Panel 1

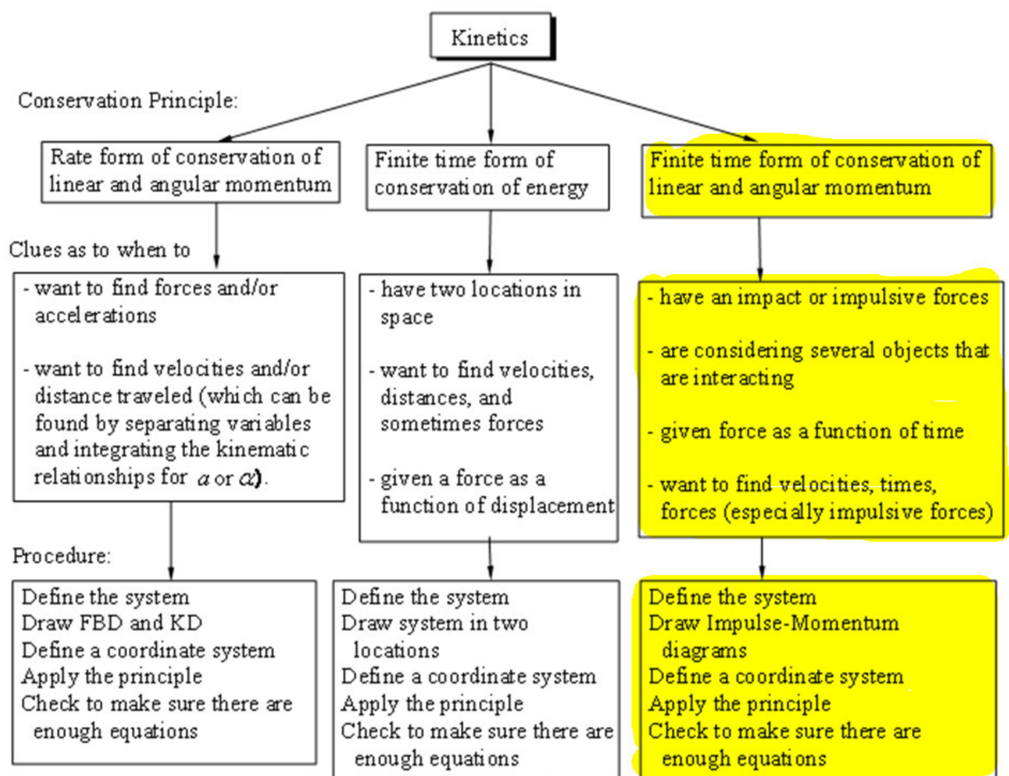
ES204 Mechanical Systems

Fixed Axis Rotation - Impact
Lecture 13

1

Dr.
Fisher

Panel 2



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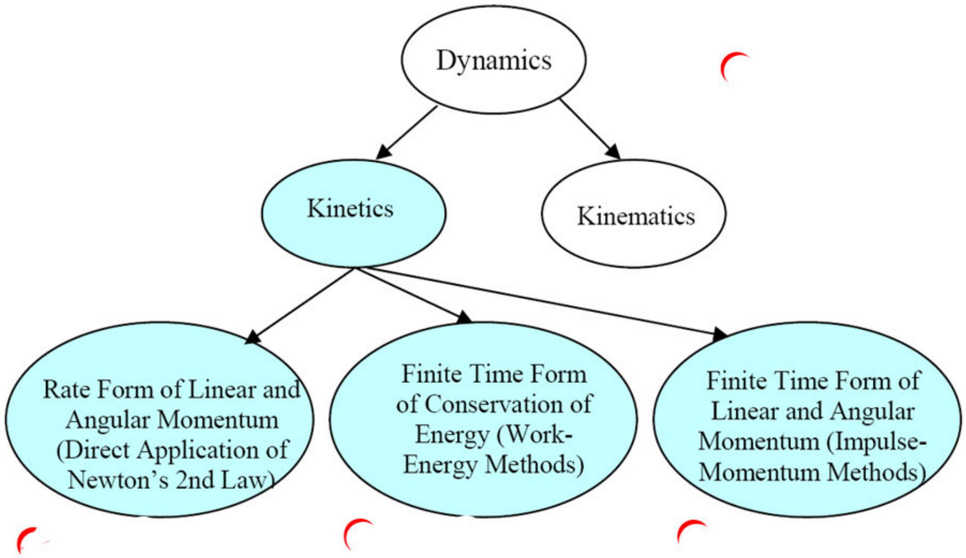
Panel 3



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Panel 4

In the text the finite-time forms of linear and angular momentum are referred to as "impulse-momentum methods". For a closed system these can be written as:

$$\Delta\vec{P}_{sys} = \int_{t_1}^{t_2} \vec{F}dt \qquad \text{and} \qquad \Delta\vec{L}_{sys_0} = \int_{t_1}^{t_2} \vec{M}_0 dt \qquad (1), (2)$$

If there are any impulsive forces (an impulsive force is a large force that acts over a small time) acting on the system then non-impulsive forces (weight, springs, etc.) can be neglected and Eq. 1-2 become

$$\Delta\vec{P}_{sys} = \sum \vec{F}_i \Delta t \qquad \text{and} \qquad \Delta\vec{L}_{sys_0} = \sum (\vec{M}_0)_i \Delta t \qquad (3), (4)$$

where $\vec{F}_i$ and $\vec{M}_i$ are external impulsive forces acting on the system. Recall for a rigid body the linear and angular momentum are

$$\vec{P}_{sys} = m\vec{v}_G \qquad \text{and} \qquad \boxed{\vec{L}_{sys_O} = I_G\vec{\omega} + \vec{r}_{G/O} \times m\vec{v}_G} \qquad (5), (6)$$

When to use

The finite time form of conservation of linear and angular momentum are typically used when:

- there is an impact or impulsive forces in the problem
- there are several interacting objects
- there is a force as a function of time
- want to find velocities, times or forces (especially impulsive forces)

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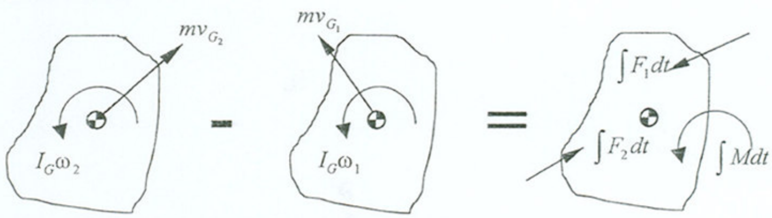
Panel 5

Procedure

Since Eqs (1-2) and Eqs. (3-4) are vector equations it is often useful to draw impulse momentum diagrams as shown below.

Picture:



In Words:	System momentum after the time interval	−	System momentum before the time interval	=	Impulses acting during the time interval
Equations:	$\vec{P}_{sys_2}$	−	$\vec{P}_{sys1}$	=	$\sum \vec{F}_i \Delta t,$
	$\left(\vec{L}_{sys_O}\right)_2$	−	$\left(\vec{L}_{sys_O}\right)_1$	=	$\sum\left(\vec{M}_O\right)_i \Delta t$

We therefore have three scalar equations

1. Linear momentum in the x-direction
2. Linear momentum in the y-direction
3. Angular momentum (moment of the momentum) about any axis

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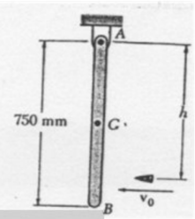
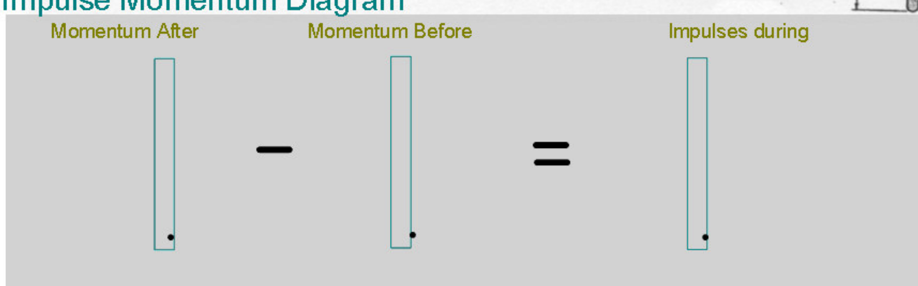
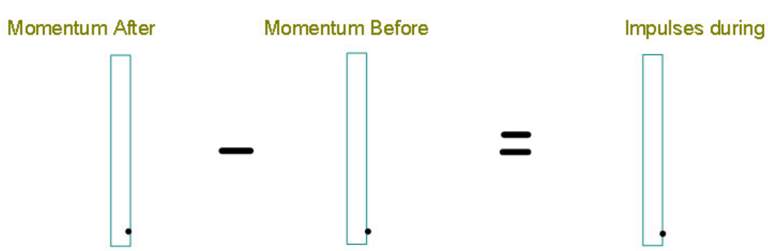
Panel 6

17.95 A 30-g bullet is fired with a horizontal velocity of 400 m/s into a 4-kg wooden beam AB suspended from a pin support at A. Knowing that h = 600 mm and that the beam is initially at rest, determine (*a*) the velocity of the mass center G of the beam immediately after the bullet becomes embedded, (*b*) the impulsive reaction at A, assuming that the bullet becomes embedded in 1 ms.



Impulse Momentum Diagram





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Panel 7

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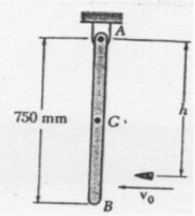

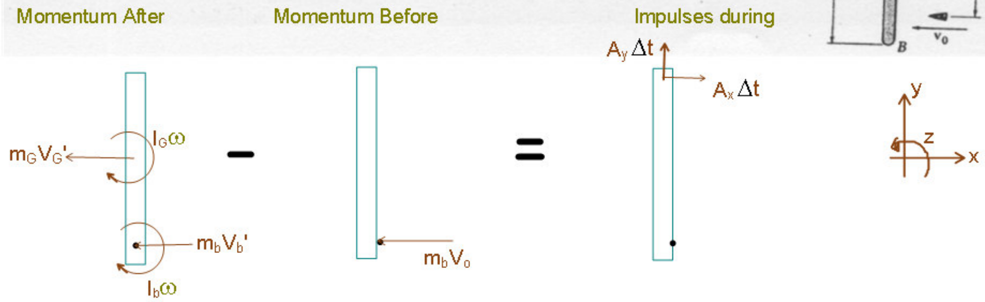

CoLM Finite: x dir P_2 - P_1 = $\Sigma F\Delta t$

CoLM Finite: y dir

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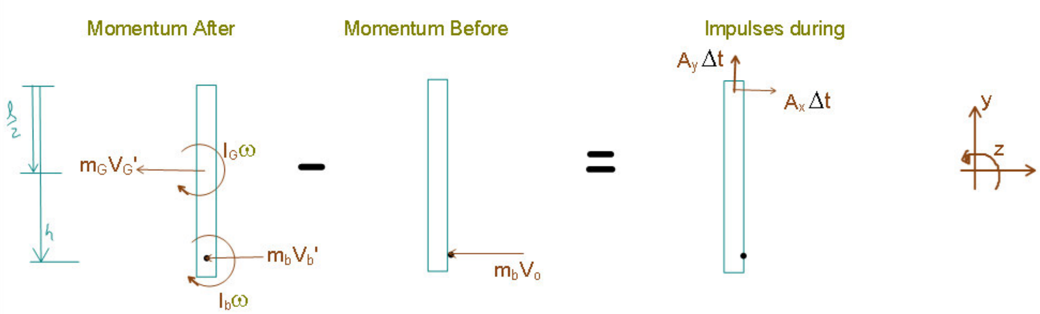
Panel 8



CoAM Finite: About which point? (Hint: make the impulse moments = 0)

L_2 - L_1 = $\Sigma M\Delta t$

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Panel 9

17.95 A 30-g bullet is fired with a horizontal velocity of 400 m/s into a 4-kg wooden beam *AB* suspended from a pin support at *A*. Knowing that *h* = 600 mm and that the beam is initially at rest, determine (*a*) the velocity of the mass center *G* of the beam immediately after the bullet becomes embedded, (*b*) the impulsive reaction at *A*, assuming that the bullet becomes embedded in 1 ms.

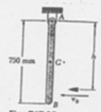
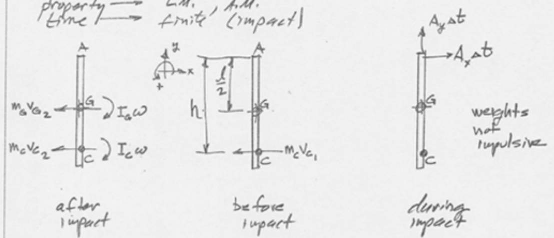


Fig. P17.95

system → bullet & beam
property → L.M., A.M.
time → finite (impact)



L.M. finite $\vec{P}_{2_{sys}} - \vec{P}_{1_{sys}} = \Sigma \vec{F} \Delta t$

x-dir $(-m_G v_{G_2} - m_c v_{c_2}) - (-m_c v_{c_1}) = A_x \Delta t$ (1)

y-dir $0 - 0 = A_y \Delta t$

A.M., finite, about point A : $\bar{L}_{A_2} - \bar{L}_{A_1} = \Sigma M_A \Delta t$

$$\left(-I_G\omega - I_c\omega - m_G v_{G_2}\frac{l}{2} - m_c v_{c_2} h\right) - (-m_c v_{c_1} h) = 0 \quad (2)$$

($I_c\omega \to 0$)

moments of inertia

$I_c = 0$ since body can be considered a particle

$I_G = \frac{1}{12} m_G l^2$ (look-up.)

kinematics

$v_{G_2} = \frac{l}{2}\omega$ (3) $v_{c_2} = h\omega$ (4)

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Panel 10

(4) eqn (4) unkn : v_{G_2} v_{c_2} ω A_x

known or computed

$m_c = .03$ kg $I_G = \frac{1}{12}(4\,kg)(.75)^2 = 0.1875$ kgm²

$m_G = 4$ kg $\Delta t = 0.001$ s

$h = .6$ m $v_{c_1} = 400$ m/s

$l = .75$ m

solve

```
> e1:= -m[g]*v[g2] - m[c]*v[c2] + m[c]*v[c1] = A[x]*dt;
> e2:= -J[g]*omega - m[g]*v[g2]*L/2 - m[c]*v[c2]*h + m[c]*v[c1]*h =
  0;
> e3:= v[g2] = L/2*omega;
> e4:= v[c2]=h*omega;
```

$$e1 := -m_g\, v_{g2} - m_c\, v_{c2} + m_c\, v_{c1} = A_x\, dt$$

$$e2 := -J_g\, \omega - \frac{1}{2} m_g\, v_{g2}\, L - m_c\, v_{c2}\, h + m_c\, v_{c1}\, h = 0$$

$$e3 := v_{g2} = \frac{1}{2} L\, \omega$$

$$e4 := v_{c2} = h\, \omega$$

```
> m[c]:= 0.03: m[g]:= 4: h:= 0.6: L:=0.75: J[g]:= 0.1875: dt:=
  0.001: v[c1]:= 400:
> solve({e1,e2,e3,e4},{v[g2],v[c2],omega,A[x]});
```

$$\{ v_{g2} = 3.549,\ v_{c2} = 5.678,\ A_x = -2366.,\ \omega = 9.464 \}$$

$A_x = 2366$ N (←)

$v_{G_2} = 3.549$ m/s (←)

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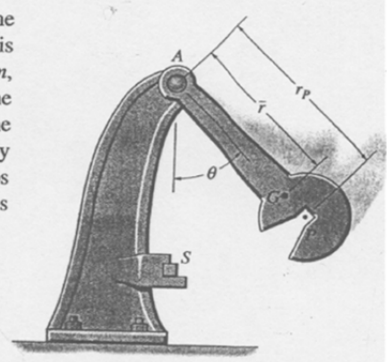
Panel 11

The Charpy impact test is used in materials testing to determine the energy absorption characteristics of a material during impact. The test is performed using the pendulum shown in Fig. 19–8*a*, which has a mass m, mass center at G, and a radius of gyration k_G about G. Determine the distance r_P from the pin at A to the point P where the impact with the specimen S should occur so that the horizontal force at the pin is essentially zero during the impact. For the computation, assume the specimen absorbs all the pendulum's kinetic energy during the time it falls and thereby stops the pendulum from swinging when $\theta = 0°$.



$I_G = mk_G^2$

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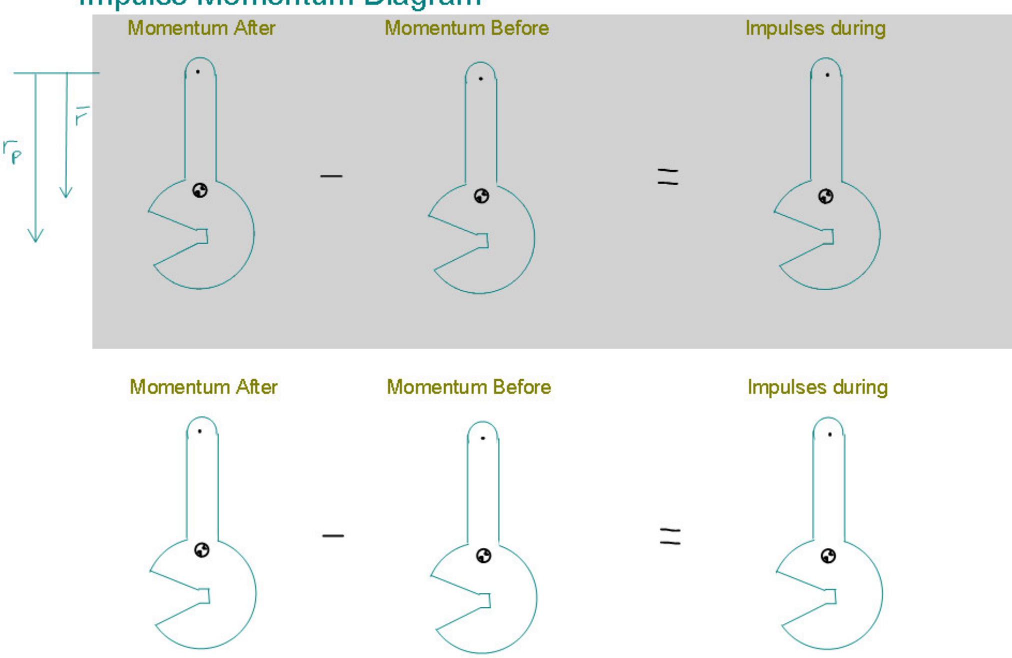
Panel 12

Impulse Momentum Diagram



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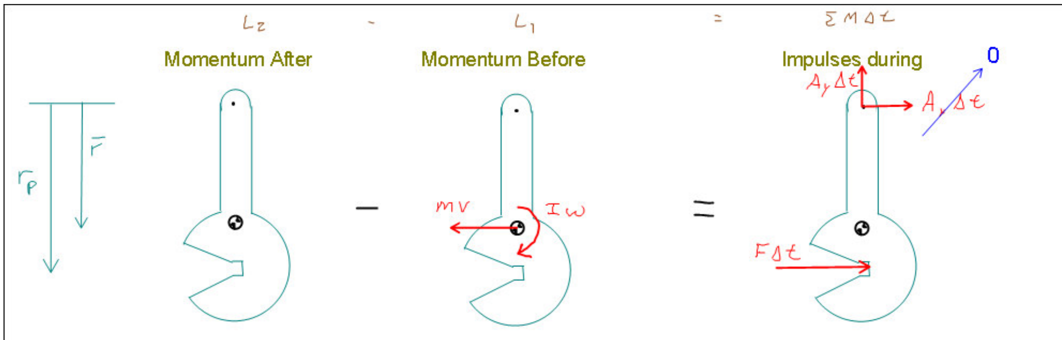
Panel 13



CoAM Finite ↻ **About point?** (Hint: make impulses = 0)

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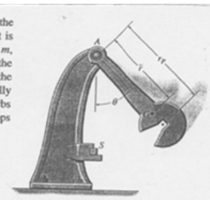
Panel 14

The Charpy impact test is used in materials testing to determine the energy absorption characteristics of a material during impact. The test is performed using the pendulum shown in Fig. 19–8*a*, which has a mass *m*, mass center at *G*, and a radius of gyration k_G about *G*. Determine the distance r_P from the pin at *A* to the point *P* where the impact with the specimen *S* should occur so that the horizontal force at the pin is essentially zero during the impact. For the computation, assume the specimen absorbs all the pendulum's kinetic energy during the time it falls and thereby stops the pendulum from swinging when $\theta = 0°$.



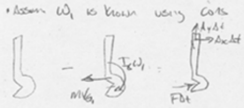
(a)

$I_G = mk_G^2$

$A_x = 0$

Use LM & AM finite time (impact) $\Delta \bar{L}_{sys_0} = \Sigma M \Delta t$; $\Delta \bar{P}_{sys} = \Sigma F \Delta t$

• Assume ω_1 is known using cons of energy.



radius of gyration

$I_G = mk_G^2$

AM about A ↻

$0 - (mv_{G_1}\bar{r} + I_G\omega_1) = -F\Delta t\, r_P$

where $v_G = \omega_1 \bar{r}$

$(I_G + m\bar{r}^2)\omega_1 = F\Delta t\, r_P$ (1)

X-direction $\xrightarrow{+}$

$0 - (-mv_G) = A_x\Delta t + F\Delta t$ ($A_x\Delta t = 0$)

$m\bar{r}\omega_1 = F\Delta t$ (2)

so

$r_P = \frac{mk_G^2 + m\bar{r}^2}{m\bar{r}} = \bar{r} + \frac{k_G^2}{\bar{r}}$

This point is called the center of percussion.

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