Panel 1

ES204 Mechanical Systems

Fixed Axis Rotation - Kinematics
Lecture 11

Dr. Fisher

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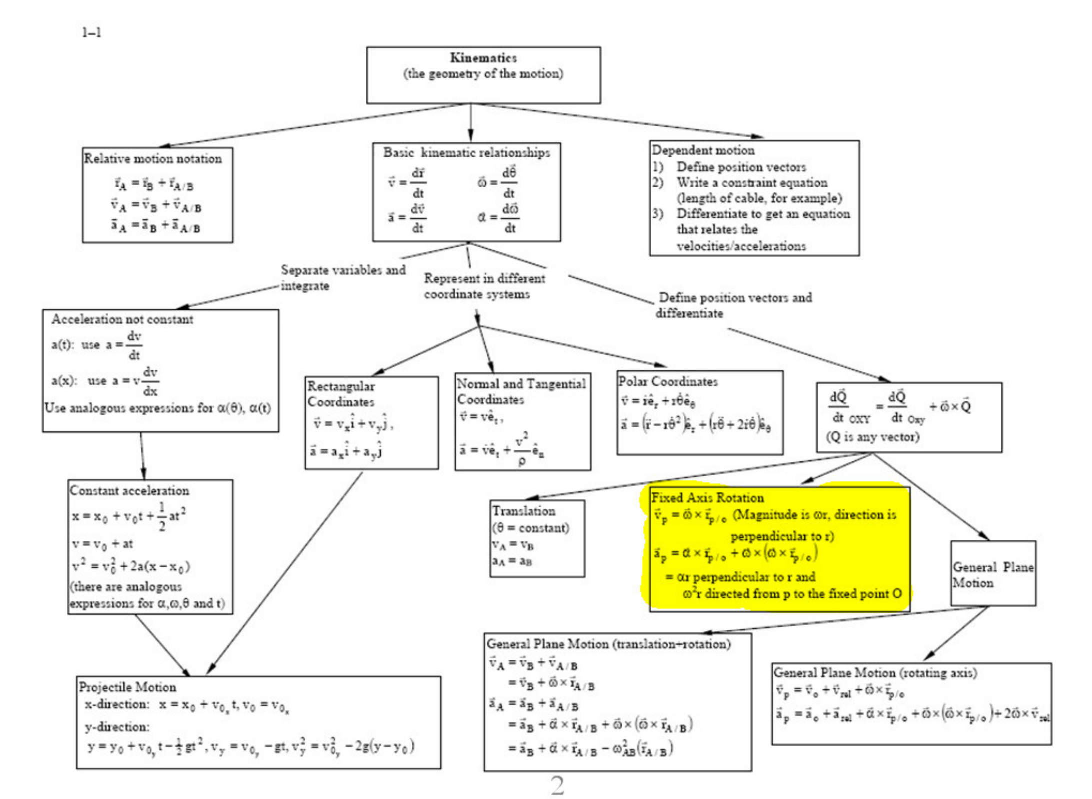
Panel 2



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Panel 3

Rigid body motion

Three types of motion

1. **Translation** (every line remains parallel to original orientation)
2. **Fixed Axis Rotation** (every point travels in a circle about a fixed point)
3. **General Plane Motion** (a combination of translation + rotation)

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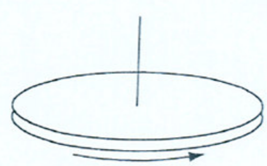
Panel 4

Rotation

Basic Kinematics

$\vec{\omega} =$

$\vec{\alpha} =$



The direction of these vectors are perpendicular to the plane of motion where the direction is given by the right hand rule (curl your fingers in the direction of the motion and your thumb gives you the direction of the vector).

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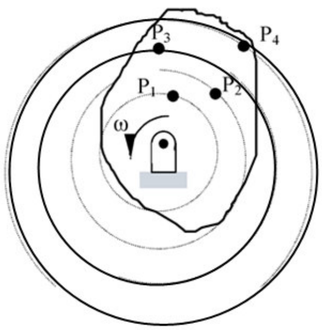
Panel 5

Fixed Axis Rotation

The description is simple: the rigid body has a hinge, joint, or pivot which is connected to a non-moving foundation. The rigid body rotates about a stationary axis passing through this fixed point. There is one point on the rigid body that has zero velocity, and it is of course this fixed point. All other points belonging to the rigid body move in circular arcs about the fixed point. The path of four points on a rigid body undergoing fixed axis rotation is shown below.



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Panel 6

Derivation using polar coordinates:

Recall from the discussion of polar coordinates

$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$$

Now if we consider the point on a rigid body undergoing fixed axis rotation we know that r does not change magnitude so $\dot{r} = \ddot{r} = 0$ and that $\dot{\theta} = \omega$, $\ddot{\theta} = \alpha$ these equations reduce to

which is identical to what we had before if we recognize that for fixed axis rotation (circular motion of every point) the unit vector in the transverse direction is identical to the tangential unit vector and the unit vector in the radial direction is equal to the negative of the unit vector in the normal direction.

These equations can also be derived using the formulas for velocity and acceleration in normal and tangential coordinates.

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Panel 7

Summary

1. There can be an acceleration component perpendicular to the radius vector with magnitude $r\alpha$. We will call it the tangential acceleration.

2. If there is an angular velocity present, there will always be an acceleration component with magnitude $r\omega^2$. The direction of this acceleration is always in exactly the opposite direction than the radius vector. This component is called the normal component.

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Panel 8

When solving problems

- Pinned joints: The point where the pin is located has the same velocity and acceleration regardless of which body you isolate as your system.
- Gears: The tangential components of acceleration of the two objects are the same, but the normal acceleration will be different.

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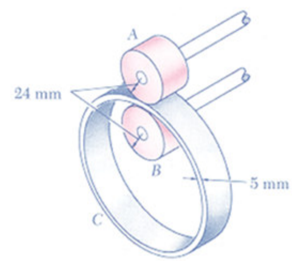
Panel 9

Ring C has an inside radius of 55 mm and an outside radius of 60 mm and is positioned between two wheels A and B, each of 24 mm outside radius. Knowing that wheel A rotates with a constant angular velocity of 300 rpm and that no slipping occurs, determine :

(a) the angular velocity of ring C and of wheel B,

(b) the acceleration of the points of A and B which are in contact with C.

(*taken from Vector Mechanics for Engineers, 5th Edition by Beer & Johnston*)



Given:

$r_A = r_B = 0.024$ m

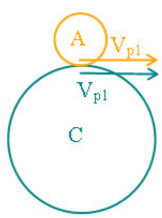
$r_{c\_ID} = 0.055$ m

$r_{c\_OD} = 0.060$ m

$\omega = 300$ rpm $= 31.42$ rad/s

A. Find the angular velocities of the other wheels

$V_{p1} = V_{p1}$



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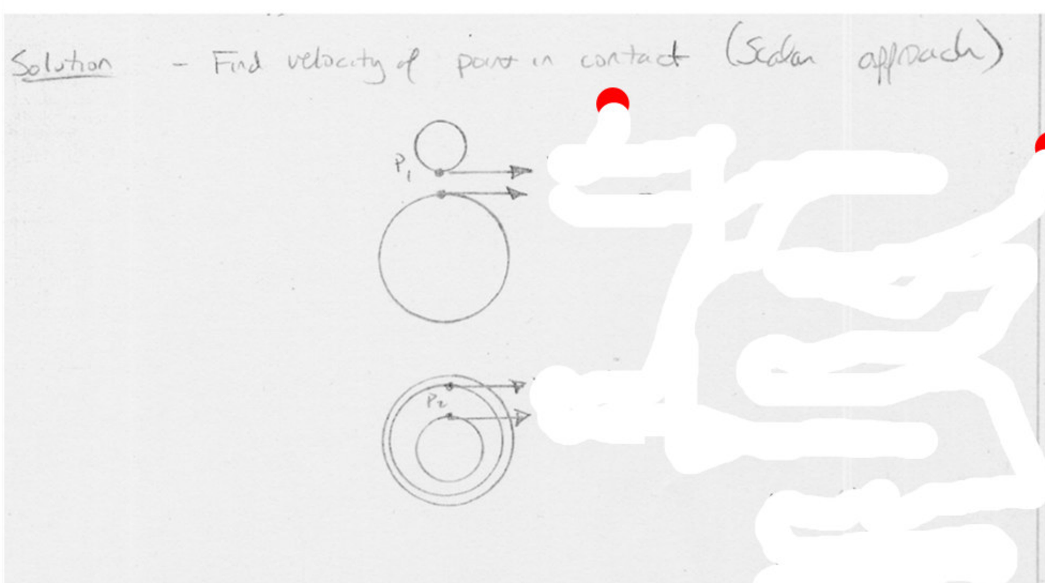
Panel 10



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Panel 11

Ring C has an inside radius of 55 mm and an outside radius of 60 mm and is positioned between two wheels A and B, each of 24 mm outside radius. Knowing that wheel A rotates with a constant angular velocity of 300 rpm and that no slipping occurs, determine :

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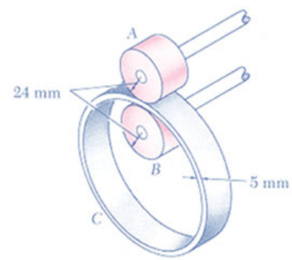


Known:

$r_A = r_B = 0.024$ m

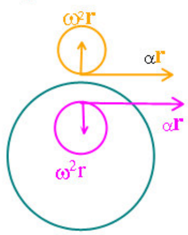
$r_{c\_ID} = 0.055$ m

$r_{c\_OD} = 0.060$ m

$\omega_a = 31.42$ rad/s

$\omega_b = 28.8$ rad/s

$\omega_c = 12.57$ rad/s



Find a_a

Find a_b

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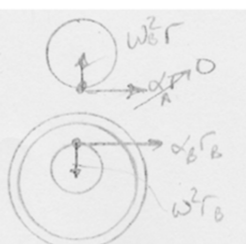
Panel 12

b) acceleration



so $\alpha_c = 0$

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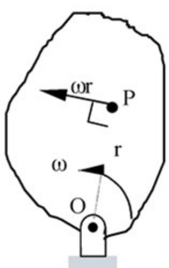
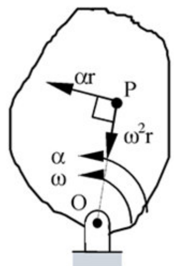
Panel 13

Visual Summary: The velocity and acceleration of a point on a rigid body undergoing fixed axis rotation is shown below.



Velocity



Acceleration

$\vec{v}_P =$ +

$\vec{a}_P =$ +

$= ($ +

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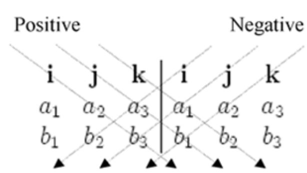
Panel 14

The cross product can be described by Sarrus's scheme. Consider the table

i	**j**	**k**	**i**	**j**	**k**
a_1	a_2	a_3	a_1	a_2	a_3
b_1	b_2	b_3	b_1	b_2	b_3



For the first three unit vectors, multiply the elements on the diagonal to the right (e.g. the first diagonal would contain **i**, a_2, and b_3). For the last three unit vectors, multiply the elements on the diagonal to the left and then negate the product (e.g. the last diagonal would contain **k**, a_2, and b_1). The cross product would be defined by the sum of these products:

$$\mathbf{a} \times \mathbf{b} = \mathbf{i}(a_2 b_3) + \mathbf{j}(a_3 b_1) + \mathbf{k}(a_1 b_2) - \mathbf{i}(a_3 b_2) - \mathbf{j}(a_1 b_3) - \mathbf{k}(a_2 b_1)$$

$$a \times b = (a_2 b_3 - a_3 b_2)\hat{i} + (a_3 b_1 - a_1 b_3)\hat{j} + (a_1 b_2 - a_2 b_1)\hat{k}$$

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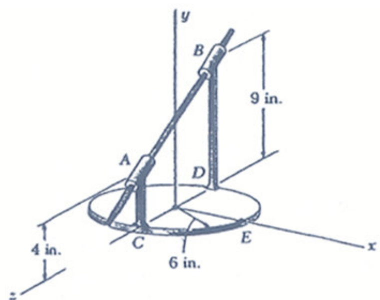
Panel 15

A circular plate of 6 in. radius is supported by two bearings A and B as shown. The plate rotates about the road joining A and B with a constant angular velocity of 26 rad/s. Knowing that, at the instant considered, the velocity of point C is directed in the positive x direction, determine :

(a) the velocity of point E,

(b) the acceleration of points E

(*taken from an unknown source*)



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Panel 16

A circular plate of 6 in. radius is supported by two bearings A and B as shown. The plate rotates about the road joining A and B with a constant angular velocity of 26 rad/s. Knowing that, at the instant considered, the velocity of point C is directed in the positive x direction, determine :

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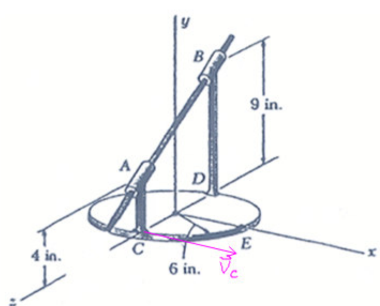

What is the general form equation for velocity?

On the figure draw the direction of omega

What's the magnitude of the omega vector?

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Panel 17

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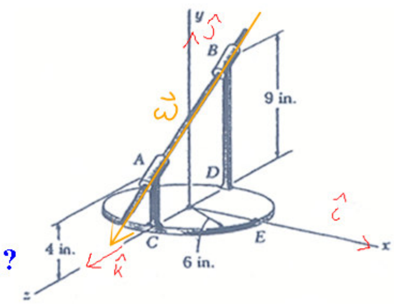

What's the unit vector in the direction of omega?

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Panel 18

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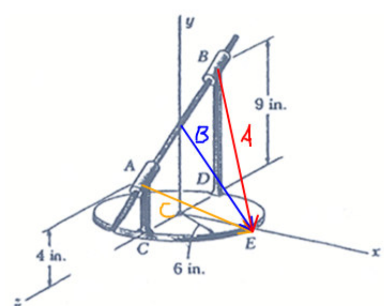
(*taken from an unknown source*)

$\vec{V}_e = \vec{\omega} \times \vec{r}_e$



Which vector is r_e?

Vote

A. It's A

B. It's B

C. It's C

D. Any of the above

E. None of the above

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Panel 19

15.20 A circular plate of 6 in. radius is supported by two bearings A and B as shown. The plate rotates about the road joining A and B with a constant angular velocity of 26 rad/s. Knowing that, at the instant considered, the velocity of point C is directed in the positive x direction, determine :
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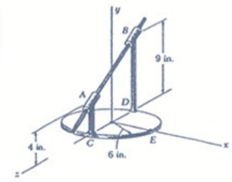

Strategy: Pure kinematics
Vectors will be helpful

Kinematics

$$\bar{v}_E = \bar{\omega} \times \bar{r}_{E/O} \qquad (1)$$

$$\bar{a}_E = \bar{\alpha} \times \bar{r}_{E/O} + \bar{\omega} \times (\bar{\omega} \times \bar{r}_{E/O}) \qquad (2)$$

We need to come up with ω and $\bar{r}_{E/O}$. Begin with the angular velocity - the plate is rotating about the axis defined by points A and B. From ConApps I (and hopefully Calc III), we can express the angular velocity vector as its magnitude and associated unit vector, hence

$$\omega = \omega \hat{e}_\omega = \omega \hat{e}_{A/B} = \omega \frac{\bar{r}_{A/B}}{|\bar{r}_{A/B}|} \qquad (3)$$

Note that the appropriate rotation sense has been included with the position vector $\bar{r}_{A/B}$ via the right-hand rule. Make sure you understand this!

Obtaining the coordinates of A and B and forming the position vector:

$$A = (0, 4, 6) \qquad B = (0, 9, -6)$$

$$\bar{r}_{A/B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j} + (A_z - B_z)\hat{k} = (0-0)\hat{i} + (4-9)\hat{j} + (6+6)\hat{k} = 0\hat{i} - 5\hat{j} + 12\hat{k}$$

$$|\bar{r}_{A/B}| = \sqrt{0^2 + 5^2 + 12^2} = 13$$

Substituting into (3)

$$\omega = 26\frac{0\hat{i} - 5\hat{j} + 12\hat{k}}{13} = 0\hat{i} - 10\hat{j} + 24\hat{k} \quad rad/s \qquad (4)$$

Now we need the position vector from the point of rotation to the point of interest E. Since the plate is rotating about an axis, we can go from any point on the axis to E. Choose point A

$$\bar{r}_{E/A} = (E_x - A_x)\hat{i} + (E_y - A_y)\hat{j} + (E_z - A_z)\hat{k} = 6\hat{i} - 4\hat{j} - 6\hat{k} \quad in \qquad (5)$$

Substituting (4) and (5) into (1) and (2) and letting Maple do our cross-products for us:

$$\bar{v}_E = \bar{\omega} \times \bar{r}_{E/A} = 156\hat{i} + 144\hat{j} + 60\hat{k} \quad in/s$$

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Panel 20

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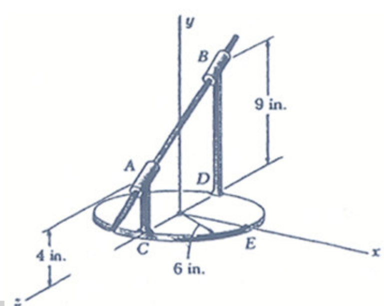

What the equation for acceleration?

Underline the vectors that are known?

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Panel 21

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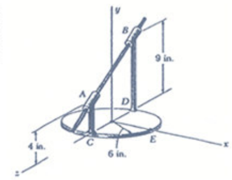

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$$\bar{v}_E = \bar{\omega} \times \bar{r}_{E/A} = 156\hat{i} + 144\hat{j} + 60\hat{k} \quad in/s$$

$$\bar{a}_E = \bar{\alpha} \times \bar{r}_{E/A} + \bar{\omega} \times (\bar{\omega} \times \bar{r}_{E/A}) = -4056\hat{i} + 3744\hat{j} + 15$$ 21 in/s^2