Panel 1

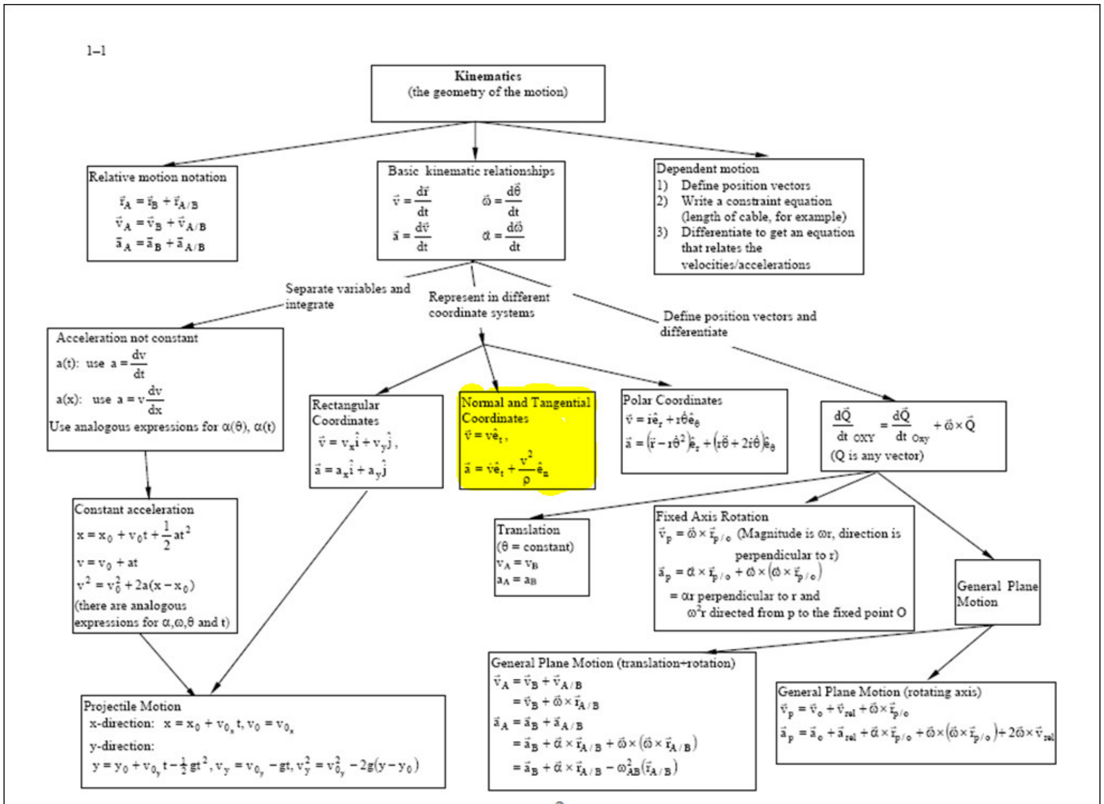
ES204 Mechanical Systems

Normal and Tangential Coordinates
Lecture 04

1

Dr. Fisher

Panel 2



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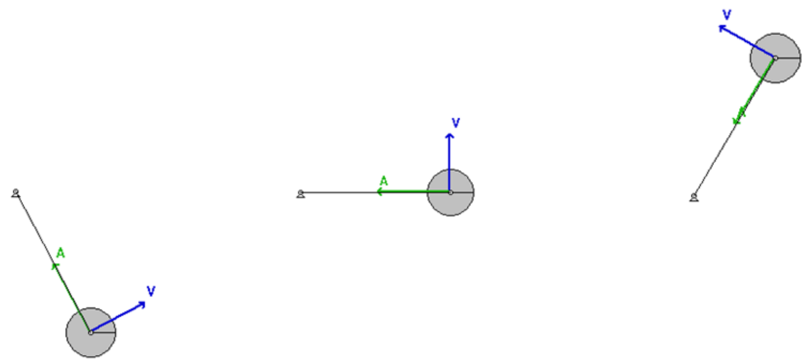
Panel 3

Some problems can be discribed easier using
normal and tangential coordinates



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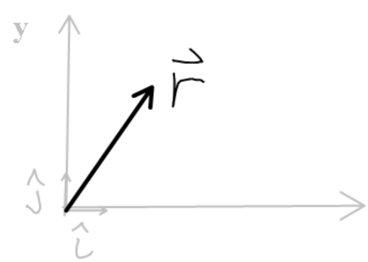
Panel 4

Different coordinate systems
Cartesian (rectangular) - review from ES201



Problems seen in ES201 - projectile motion, blocks on inclines, etc.

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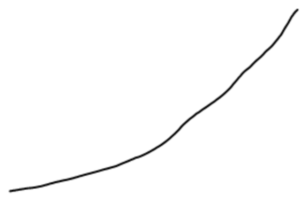
Panel 5

Normal - Tangential coordinates

$\hat{e}_t$ = **unit vector in tangential direction**



$\hat{e}_n$ = **unit vector in normal direction**

Velocity

← **velocity is in the tangential direction**

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Panel 6

Acceleration (proof in text)

$$\vec{a} = \dot{v}\,\hat{e}_t + v\,\dot{\hat{e}}_t$$

v = particle speed

ρ = radius of curvature

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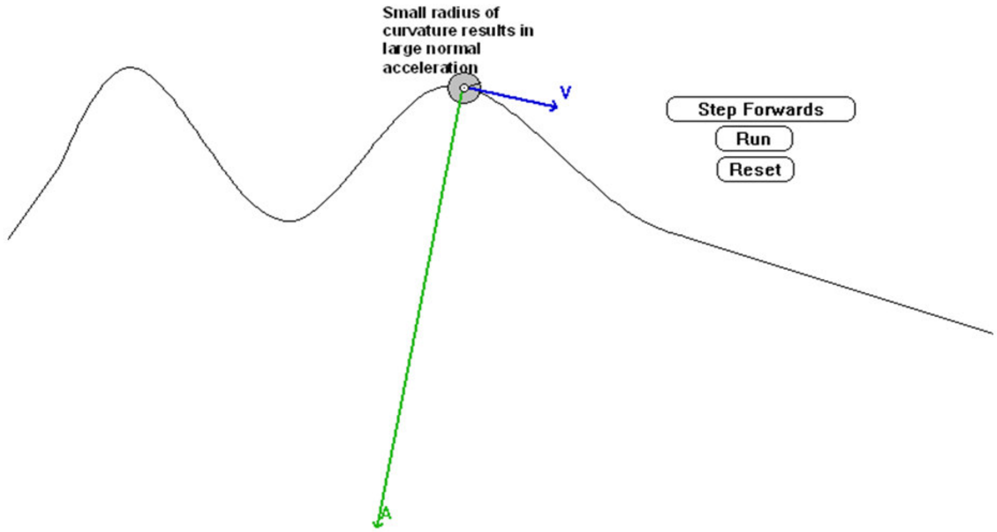
Panel 7

Concepts:

- The direction of the velocity is always tangent to the path
- The direction of the acceleration is not tangent to the path
    - there are normal and tangential components of acceleration
    - when the radius of curvature is small the normal acceleration is large
    - the tangential acceleration reflects a change in speed
        (at < 0 slowing down, at > 0 speeding up, notice when going up or down a hill)
    - at points of inflection the normal acceleration is zero

$$a_n = \frac{v^2}{\rho}, \quad a_t = \frac{dv}{dt}$$



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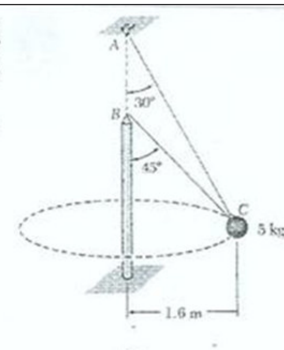
Panel 8

12.39 Two wires AC and BC are tied at C to a sphere which revolves at a constant speed v in the horizontal circle shown. Determine the range of the allowable values of v if the tension in either of the wires is not to exceed 40 N.
(*taken from Vector Mechanics for Engineers, 5th Edition by Beer & Johnston*)



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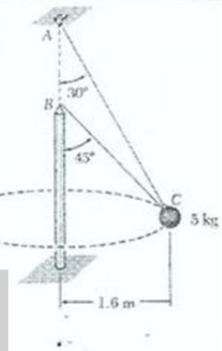
Panel 9

12.39 Two wires AC and BC are tied at C to a sphere which revolves at a constant speed *v* in the horizontal circle shown. Determine the range of the allowable values of *v* if the tension in either of the wires is not to exceed 40 N.
(*taken from Vector Mechanics for Engineers, 5th Edition by Beer & Johnston*)



FBD **KD**

FBD **KD**

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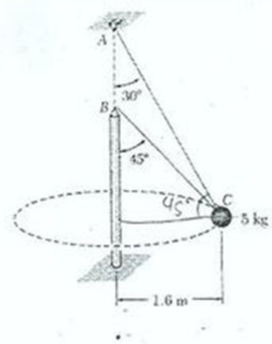
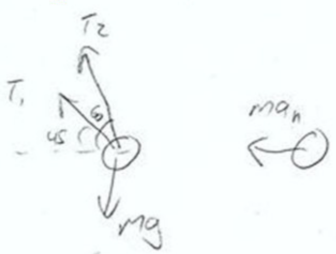
Panel 10

12.39 Two wires AC and BC are tied at C to a sphere which revolves at a constant speed *v* in the horizontal circle shown. Determine the range of the allowable values of *v* if the tension in either of the wires is not to exceed 40 N.
(*taken from Vector Mechanics for Engineers, 5th Edition by Beer & Johnston*)



Rate of change of Linear momentum equations



X-dir

Y-dir

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Panel 11

Solve for T1 and T2 in terms of v

solution below

Greater velocity will make the tension in the lower cable greater.

- Think about it, if was going really fast, the lower cable would be horizontal and the top cable would go slack. So it can only go so fast.

Lower velocity will make the tension in the upper cable great.

- Since ball is 5 kg if the ball was stopped the lower cable would go slack and the top cable would hold all 5 kg's (nearly 50 N). So it must go at least fast enough to force the lower cable to take some weight.

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Panel 12

$$T_1 \cos 45^\circ + T_2 \cos 60^\circ = m a_n = m \frac{v^2}{1.6}$$

$$T_1 \sin 45^\circ + T_2 \sin 60^\circ - mg = 0$$

$$T_1 = -94.76 + 10.46\, v^2$$

$$T_2 = 134 - 8.54\, v^2$$

$T_1 = 40$ $v = 3.59$ m/s Above that v break

$T_2 = 40$ $v = 3.32$ m/s Below that v break

$$3.32 \text{ m/s} < v < 3.59 \text{ m/s}$$

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Panel 13

Lecture 1-5 Two rope example problem

```
> restart;
> m:=5:
  g:=9.81:
> r:=1.6:
> eq1:=tb*cos(45/180*Pi)+ta*cos(60/180*Pi)=m*v^2/r;
```

$$eq1 := \frac{tb\sqrt{2}}{2} + \frac{ta}{2} = 3.125000000\, v^2 \qquad (1)$$

```
> eq2:=tb*sin(45/180*Pi)+ta*sin(60/180*Pi)-m*g=0;
```

$$eq2 := \frac{tb\sqrt{2}}{2} + \frac{ta\sqrt{3}}{2} - 49.05 = 0 \qquad (2)$$

```
> q:=solve({eq1,eq2},{ta,tb});
```

$$q := \{ta = 134.0070921 - 8.537658774\, v^2,\ tb = -94.75732356 + 10.45645380\, v^2\} \qquad (3)$$

```
> assign(q);
> plot([ta,tb,40],v=0..6,0..140,color=[red,green,black]);
```

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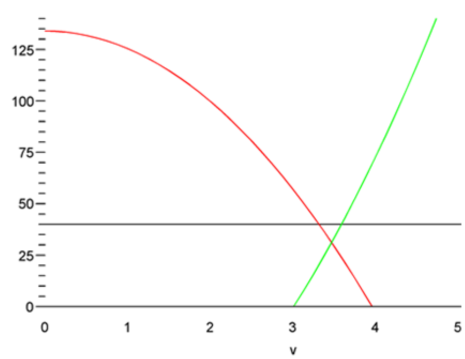
Panel 14




```
> eq3:=ta=40:
> eq4:=tb=40:
> ta_lim:=fsolve(eq3,v,v=0..6);
```

$$ta\_lim := 3.318263828 \qquad (4)$$

```
> tb_lim:=fsolve(eq4,v,v=0..6);
```

$$tb\_lim := 3.589913440 \qquad (5)$$

```
>
```

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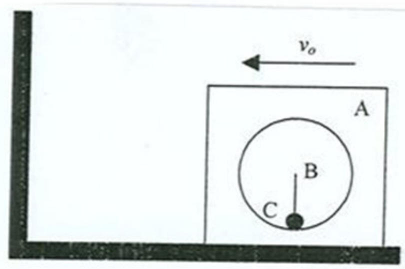
Panel 15

Given: Sphere C and Block A are both moving to the left with a velocity of v_o when the block is suddenly stopped by the wall. Point B is attached to block A.

Find: The smallest velocity for which the sphere will swing in a full circle about the pivot B if
a) BC is a slender rod of negligible mass
b) BC is a cord



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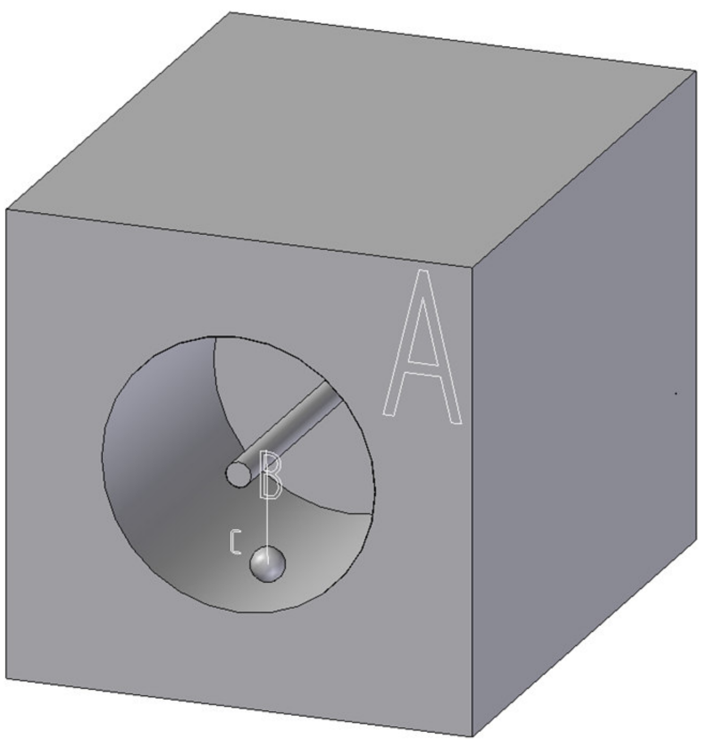
Panel 16



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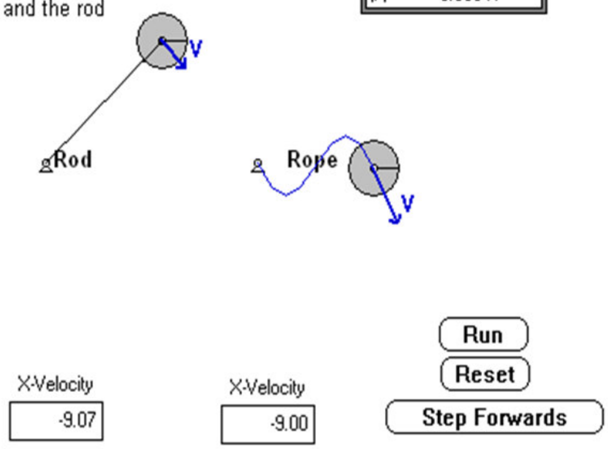
Panel 17

Concepts

Conservation of energy
Different criteria for the rope and the rod

Tension of Rope
|F| 0.000 N



Determine the minimum velocity for the ball to complete a complete circle in the two cases shown

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Panel 18

How do we want to approach this problem?

What is the difference between the rope and the rod?

Rope

Rod

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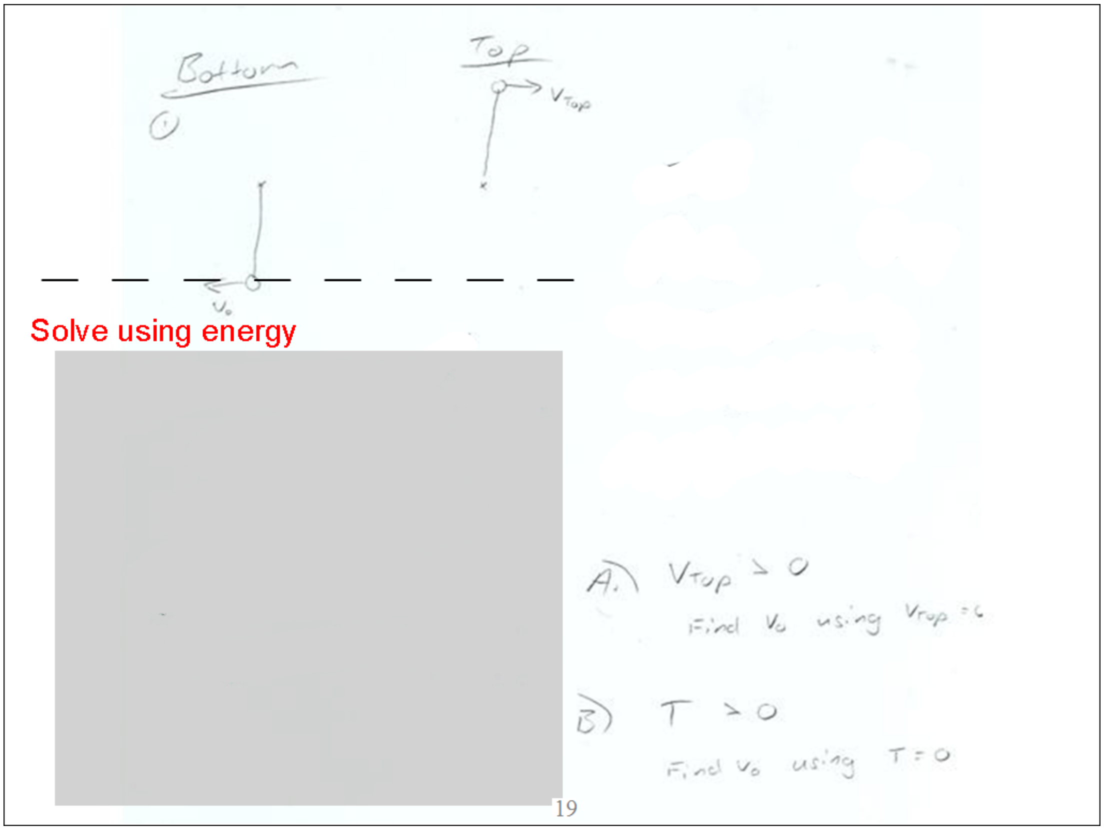
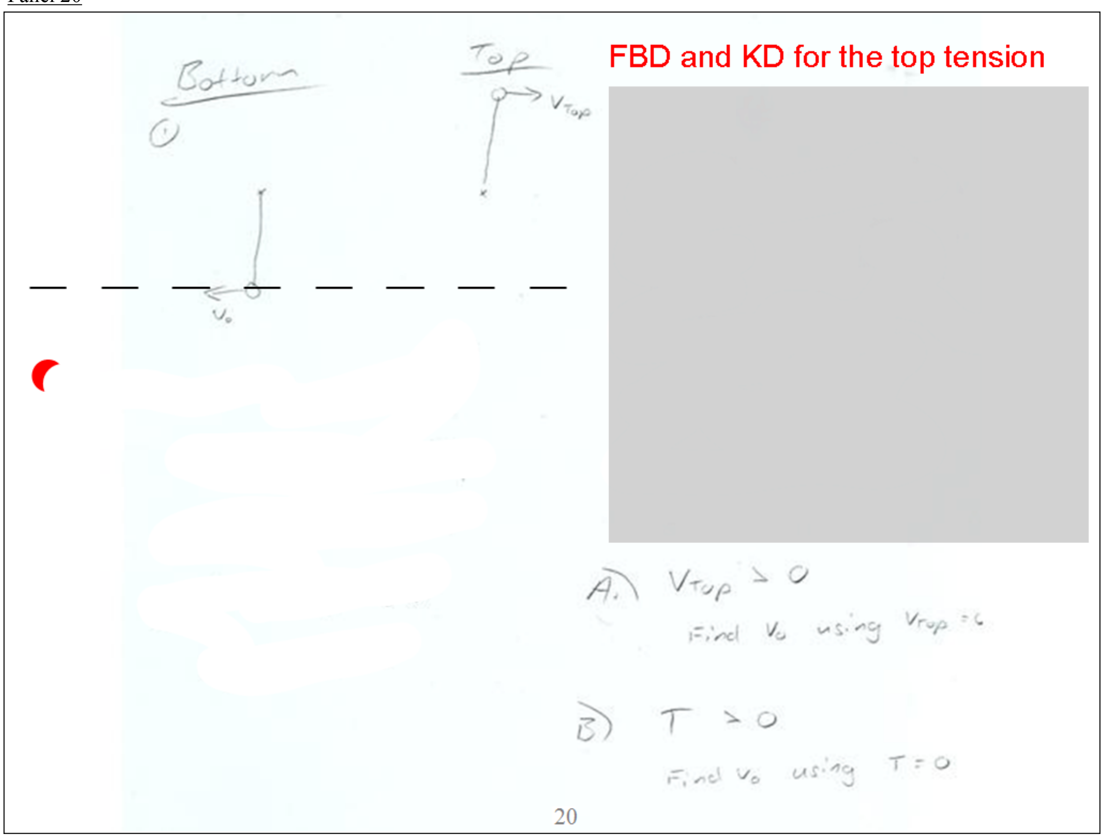
Panel 19



Panel 20



Panel 21

Solve for Vo

A.) $V_{top} > 0$

$$4g\ell + V_{Top}^2 = V_0^2$$

B.) $T > 0$:

$$T^{\to 0} + mg = m\frac{v^2}{\ell}$$

$$4g\ell + V_{top}^2 = V_0^2$$

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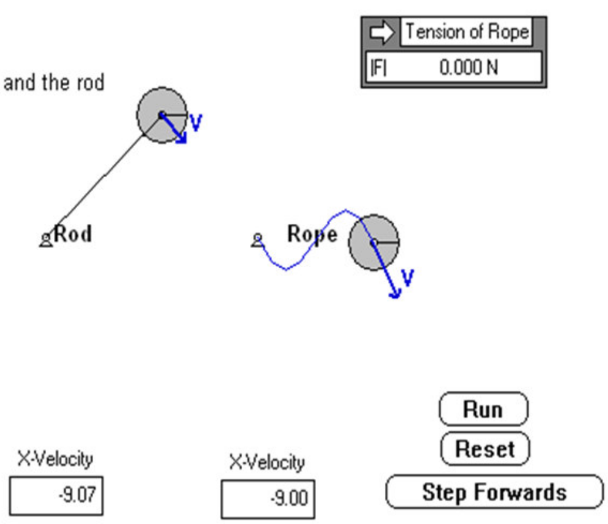
Panel 22

Concepts

Conservation of energy

Different criteria for the rope and the rod



Determine the minimum velocity for the ball to complete a complete circle in the two cases shown

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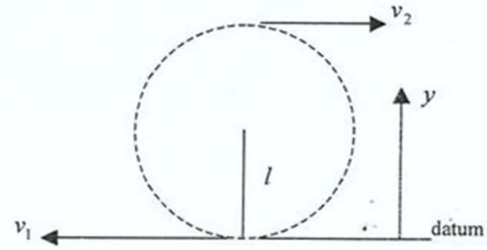
Panel 23

From the discussion in class, we found that for a rod and a rope swinging in a vertical circle, the criteria for each to make a full revolution was that for the rod the velocity at the top had to be just slightly greater than zero while for the rope the tension at the top had to be just slightly greater than zero. Let's see if we can use kinetics to compare the required velocities at the bottom of the circle:



Use COE(FT) to relate an object at two different times and locations

$$\Delta E_{sys} = W = 0 \quad \Rightarrow \quad E_{K2} + E_{G2} = E_{K1} + E_{G1}$$

$$\frac{1}{2}mv_1^2 + 0 = \frac{1}{2}mv_2^2 + mg(2l)$$

Solving for v_1

$$v_1 = \sqrt{v_2^2 + 4gl} \qquad (1)$$

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Panel 24

For the slender rod, the velocity at the top, v_2, must be just slightly greater than zero, thus substituting v_2=0 into (1) will give us the velocity that v_1 must be just slightly greater than:

$$\boxed{v_{1,rod} = \sqrt{4gl}}$$

For the rope, the tension at the top must be just slightly greater than zero, thus we need to use COLM(RF) and kinetics to relate velocity to tension:

$$\sum F_n = ma_n \quad \Rightarrow \quad mg = ma_n = m\frac{v_2^2}{\rho} \quad \therefore \quad v_2^2 = g\rho$$

Substituting into (1) and recognizing the radius of curvature to be l,

$$\boxed{v_{1,rope} = \sqrt{5gl}}$$

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