Panel 1

ES204 Mechanical Systems

General Plane Motion
Vector Approach to Velocity Kinematics
Lecture 15

Dr. Fisher

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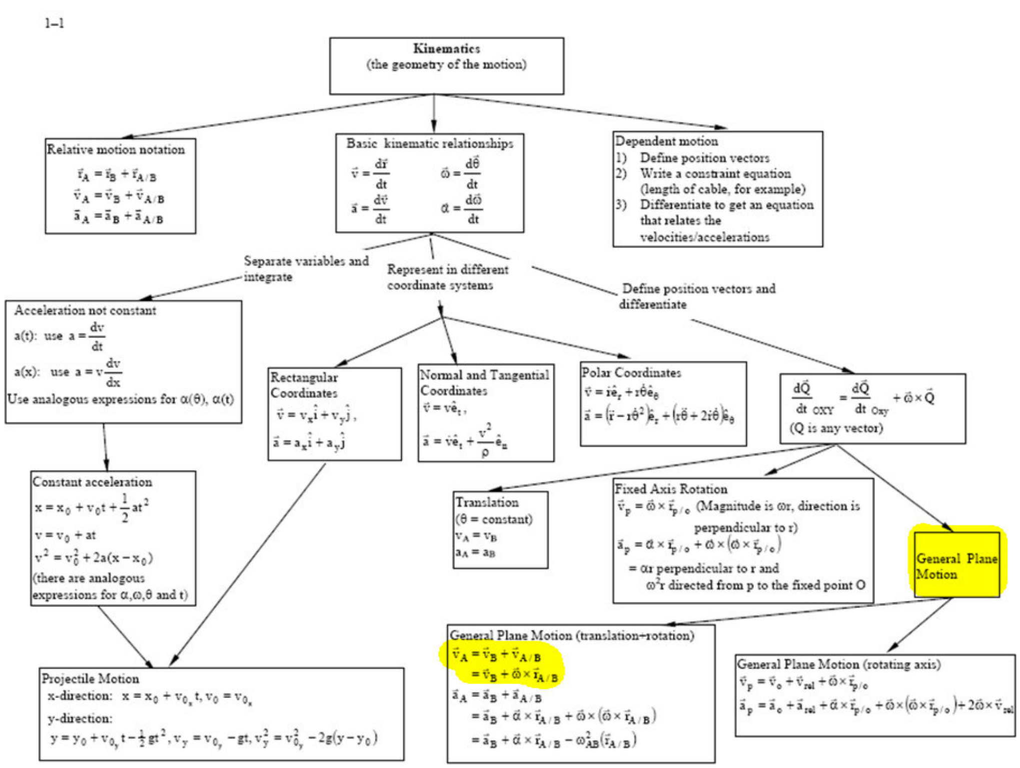
Panel 2



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Panel 3

Rigid body motion

Three types of motion

1. **Translation** (every line remains parallel to original orientation)
2. **Fixed Axis Rotation** (every point travels in a circle about a fixed point)
3. **General Plane Motion** (a combination of translation + rotation)

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Panel 4

Three types of plane motion

Type of motion	How to find the velocity of a point on the rigid body
1. Translation	$\vec{V}_a = \vec{V}_b$ (All points have same velocity)
2. Fixed axis rotation	$\vec{V}_p = \vec{\omega} \times \vec{r}_{p/o}$ Velocity determined by omega and distance to the fixed pt of rotation
3. General plane motion	• Instantaneous center of velocity o Scalar approach o Need to know the directions of the velocities of two points • Vector algebra approach o Write position vectors o Use $\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$ and equate components o Get more equations by looking at another object

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Panel 5

Starting with relative motion

$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

If two points A and B are on the same rigid body

$$\vec{v}_{B/A} = \vec{\omega} \times \vec{r}_{B/A}$$

Magnitude = $\omega\, r_{B/A}$ for plane motion

Direction = perpendicular to ω and $r_{B/A}$

So, for general plane motion

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

General equation to relate two velocity of two points on the same rigid body

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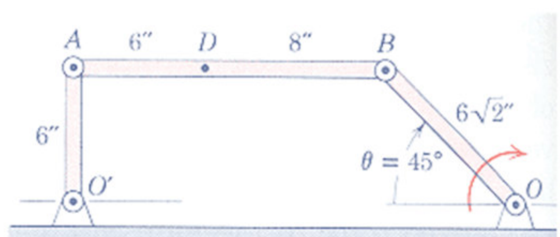
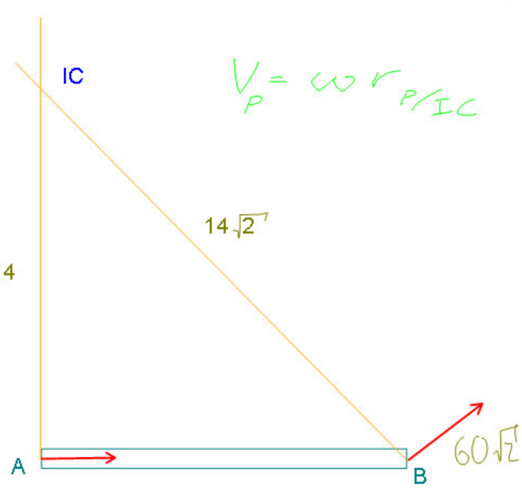
Panel 6

Arm OB of the linkage has a clockwise angular velocity of 10 rad/s in the position shown where θ=45°. Determine :

(a) the velocity of point A,
(b) the velocity of point D,
(c) the angular velocity of link AB

(*taken from Engineering Mechanics, 3rd Edition by Meriam & Kraige*)





$V_P = \omega\, r_{P/IC}$

$V_B = \omega_{AB}\, r_{B/IC}$

$60\sqrt{2} = \omega_{AB}(14\sqrt{2})$

$\omega_{AB} = 4.28$ rad/s ↺

$V_A = \omega_{AB}\, r_{A/IC}$

$V_A = 4.28\,(14)$

$V_A = 60$ in/s

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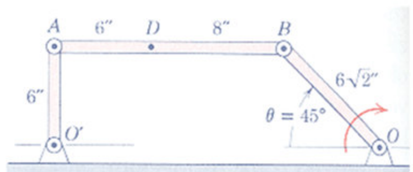
Panel 7

ROSE-HULMAN INSTITUTE OF TECHNOLOGY
Department of Mechanical Engineering

ES 204 Mechanical Systems

Example Problem - Le 13

Ex. Arm OB of the linkage has a clockwise angular velocity of 10 rad/s in the position shown where θ=45°. Determine :
(a) the velocity of point A,
(b) the velocity of point D,
(c) the angular velocity of link AB
(*taken from Engineering Mechanics, 3rd Edition by Meriam & Kraige*)



Vector Approach (Relative Motion)

Strategy:

1. Solve for $\bar{v}_B$ knowing ω_{OB} and $\bar{r}_{B/O}$
2. Knowing $\bar{v}_B$ and $\bar{r}_{A/B}$, solve for $\bar{v}_A$ and ω_{AB}
3. Knowing $\bar{v}_B$ and $\bar{r}_{D/B}$, solve for $\bar{v}_D$

Part 1:

$$\bar{v}_B = \bar{v}_O + \omega_{OB} \times \bar{r}_{B/O}$$

Since O is hinged and therefore the point of rotation, $\bar{v}_O = 0$. From the diagram, $\omega = -10\hat{k}\ rad/s$ and $\bar{r}_{B/O} = -6\hat{i} + 6\hat{j}\ in$. Thus

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Panel 8

$$\bar{v}_B = \left(-10\hat{k}\right)\times\left(-6\hat{i} + 6\hat{j}\right) = 60\hat{i} + 60\hat{j}\ in/s \qquad (1)$$

Part 2:

$$\bar{v}_A = \bar{v}_B + \bar{\omega}_{AB} \times \bar{r}_{A/B}$$
$$= v_{B,x}\hat{i} + v_{B,y}\hat{j} + \left(\omega_{AB}\hat{k}\right)\times\left(r_{A/B,x}\hat{i} + r_{A/B,y}\hat{j}\right)$$
$$v_{A,x}\hat{i} + v_{A,y}\hat{j} = v_{B,x}\hat{i} + v_{B,y}\hat{j} - \omega_{AB} r_{A/B,x}\hat{j} + \omega_{AB} r_{A/B,y}\hat{i}$$

From the diagram, $v_{A,y} = 0$ and $\bar{r}_{A/B} = -14\hat{i} + 0\hat{j}\ in$. Thus we can write the last equation from above in component form:

$$\hat{i}: \quad v_{A,x} = v_{B,x} - \omega_{AB} r_{A/B,y}$$
$$v_{A,x} = v_{B,x} - 0 \qquad (2)$$
$$\hat{j}: \quad v_{A,y} = v_{B,y} + \omega_{AB} r_{A/B,x}$$
$$0 = v_{B,y} + \omega_{AB}(-14) \qquad (3)$$

Solving the two equations (2,3) for the two unknowns ($v_{A,x}$, ω_{AB}):

$$v_{A,x} = 60\ in/s,\ v_{A,y} = 0\ in/s \quad \Rightarrow \quad \bar{v}_A = 60\hat{i}\ in/s$$
$$\omega_{AB} = 4.28 \quad \Rightarrow \quad \omega_{AB} = 4.28\hat{k}\ rad/s$$

Part 3:

Vector Algebra Example *Page 1 of 2*

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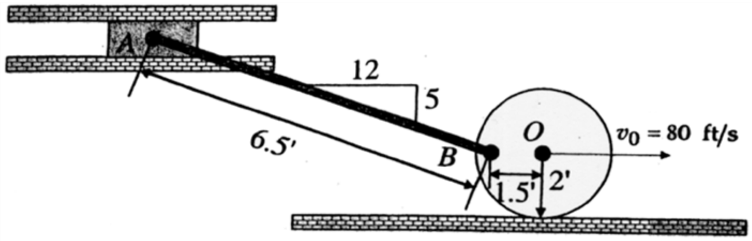
Panel 9



For the train wheel shown, find the velocity of the piston and the angular velocity of push arm AB when the velocity of the train is 80 ft/s. Assume no slip

Classify each motion as translation, fixed axis rotation, or general plane motion:

Piston A

Push arm AB

The wheel

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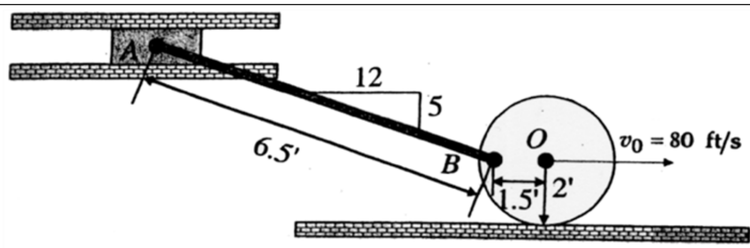
Panel 10



For the train wheel shown, find the velocity of the piston and the angular velocity of push arm AB when the velocity of the train is 80 ft/s. Assume no slip

So we know Vo and we want V_A so it seems logical that we'll relate the two via V_B
Write the two general form velocity equations that we'll be using

Relating V_B to Vo

Relating V_A to V_B

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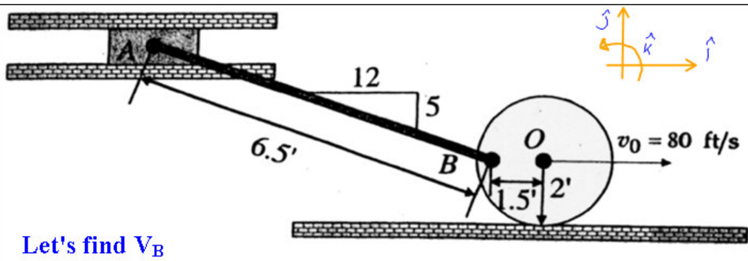
Panel 11



For the train wheel shown, find the velocity of the piston and the angular velocity of push arm AB when the velocity of the train is 80 ft/s. Assume no slip

Let's find V_B

$\vec{V}_B = \vec{V}_O + \vec{\omega}_{BO} \times \vec{r}_{B/O}$

$\vec{V}_O =$

$\vec{\omega}_{BO} =$

$\vec{r}_{B/O} =$

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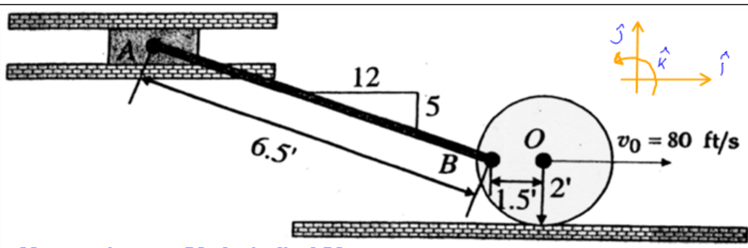
Panel 12



For the train wheel shown, find the velocity of the piston and the angular velocity of push arm AB when the velocity of the train is 80 ft/s. Assume no slip

Now we've got V_B let's find V_A

$\vec{V}_B = 80\hat{i} + 60\hat{j}$

$\vec{\omega}_{AB} =$

$\vec{r}_{A/B} =$

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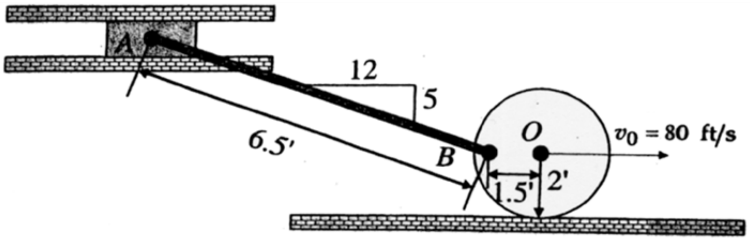
Panel 13



For the train wheel shown, find the velocity of the piston and the angular velocity of push arm AB when the velocity of the train is 80 ft/s. Assume no slip

Quick summary:

We knew Vo so we found V_b since it's on the wheel

V_b is also on the arm so we related it to V_A

Relating V_B to Vo

$$\vec{V}_B = \vec{V}_O + \vec{\omega}_{BO} \times \vec{r}_{B/O}$$

Relating V_A to V_B

$$\vec{V}_A = \vec{V}_B + \vec{\omega}_{BA} \times \vec{r}_{A/B}$$

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