Panel 1

ES204 Mechanical Systems

General Plane Motion
Energy Examples
Lecture 16

Dr. Fisher

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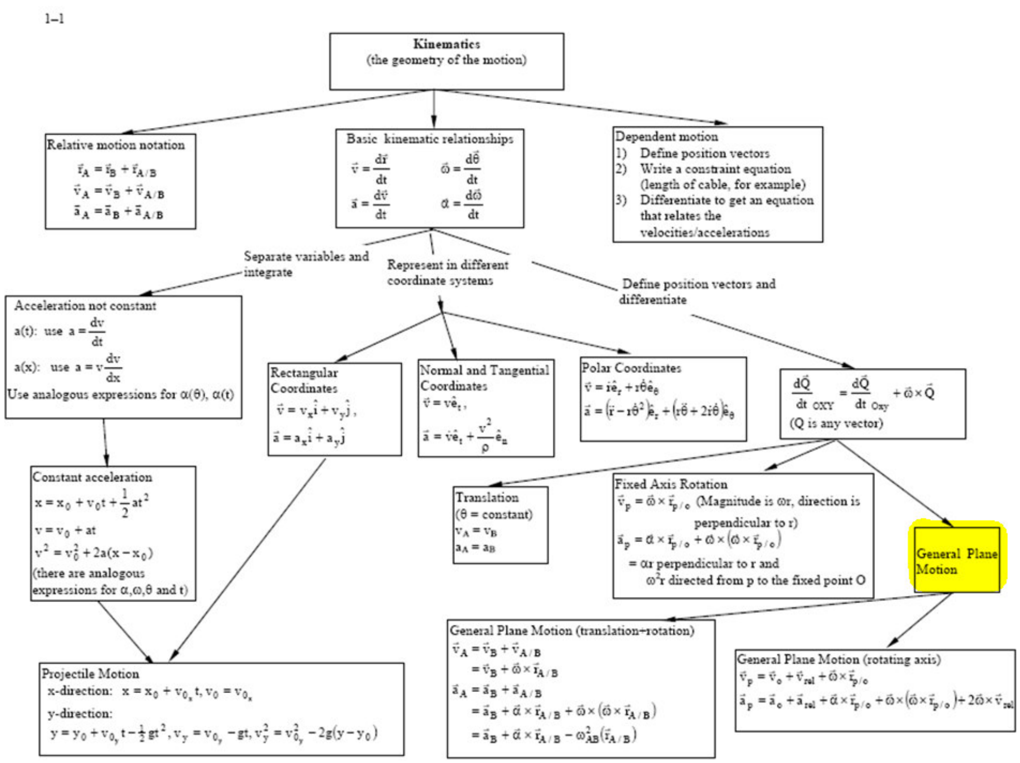
Panel 2



2

Panel 3

Three types of plane motion

Type of motion	How to find the velocity of a point on the rigid body
1. Translation	$\vec{V}_a = \vec{V}_b$ (All points have same velocity)
2. Fixed axis rotation	$\vec{V}_p = \vec{\omega} \times \vec{r}_{p/o}$ Velocity determined by omega and distance to the fixed pt of rotation
3. General plane motion	• Instantaneous center of velocity o Scalar approach o Need to know the directions of the velocities of two points • Vector algebra approach o Write position vectors o Use $\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$ and equate components o Get more equations by looking at another object

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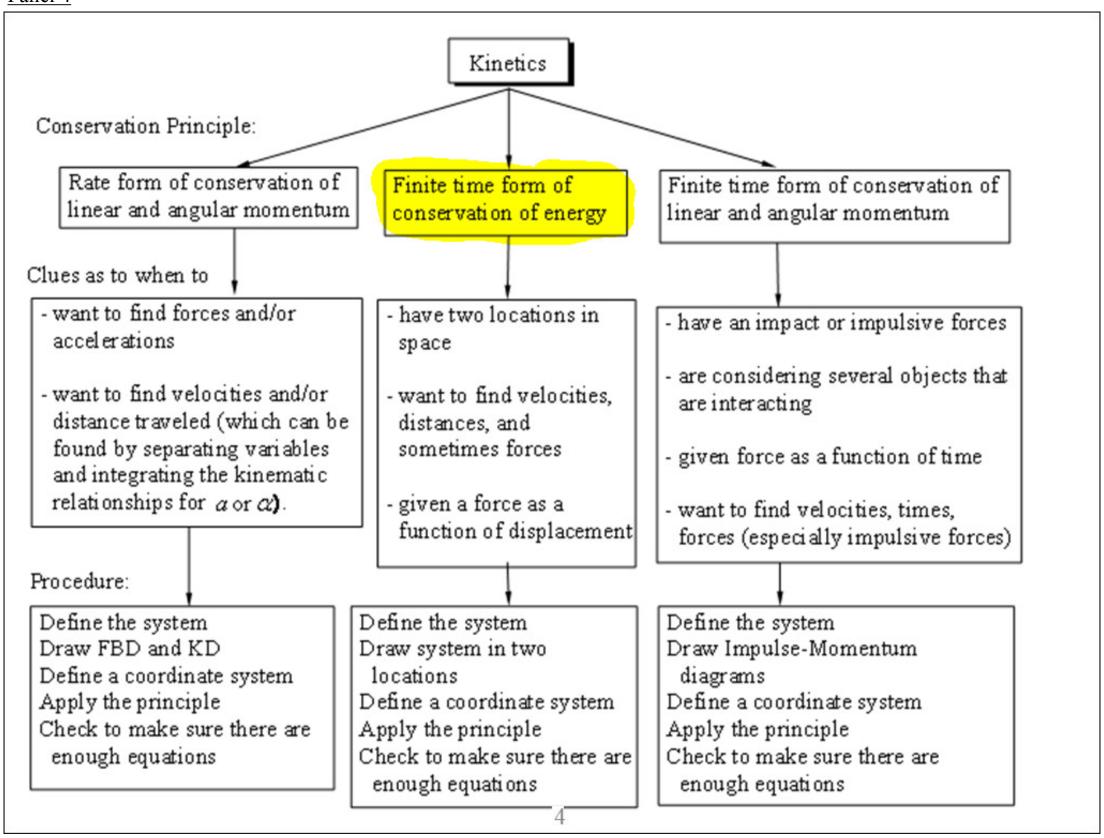
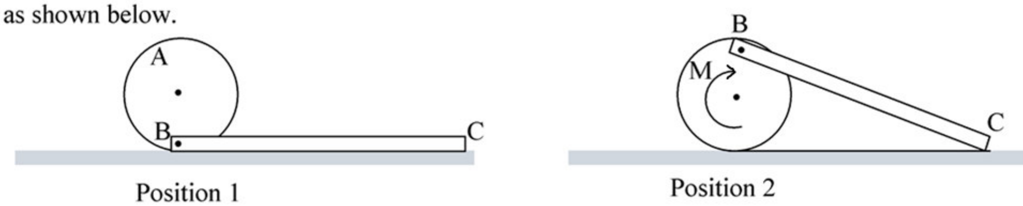
Panel 4



Panel 5

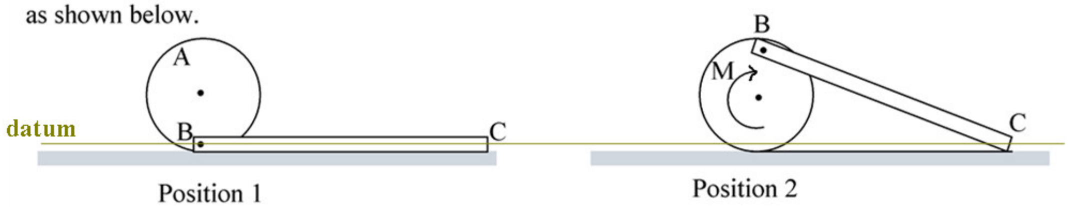
Disk A ($I_G = 2$ kg-m^2, m = 16 kg) is pinned to bar BC (L = 1 m, m = 12 kg, $I_G = 1$ kg-m^2) and is initially at rest in position 1 shown when a constant clockwise moment M = 100 N-m is applied. The distance between the center of the disk and B is 0.45 m and the radius of the disk is 0.5 m. Assuming the disk rolls without slipping on the ground and the friction between the end of the bar and the ground is negligible, what is the angular velocity of the disk when the bar is in position 2 as shown below.



5

Panel 6

Disk A ($I_G = 2$ kg-m^2, m = 16 kg) is pinned to bar BC (L = 1 m, m = 12 kg, $I_G = 1$ kg-m^2) and is initially at rest in position 1 shown when a constant clockwise moment M = 100 N-m is applied. The distance between the center of the disk and B is 0.45 m and the radius of the disk is 0.5 m. Assuming the disk rolls without slipping on the ground and the friction between the end of the bar and the ground is negligible, what is the angular velocity of the disk when the bar is in position 2 as shown below.

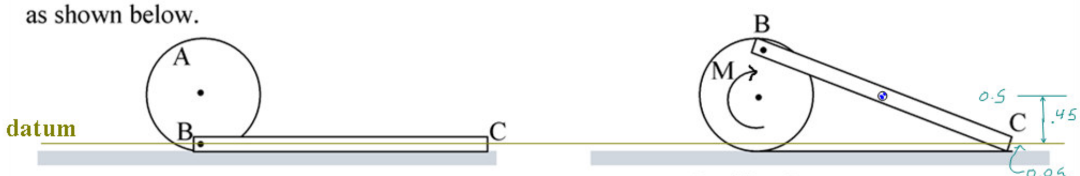


Write the general form energy equation and start breaking it into components

6

Panel 7

Disk A (I_G = 2 kg-m², m = 16 kg) is pinned to bar BC (L = 1 m, m = 12 kg, I_G = 1 kg-m²) and is initially at rest in position 1 shown when a constant clockwise moment M = 100 N-m is applied. The distance between the center of the disk and B is 0.45 m and the radius of the disk is 0.5 m. Assuming the disk rolls without slipping on the ground and the friction between the end of the bar and the ground is negligible, what is the angular velocity of the disk when the bar is in position 2 as shown below.



So we've got: Position 1 Position 2

$E_{g2} + E_{K2} = M\theta$

What's the equation for E_{k2}?

E_{g2} is just :

and $M\theta$ is just .

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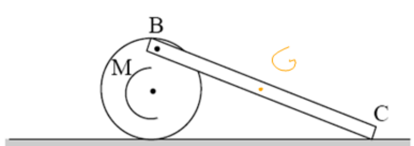
Panel 8



$E_{K2} = 3\omega_w^2 + 6V_{BC}^2 + \frac{1}{2}\omega_{BC}^2$

So we must relate the velocity terms so that we have only 1 variable

V_{BC} is the velocity at the center of gravity so I labelled that point G for convenience

The two bodies are connected at B so V_B will be valuable

Find V_B in terms of ω_w

Write the vector approach equation that relates V_G to V_B

What is ω_{BC}
Find the IC on the diagram above

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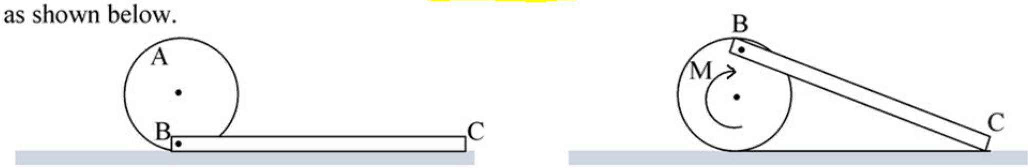
Panel 9

Disk A ($I_G = 2$ kg-m^2, m = 16 kg) is pinned to bar BC (L = 1 m, m = 12 kg, $I_G = 1$ kg-m^2) and is initially at rest in position 1 shown when a constant clockwise moment M = 100 N-m is applied. The distance between the center of the disk and B is 0.45 m and the radius of the disk is 0.5 m. Assuming the disk rolls without slipping on the ground and the friction between the end of the bar and the ground is negligible, what is the angular velocity of the disk when the bar is in position 2 as shown below.



Position 1 Position 2

$$E_{g_2} + E_{K_2} = W$$

$$12(.45)g + 3\omega_w^2 + 6(.95\omega_w)^2 = 100\pi$$

$$\boxed{\omega = 5.6 \ \text{rad/s}}$$

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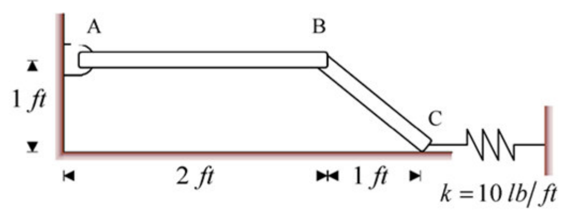
Panel 10

Ex. Bar AB weighs 10 lb and bar BC weighs 6 lb. If the system is released from rest in the position shown, what are the angular velocities of the bars at the instant just before joint B hits the smooth floor?



Known:

$m_{AB} = 0.311 \dfrac{lb\,s^2}{ft}$ $\quad m_{BC} = 0.186 \dfrac{lb\,s^2}{ft}$ $\quad I_{G,AB} = 0.104\, lb\,s^2\,ft$ $\quad I_{G,BC} = 0.031\, lb\,s^2\,ft$

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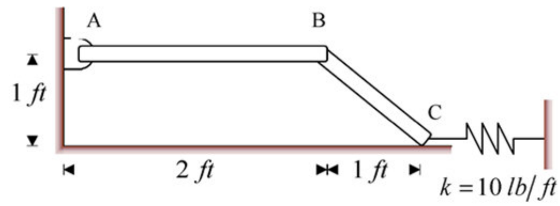
Panel 11

Ex. Bar AB weighs 10 lb and bar BC weighs 6 lb. If the system is released from rest in the position shown, what are the angular velocities of the bars at the instant just before joint B hits the smooth floor?



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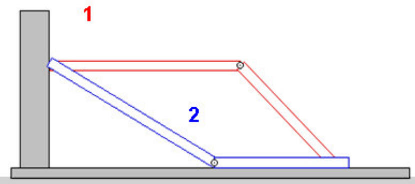

Write the general form equation, break it into components, and indicate which components equal zero

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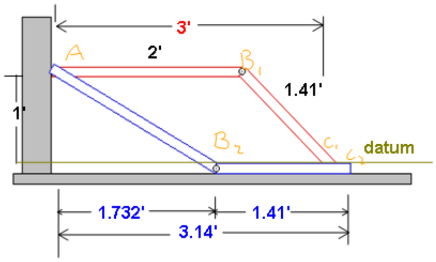
Panel 12



$E_{sys_2} - E_{sys_1} = W$

$E_{g_2} =$

$E_{s_2} =$

$E_{g_1} =$

What's E_{k2}?

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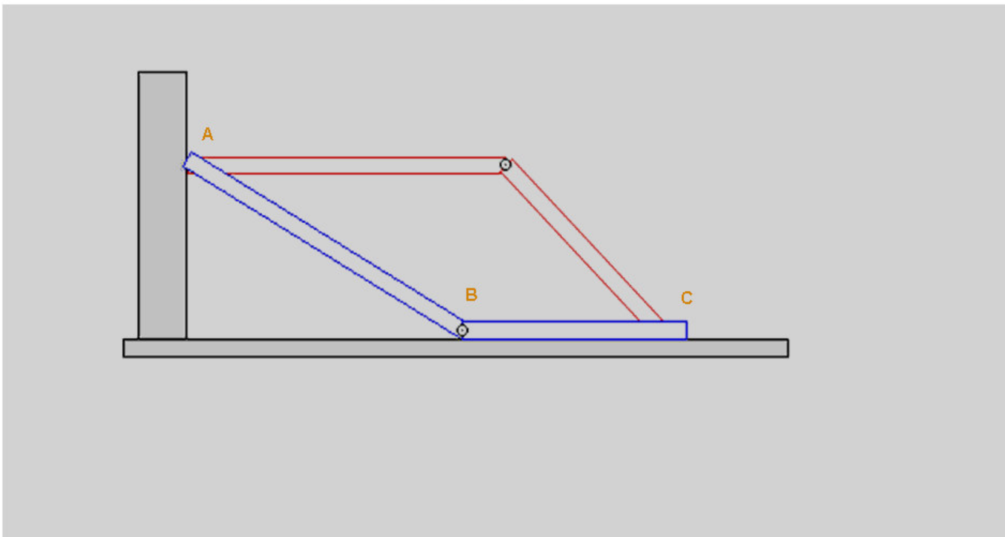
Panel 13

Need to relate omegas to v's

Find the IC of link BC at position 2



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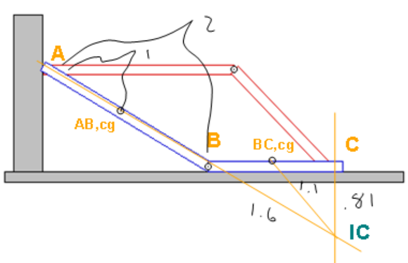
Panel 14



When relating the omegas and velocities at the cg's:

- Relate the object omega to the velocity at the cg
- Relate the omega of object 1 to omega of object 2

Relate the omegas using the common pt

$$V_B = V_B$$

$$\omega_{AB} r_{B/IC_{AB}} = \omega_{BC} r_{B/IC_{BC}}$$

Relate $V_{AB,cg}$ to ω_{AB}

Relate $V_{BC,cg}$ to ω_{BC}

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Panel 15

That's it start plugging into Maple

$$E_{sys_2} - E_{sys_1} = W$$

$$E_{g_2} + E_{K_2} + E_{s_2} - E_{g_1} - E_{K_1} - E_{s_1} = W$$

$E_{g_2} =$

$E_{s_2} =$

$E_{g_1} =$

$E_{K_2} =$

$V_{AB_{cg}} =$

$V_{BC_{cg}} =$

I prefer to think about it was 4 equations (energy plus 3 kinematics)
but call it as many equations as you like still works out

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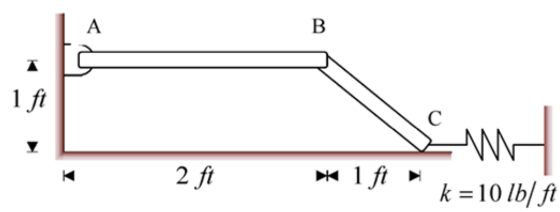
Panel 16

Ex. Bar AB weighs 10 lb and bar BC weighs 6 lb. If the system is released from rest in the position shown, what are the angular velocities of the bars at the instant just before joint B hits the smooth floor?



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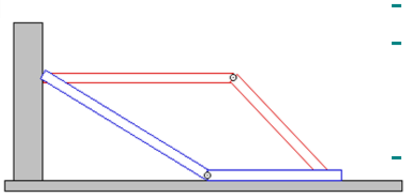
Summary
- Pick kinetic strategy (Energy)
- Draw energy diagram with datum
- Write general form equation for energy
- Break into components
- Determine which are zero
- Write equation for remaining terms
- Reduce kinematics to a single variable
  - Relate omegas to velocity at cg
  - Using point on both, relate omegas
- Plug in everything and solve



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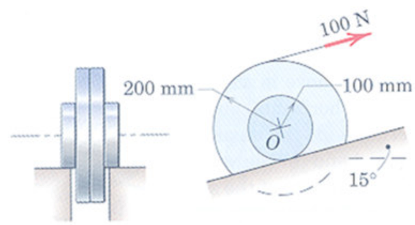
Panel 17

The wheel rolls up the incline on its hubs without slipping and is pulled by the 100-N force applied to the cord wrapped around its outer rim. If the wheel start from rest, compute its angular velocity after its center has moved a distance of 3-m up the incline. The wheel has a mass of 40-kg with a center of mass at O and has a centroidal radius of gyration of 150-mm. (*taken from Engineering Mechanics, 4th Edition by Meriam & Kraige*)



Similar to a homework problem

Solution available on the website

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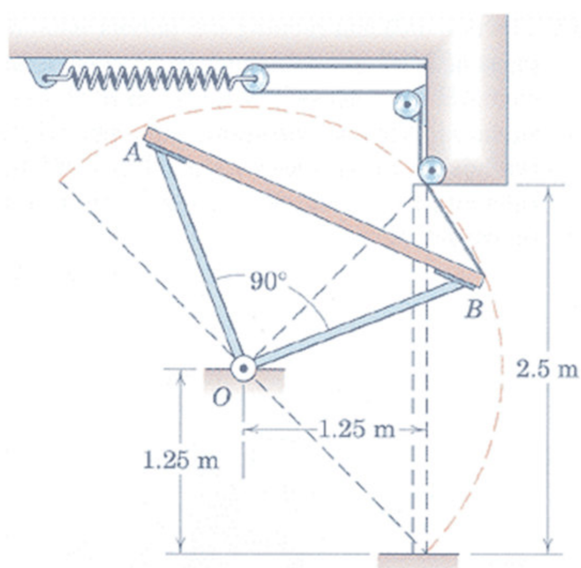
Panel 18

AB is the cross section of a garage door which is a rectangular 2.5m by 5m panel of uniform thickness with a mass of 200 kg. The door is supported by the struts of negligible mass and hinged at O. Two spring-and-cable assemblies, one on each side of the door, control the movement. When the door is in the horizontal open position, each spring is unextended. If the door is given a slight push from the open position and allowed to fall, determine the spring constant *k* for each spring which will limit the angular velocity of the door to 1.5 rad/s when edge B strikes the floor.
(*taken from Engineering Mechanics, 4th Edition by Meriam & Kraige*)



This one brings back some dependent motion to add to the problem

Solution available on the website

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