Panel 1

# ES204 Mechanical Systems

## Radial and Transverse Coordinates
## Lecture 05

Dr. Fisher

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Panel 2

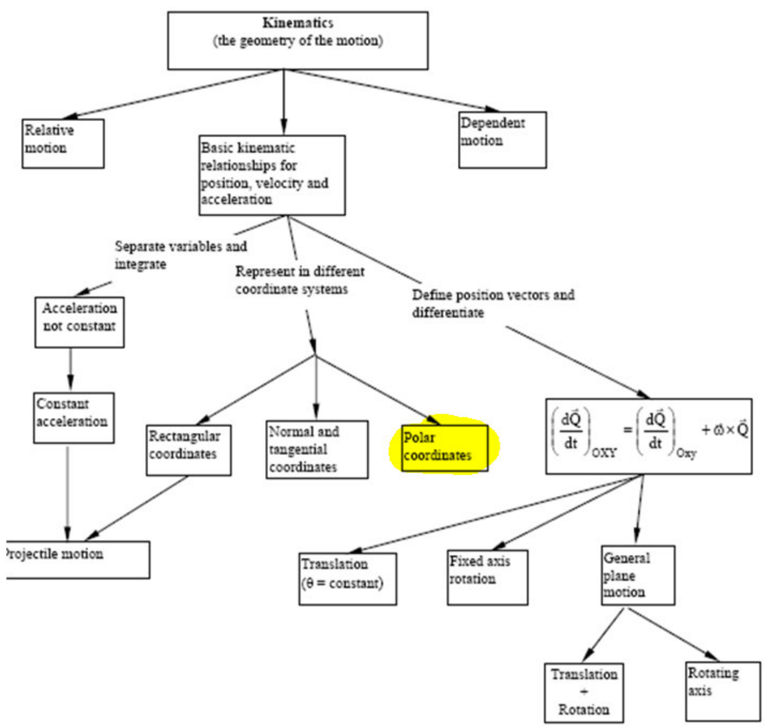


**Figure 1.5** Concept Map for Kinematics

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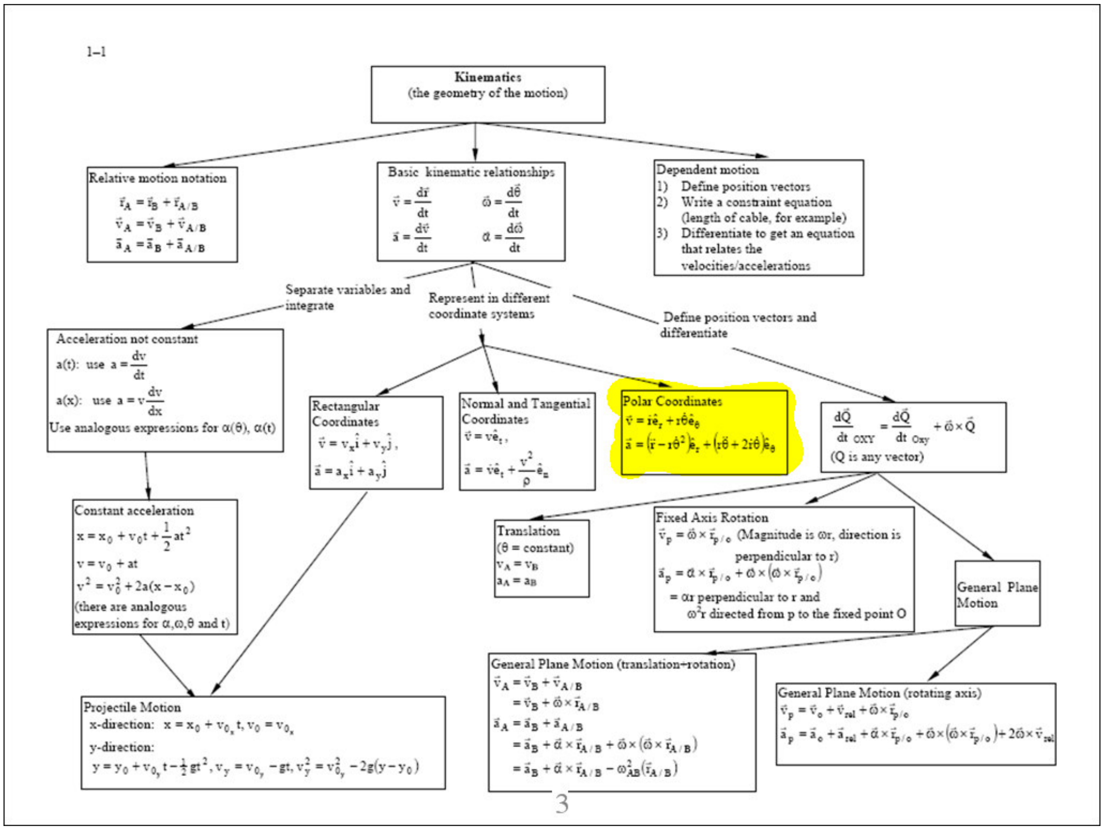
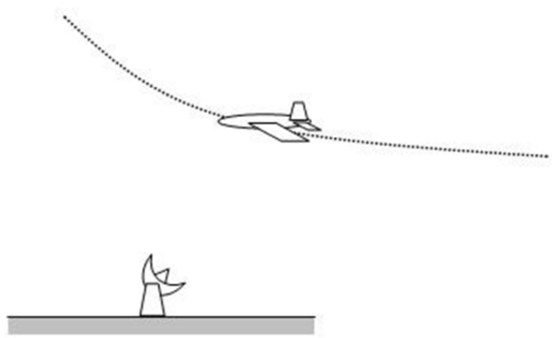
Panel 3




Panel 4

# Radial and Transverse



Particle position given by r, $\theta$

$\hat{e}_r$ = unit vector in the radial direction

$\hat{e}_\theta$ = unit vector in the transverse direction

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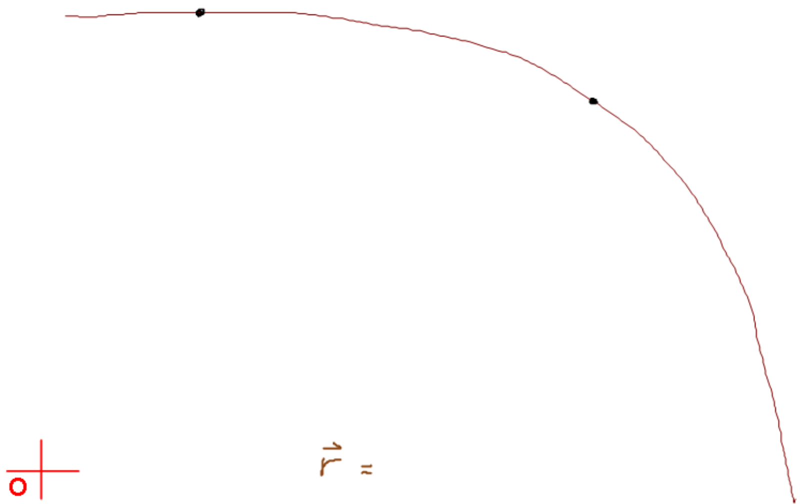
Panel 5

## Radial and Transverse Components

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Panel 6

## Radial -Transverse coordinates (cont.)

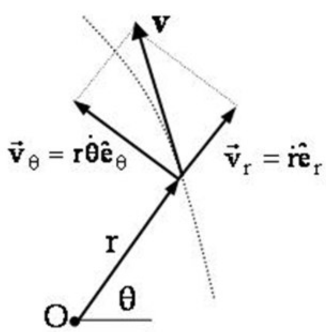
So we get $\vec{v} = \frac{dr}{dt}\hat{e}_r + r\frac{d\hat{e}_r}{dt} = \dot{r}\hat{e}_r + r\dot{\hat{e}}_r$

$$=$$

$$v_r = \dot{r}$$

$$v_\theta = r\dot{\theta}$$

$$|\vec{v}| = \sqrt{v_r^2 + v_\theta^2}$$

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Panel 7

# Radial -Transverse coordinates (cont.)

Acceleration: $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\dot{r}\hat{e}_r + r\dot{\hat{e}}_r)$

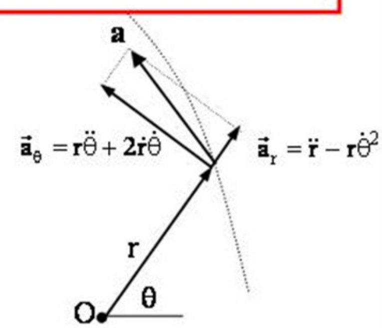
$$\ddot{r}\hat{e}_r + \dot{r}\dot{\hat{e}}_r \qquad \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\dot{\hat{e}}_\theta$$

$$\dot{r}\dot{\theta}\hat{e}_\theta \qquad -r\dot{\theta}^2\hat{e}_r$$

$a_r = \ddot{r} - r\dot{\theta}^2 =$ radial acceleration

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} =$ tangential acceleration

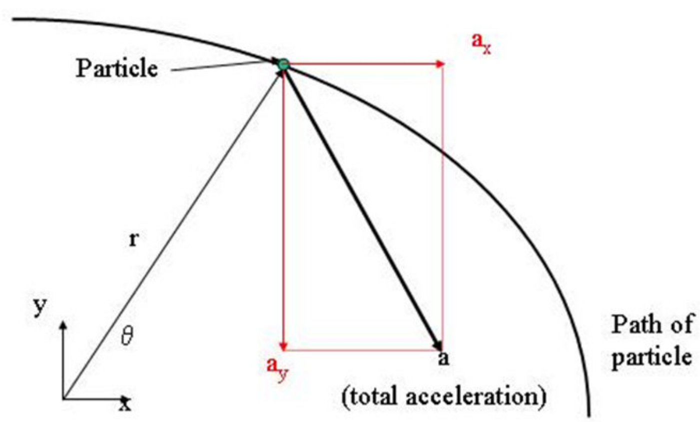
$|\vec{a}| = \sqrt{a_r^2 + a_\theta^2}$




**Note:** $a_r$ is not the time derivative of $v_r$

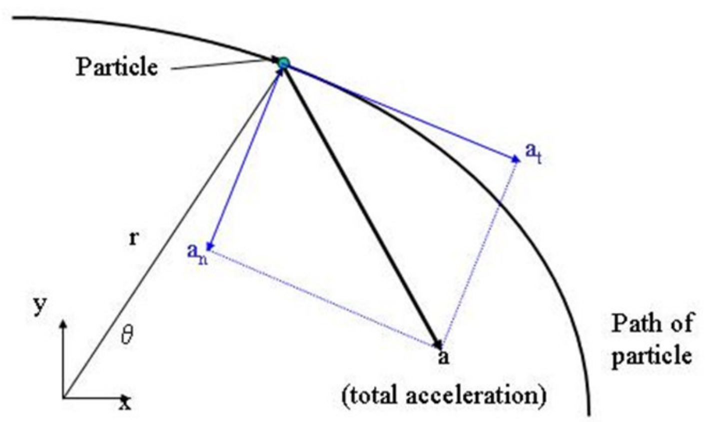
$a_\theta$ is not the time derivative of $v_\theta$

Panel 8




| Coordinate System | Position | Velocity | Acceleration |
|---|---|---|---|
| Cartesian | $\vec{r} = x\hat{i} + y\hat{j}$ | $\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j}$ | $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$ |
| Normal and Tangential | N/A | $\vec{v} = v\hat{e}_t$ | $\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$ |
| Radial and Transverse | $\vec{r} = r\hat{e}_r$ | $\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$ | $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta = a_r\hat{e}_r + a_\theta\hat{e}_\theta$ |

Panel 9




| Coordinate System | Position | Velocity | Acceleration |
|---|---|---|---|
| Cartesian | $\vec{r} = x\hat{i} + y\hat{j}$ | $\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j}$ | $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$ |
| Normal and Tangential | N/A | $\vec{v} = v\hat{e}_t$ | $\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$ |
| Radial and Transverse | $\vec{r} = r\hat{e}_r$ | $\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$ | $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta = a_r\hat{e}_r + a_\theta\hat{e}_\theta$ |

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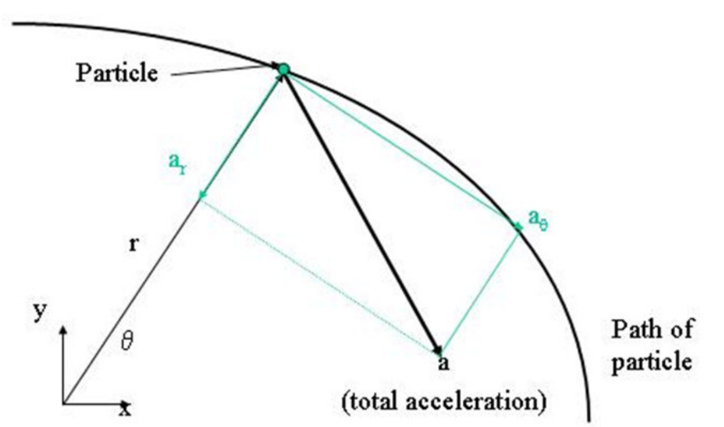
Panel 10




| Coordinate System | Position | Velocity | Acceleration |
|---|---|---|---|
| Cartesian | $\vec{r} = x\hat{i} + y\hat{j}$ | $\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j}$ | $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$ |
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| Radial and Transverse | $\vec{r} = r\hat{e}_r$ | $\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$ | $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta = a_r\hat{e}_r + a_\theta\hat{e}_\theta$ |

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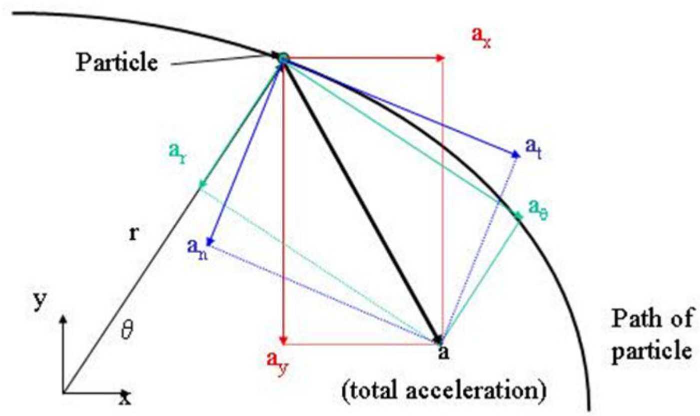
Panel 11




| Coordinate System | Position | Velocity | Acceleration |
|---|---|---|---|
| Cartesian | $\vec{r} = x\hat{i} + y\hat{j}$ | $\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j}$ | $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$ |
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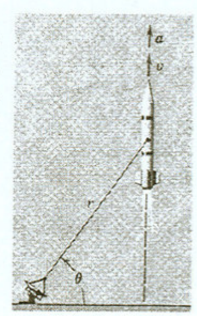
Panel 12

**2.137** The rocket is fired vertically and tracked by the radar shown. When $\theta = 60^\circ$, other corresponding measurements give the values of $r = 30{,}000\ ft$, $\ddot{r} = 70\ ft/s^2$, and $\dot{\theta} = 0.02\ rad/s$. Calculate the velocity and acceleration of the rocket at this position.

**Ans.** $v = 1200\ ft/s$

$a = 67.0\ ft/s^2$

(*taken from Dynamics, 3rd Edition by Merriam & Kraige*)



**Velocity**

$\vec{V} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$V_r = \dot{r}$

$V_r =$

$V_\theta = r\dot{\theta}$

$V_\theta =$

**Acceleration**

$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

$a_r = \ddot{r} - r\dot{\theta}^2$

$a_r =$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$a_\theta =$

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Panel 13

**2.137** The rocket is fired vertically and tracked by the radar shown. When $\theta = 60^\circ$, other corresponding measurements give the values of $r = 30{,}000\ ft$, $\ddot{r} = 70\ ft/s^2$, and $\dot{\theta} = 0.02\ rad/s$. Calculate the velocity and acceleration of the rocket at this position.

**Ans.** $v = 1200\ ft/s$

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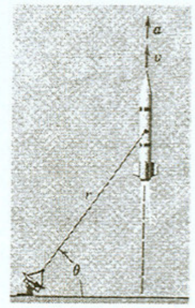

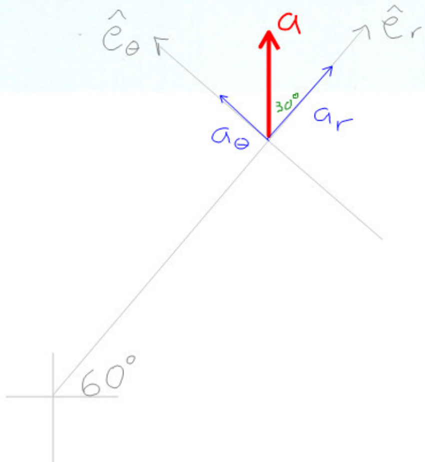


Use the formulas for velocity and acceleration to find either the radial or transverse components

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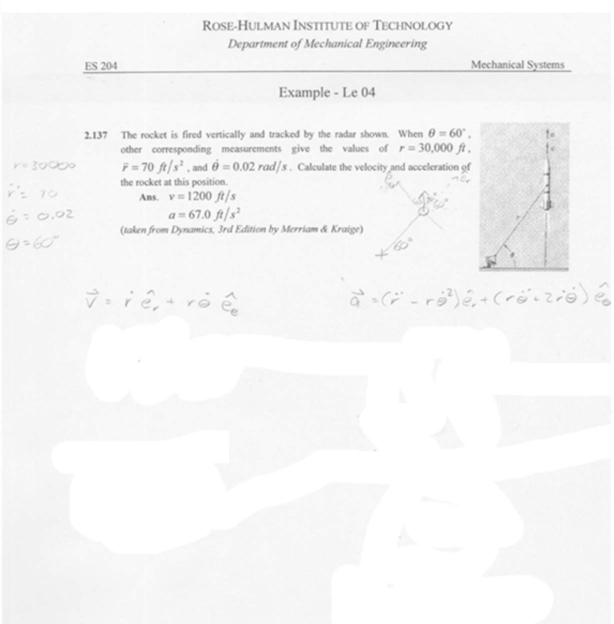
Panel 14



ROSE-HULMAN INSTITUTE OF TECHNOLOGY

*Department of Mechanical Engineering*

ES 204 Mechanical Systems

Example - Le 04

**2.137** The rocket is fired vertically and tracked by the radar shown. When $\theta = 60^\circ$, other corresponding measurements give the values of $r = 30{,}000\ ft$, $\ddot{r} = 70\ ft/s^2$, and $\dot{\theta} = 0.02\ rad/s$. Calculate the velocity and acceleration of the rocket at this position.

**Ans.** $v = 1200\ ft/s$

$a = 67.0\ ft/s^2$

(*taken from Dynamics, 3rd Edition by Merriam & Kraige*)

$r = 30000$

$\ddot{r} = 70$

$\dot{\theta} = 0.02$

$\theta = 60^\circ$

$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

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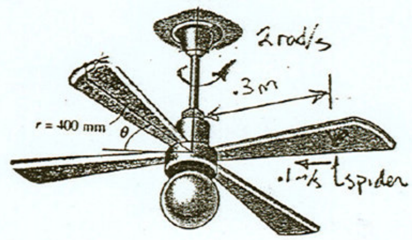
Panel 15

A spider is crawling towards the center of a rotating fan at a constant speed of .1 m/s relative to the fan. The fan is rotating at a constant 2 rad/s. The spider is a very rare jumping spider that can jump with a take-off speed of [illegible] 1.0 m/s when standing still.

a) At the instant shown what is the velocity and acceleration of the spider? (15 pts)

b) If the spider wants to jump onto the light in the center of the fan in what direction should the spider jump? Assume the spider leaves the fan with a horizontal velocity. (10 pts)




$r =$

$\dot{r} =$

$\ddot{r} =$

$\dot{\theta} =$

$\ddot{\theta} =$

**Velocity**

$\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$

m/s

**Acceleration**

$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{e}_\theta$

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Panel 16

A spider is crawling towards the center of a rotating fan at a constant speed of .1 m/s relative to the fan. The fan is rotating at a constant 2 rad/s. The spider is a very rare jumping spider that can jump with a take-off speed of [illegible] 1.0 m/s when standing still.

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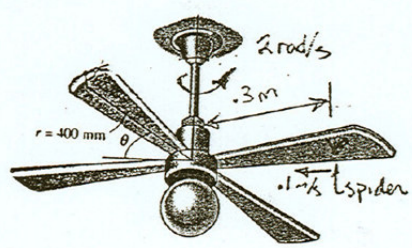


Solve for part b)

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Panel 17

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ES 204 Mechanical Systems

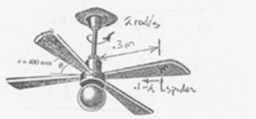
Example - Le 04

A spider is crawling towards the center of a rotating fan at a constant speed of .1 m/s relative to the fan. The fan is rotating at a constant 2 rad/s. The spider is a very rare jumping spider that can jump with a take-off speed of ~~6~~ 1.0 m/s when standing still.

a) At the instant shown what is the velocity and acceleration of the spider? (15 pts)

b) If the spider wants to jump onto the light in the center of the fan in what direction should the spider jump? Assume the spider leaves the fan with a horizontal velocity. (10 pts)

c) At the instant shown what is the radius of curvature of the spiders path? (5 pts)




Knowns

$\dot{\theta} = 2$ rad/s  $\ddot{\theta} = 0$  $\dot{r} = -0.1$ m/s  $r = 0.3$ m  $\ddot{r} = 0$

Velocity

$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$= (-0.1)\hat{e}_r + (0.3)(2)\hat{e}_\theta$

$\vec{v} = -0.1\hat{e}_r + 0.6\hat{e}_\theta$ m/s

Acceleration

$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

$= (0 - 0.3(2)^2)\hat{e}_r + ((0.3)(0) + 2(-0.1)(2))\hat{e}_\theta$

$\vec{a} = -1.2\hat{e}_r - 0.4\hat{e}_\theta$ m/s²

b) $V_{spider} = V_{fan} + V_{spider/fan}$

$V_{spider/fan} = 1$  $V_{fan} = V_\theta = 0.6$

$\sin\theta = \frac{0.6}{1}$

$\theta = 36.9°$

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Part A and B
Solution