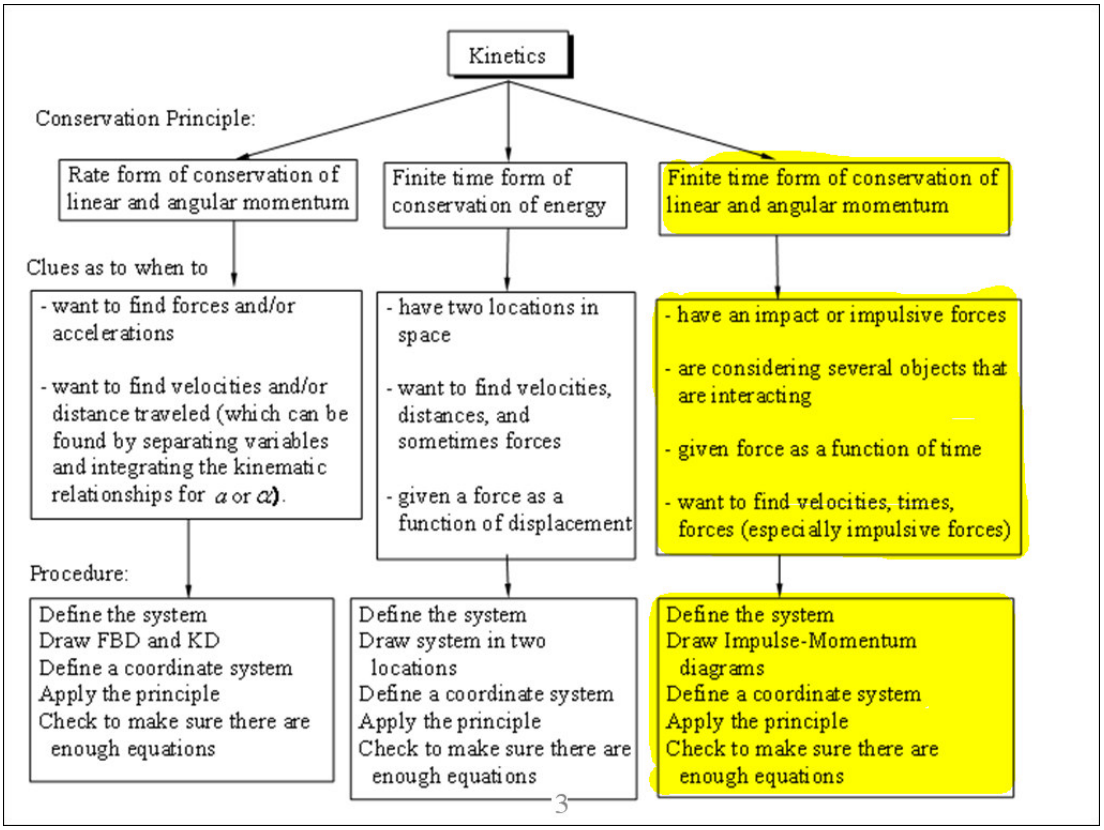
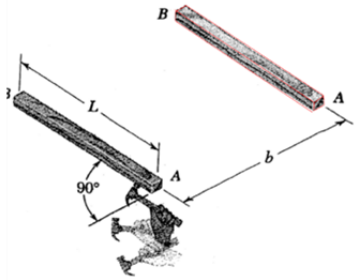
Panel 3



Panel 4



A uniform slender rod AB is at rest on a frictionless horizontal table when end A of the rod is struck by a hammer which delivers an impulse that is perpendicular to the rod. In the subsequent motion, determine the distance b through which the rod will move each time it completes a full revolution.

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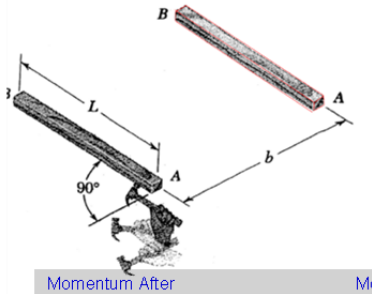
Panel 5



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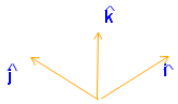
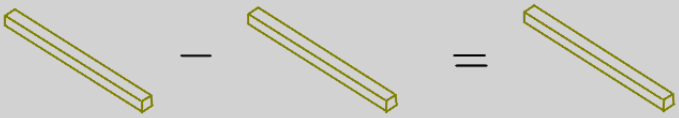


Momentum After Momentum Before Impulses During



CoLM and CoAM Finite equations

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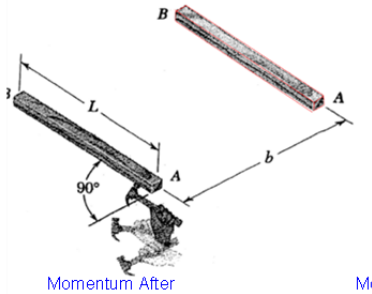
Panel 6



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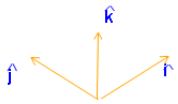

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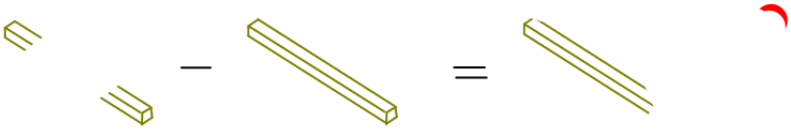

CoLM and CoAM Finite equations

$P_2 - P_1 = \Sigma F \Delta t$

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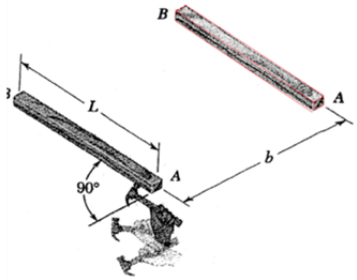
Panel 7



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$$\omega = \frac{6V}{L}$$

Find time needed to make a revolution

Find the distance travelled in that much time

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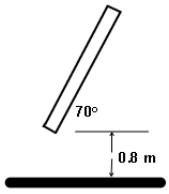
Panel 8



A slender bar 1.5 meters long and weighing 10 kg is dropped onto a horizontal surface as shown. Assume the friction between the bar and the ground is very small. If the bar does not bounce determine the angular velocity of the bar after impact.

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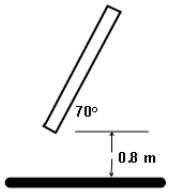
Panel 9



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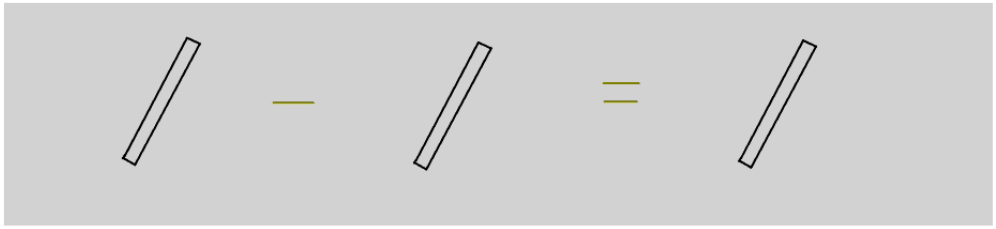
You'd need use energy to solve for velocity before impact first

$V_o = \sqrt{2gh} = 3.96$ m/s

Momentum After — Momentum Before — Impulses During



CoLM and CoAM Finite equations

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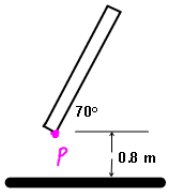
Panel 10



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Momentum After — Momentum Before — Impulses During

CoLM and CoAM Finite equations

CoLM $\uparrow$

$m V_x' - 0 = 0$

$V_x' = 0$

CoAM about P $\circlearrowleft +$

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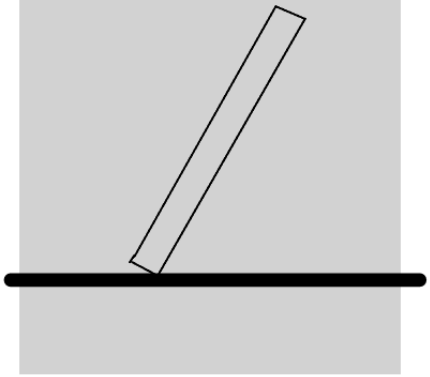
Panel 11

Need to relate ω and V_y'

Find IC (or use vector approach if you like)

Relate V_y' to ω



Recall from the problem:
It doesn't bounce
No friction, just slides out

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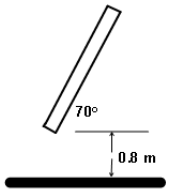
Panel 12



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Summary
- Impact setup same as always just motion is GPM
- Setup CoLM and CoAM equations as needed
- Reduce number of variables by relating ω's and V's

Eq	Unk
1	V_y'
2	ω

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