

Panel 1

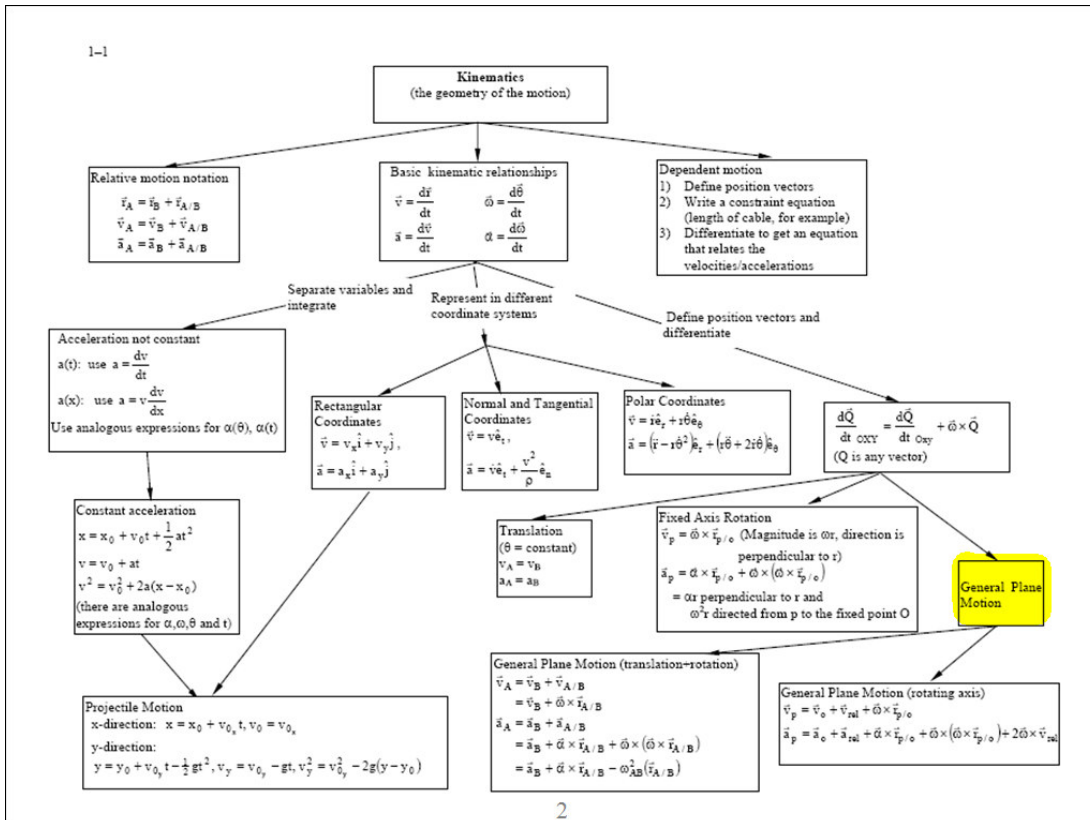
ES204 Mechanical Systems

General Plane Motion Instantaneous Centers Lecture 14

Dr. Fisher

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Panel 2



Panel 3

Rigid body motion

Three types of motion

1. **Translation** (every line remains parallel to original orientation)
2. **Fixed Axis Rotation** (every point travels in a circle about a fixed point)
3. **General Plane Motion** (a combination of translation + rotation)

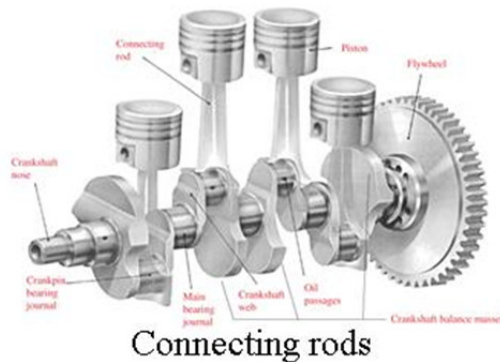
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Panel 4

There is general plane motion all around us



Wheels of a car



Connecting rods



Robots



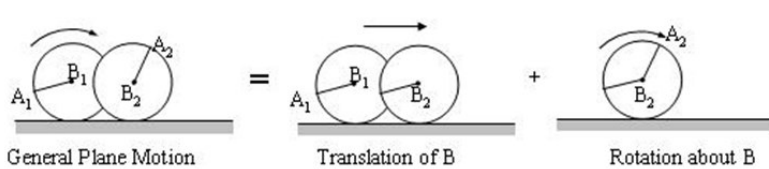
Biomechanics

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Panel 5

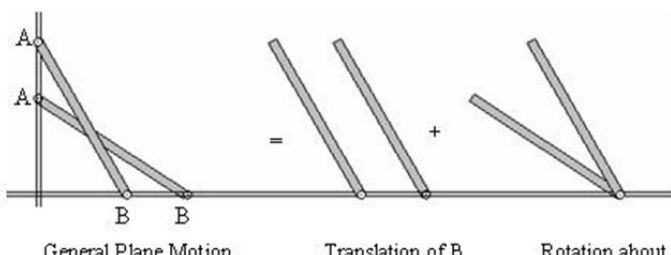
What is general plane motion?

General plane motion is a combination of a translation and a rotation.



$$\vec{v}_A = \vec{v}_B + \vec{v}_{B/A}$$

$$= \vec{v}_B + \vec{\omega} \times \vec{r}_{B/A}$$



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Panel 6

Summary

If two points A and B are on the same rigid body

$$\vec{v}_{B/A} = \vec{\omega} \times \vec{r}_{B/A}$$

Magnitude = $\omega r_{B/A}$ for plane motion
 Direction = perpendicular to ω and $r_{B/A}$

So, for general plane motion

$\vec{v}_B =$

Translation

Rotation

General equation to relate two velocity of two points on the same rigid body

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Panel 7

Three types of plane motion

Type of motion	How to find the velocity of a point on the rigid body
1. Translation	
2. Fixed axis rotation	
3. General plane motion	

Panel 8

The Instantaneous Center of velocity (IC)

The instantaneous center of velocity (IC) is a unique reference point which momentarily has a velocity of zero. Thus, as far as velocities are concerned, the body seems to rotate about the instantaneous center, that is, the velocity of any point on the rigid body is simply the angular velocity of the rigid body times the distance to the IC ($v_p = \omega r_{p/IC}$)

Direction of the velocity of B

Direction of the velocity of C

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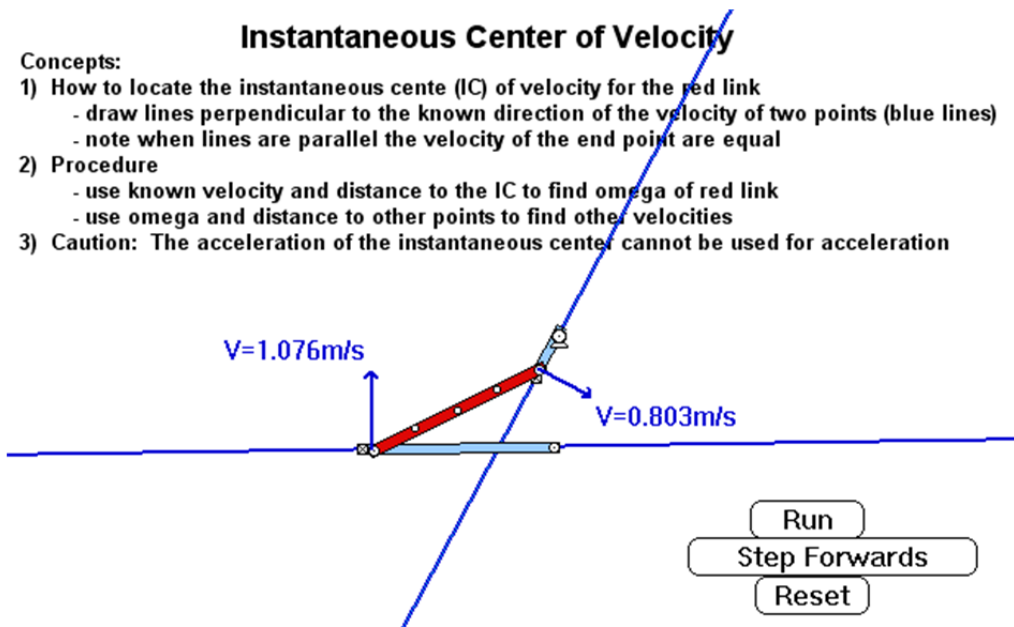
Panel 9

Working Model Example:

Instantaneous Center of Velocity

Concepts:

- 1) How to locate the instantaneous centre (IC) of velocity for the red link
 - draw lines perpendicular to the known direction of the velocity of two points (blue lines)
 - note when lines are parallel the velocity of the end point are equal
- 2) Procedure
 - use known velocity and distance to the IC to find omega of red link
 - use omega and distance to other points to find other velocities
- 3) Caution: The acceleration of the instantaneous center cannot be used for acceleration

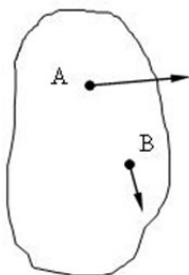


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Panel 10

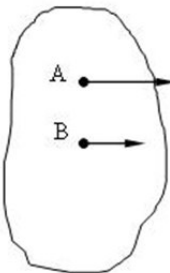
Find the IC for these cases

Case 1



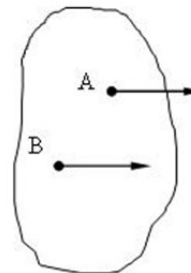
General Plane motion

Case 2



Fixed axis rotation or an instant of general plane motion

Case 3



Translation or an instant of general plane motion

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Panel 11

Observations

Notes:

- The instantaneous center may lie off the body
- Once the instantaneous center is found the absolute velocity of any point on the body is readily determined
- The acceleration of the instantaneous center is not zero (in general) so it cannot be used to find the acceleration of points on the rigid body.
- For rolling without slipping on a fixed surface the IC is the point of contact.

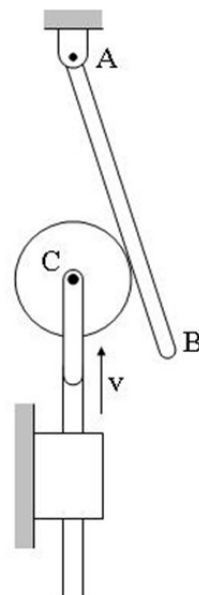


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Panel 12

Concept question

Where is the instantaneous center of velocity of disk C?



Assume no slipping between AB and disk C

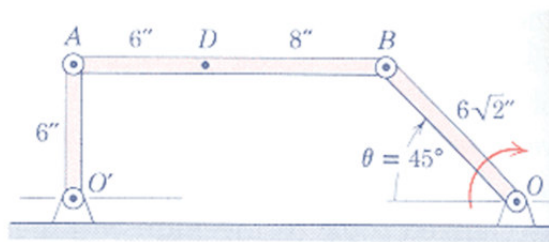
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Panel 13

Arm OB of the linkage has a clockwise angular velocity of 10 rad/s in the position shown where $\theta=45^\circ$. Determine :

- (a) the velocity of point A,
- (b) the velocity of point D,
- (c) the angular velocity of link AB

(taken from *Engineering Mechanics, 3rd Edition* by Meriam & Kraige)



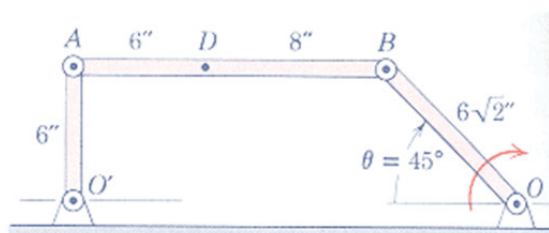
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Panel 14

Arm OB of the linkage has a clockwise angular velocity of 10 rad/s in the position shown where $\theta=45^\circ$. Determine :

- (a) the velocity of point A,
- (b) the velocity of point D,
- (c) the angular velocity of link AB

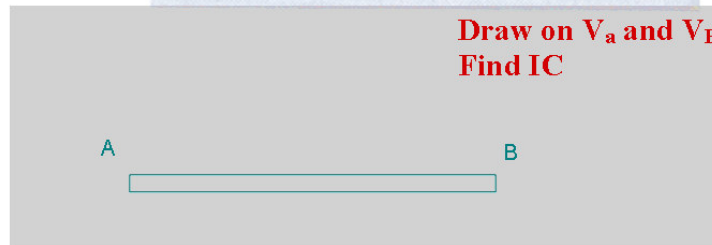
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Prodedure:

- Find V_B using fixed axis
- Draw dir of V_a
- Find IC
- Find ω_{AB}
- Find V_a and V_d

**Draw on V_a and V_B
Find IC**



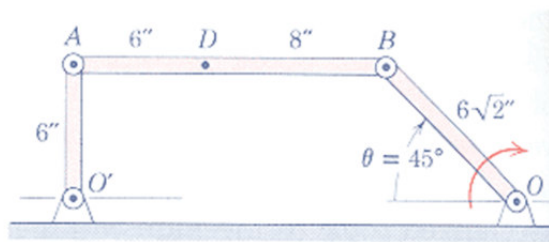
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Panel 15

Arm OB of the linkage has a clockwise angular velocity of 10 rad/s in the position shown where $\theta=45^\circ$. Determine :

- (a) the velocity of point A,
- (b) the velocity of point D,
- (c) the angular velocity of link AB

(taken from *Engineering Mechanics, 3rd Edition* by Meriam & Kraige)



Find omega
Find Va

$$V_P = \omega r_{P/IC}$$



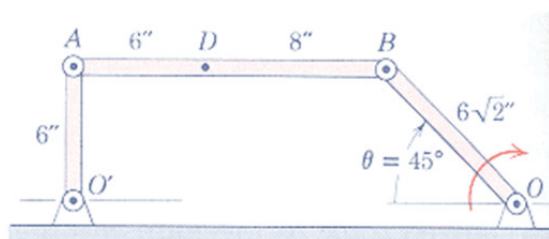
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Panel 16

Arm OB of the linkage has a clockwise angular velocity of 10 rad/s in the position shown where $\theta=45^\circ$. Determine :

- (a) the velocity of point A,
- (b) the velocity of point D,
- (c) the angular velocity of link AB

(taken from *Engineering Mechanics, 3rd Edition* by Meriam & Kraige)

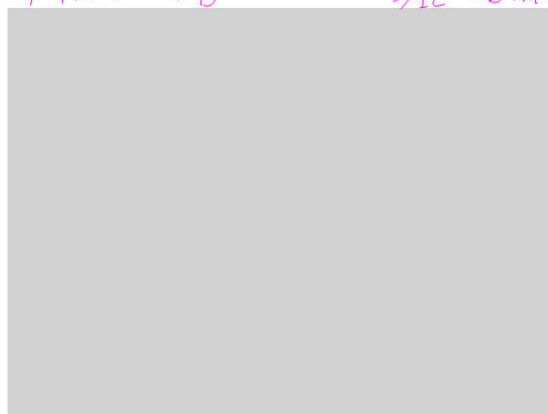


$$V_B = \omega_{AB} r_{B/IC}$$

Find V_O

$$r_{O/IC} = \sqrt{6^2 + 14^2}$$

$$V_A = \omega_{AB} r_{A/IC}$$



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