Panel 1

ES204 Mechanical Systems

Dependent Motion
Lecture 03

Dr. Fisher

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Panel 2

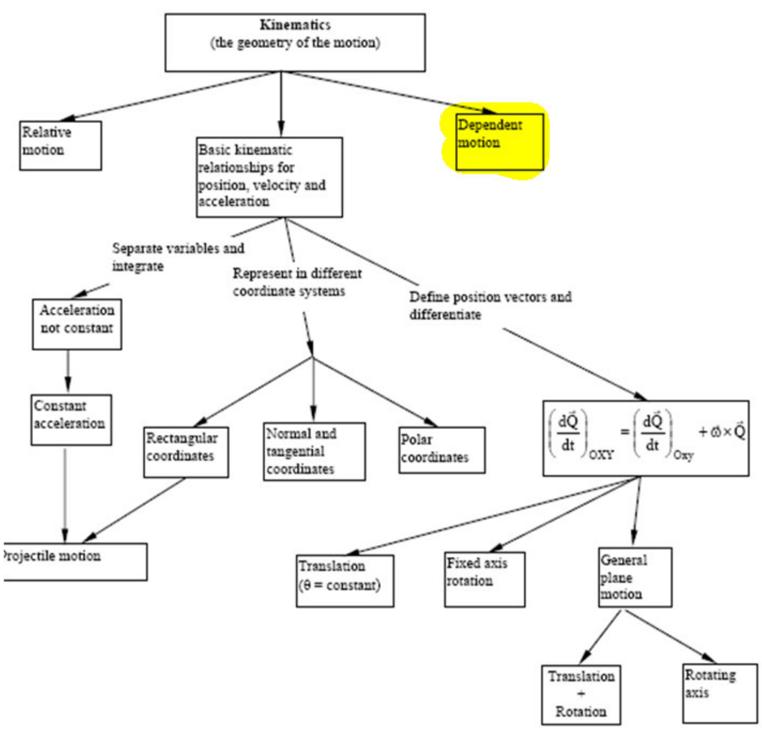

Figure 1.5 Concept Map for Kinematics

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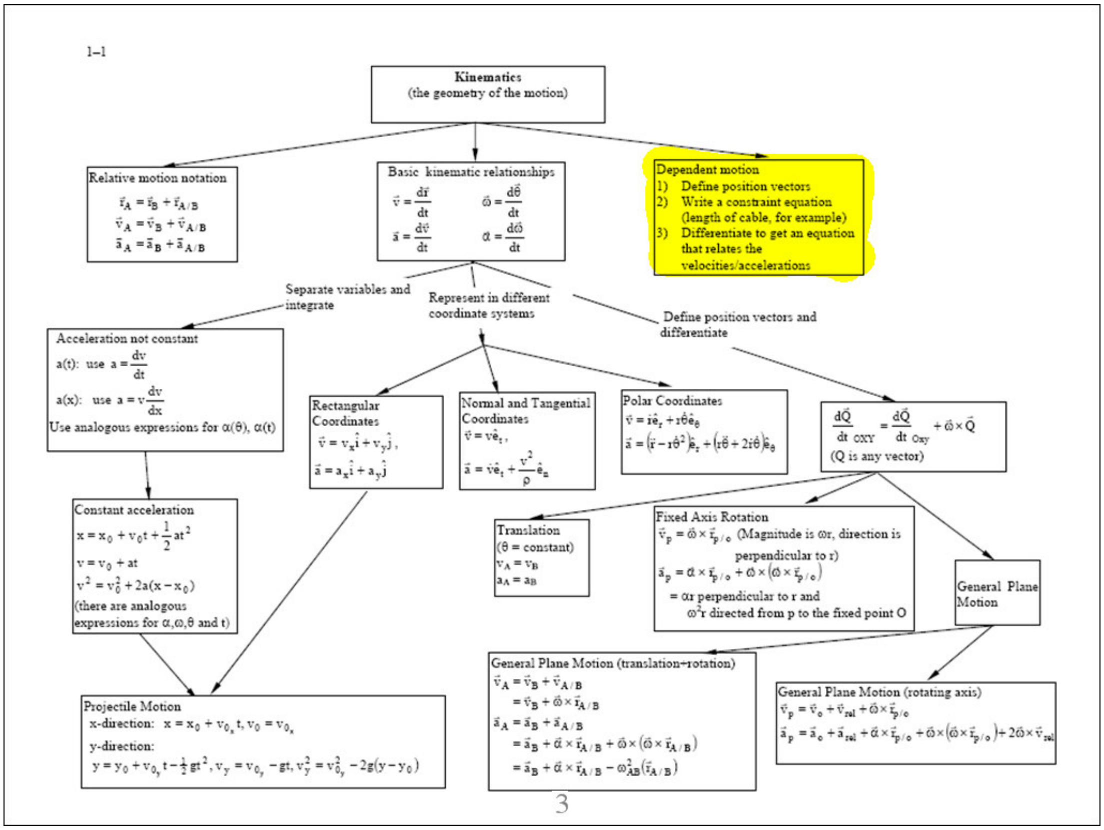
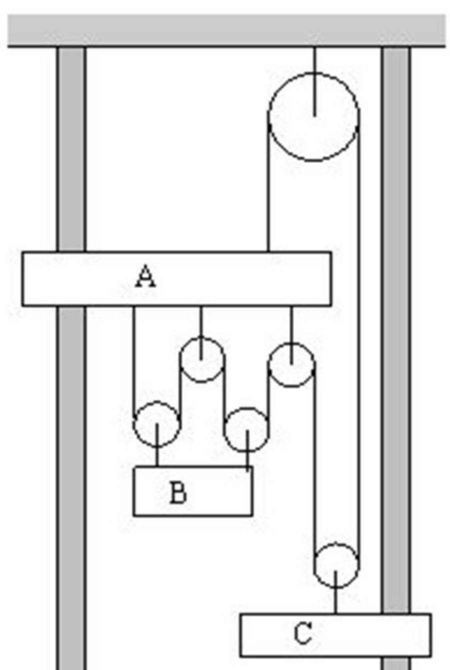
Panel 3



Panel 4

What is dependent motion?



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Panel 5

Sometimes the motion of two or more objects are related because of a constraint. To relate the motion of the objects a constraint equation needs to be determined. A common example of dependent motion is when objects are connected by an inextensible cable.

The following procedure can be very helpful when approaching problems of this type:

1. Set up a datum for each position vector and draw the position vectors
2. Determine a constraint equation such as the length of cable connecting the objects.
3. Differentiate the constraint equation to determine velocity and acceleration constraint equations.

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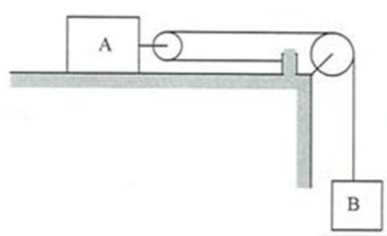
Panel 6

Example 1

Known: Two objects are connected by a cable
Find: Determine a relationship between the acceleration of the two objects
Given: See picture
Analysis:
Strategy: Determine a constraint equation

Step 1: Define datums and position vectors as shown.



Step 2: Write constraint equation: L =

Step 3: Differentiate:

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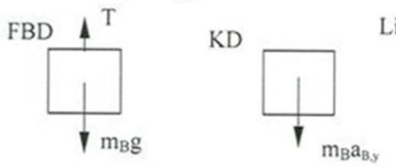
Panel 7

What if you dependent motion problem involves kinetics?

1) Note that in these equations the positive direction for a_A and a_B are to the right and down respectively. This is important if a constraint equation is used in conjunction with conservation of linear momentum. For example if we applied conservation of linear momentum to block B we would get:



Linear momentum in the y-direction

$$+\downarrow \Sigma F_y = ma_y$$

$$m_Bg - T = m_Ba_{B,y}$$

If, in the linear momentum equation, up had been defined to be positive then the equation obtained would be inconsistent with the constraint relationship derived.

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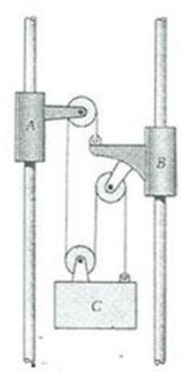
Panel 8

Start by implementing the 3 steps for Dependent motion problems...

11.48 Collar A starts from rest at t=0 and moves upward with a constant acceleration of 3.6 in/s^2. Knowing that collar B moves downward with a constant velocity of 16 in/s, determine:

(a) the time at which the velocity of block C is zero,
(b) the corresponding position of block C.

(*taken from Vector Mechanics for Engineers, 5th Edition by Beer & Johnston*)



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Panel 9

11.48 Collar A starts from rest at $t=0$ and moves upward with a constant acceleration of 3.[illegible] in/s². Knowing that collar B moves downward with a constant velocity of 16 in/[illegible] determine:

(a) the time at which the velocity of block C is zero,

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Panel 10

Find:

Givens:

	Velocity	Acceleration
Block A		
Block B		
Block C		

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Panel 11

What can we know about the acceleration and velocity of Block C?

Velocity of C

$0 = -2V_a - V_B + 3V_c$

Acceleration of C

$0 = -2a_a - a_B + 3a_c$

either works

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Panel 12

What do we know about acceleration and velocity?

Velocity of C

$0 = -2V_a - V_B + 3V_c$

Acceleration of C

$0 = -2a_a - a_B + 3a_c$

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Panel 13

Once you've got an equation for velocity the rest is gravy. You need to solve for time using V=0, then enter that time into a position equation,

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Panel 14

Dependent Motion:

Based on the above selection of datum and position vectors, make sure you can come up with

$$L = (y_B - y_A) + (y_C - y_A) + 2(y_C - y_B) + Const$$
$$= -2y_A - y_B + 3y_C + Const$$

Differentiating:

$$0 = -2v_A - v_B + 3v_C \quad (1)$$
$$0 = -2a_A - a_B + 3a_C \quad (2)$$

We are interested in block C, what can we find out? How about acceleration from (2)

$$a_C = -2.4 \ in/s^2 \quad (3)$$

Knowing acceleration, we can get velocity

$$a_C = \frac{dv_C}{dt} \Rightarrow a_C dt = dv_C \Rightarrow \int_0^t -2.4dt = \int_{v_{Co}}^{v_C} dv_C \Rightarrow -2.4t = v_C - v_{Co} \quad (4)$$

where v_{Co} is the initial velocity of C. We can get the initial velocity of C from equation (1) as v_{Co}=5.33 in/s, thus we can solve (4) for the time when v_C=0 to be

$$\boxed{t_{v_C=0} = 2.22 \ s}$$

I'll leave it to you to use the kinematic relationship between position and velocity along with (4) to come up with the distance traveled from C's original position to be

$$\boxed{x = 5.93 \ in}$$

what is the direction?

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